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Nonlinear static characteristics of piezoelectric unimorph bending micro actuators

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Abstract: The nonlinear static characteristic of a piezoelectric unimorph cantilever micro actuator driven by a strong applied electric field is studied based on the couple stress theory. The cantilever actuator consists of a piezoelectric layer, a passive (elastic) layer and two electrode layers. First, the nonlinear static characteristic of the actuator caused by the electrostriction of the piezoelectric layer under a strong applied electric field is analyzed using the Rayleigh-Ritz method. Secondly, since the thickness of the cantilever beam is in micro scale and there exists a size effect, the size dependence of the deformation behavior is evaluated using the couple stress theory. The results show that the nonlinearities of the beam deflection increase along with the increase of the applied electric field which means that softening of the micro beam rigidity exists when a strong external electric field is applied. Meanwhile, the optimal value of the thickness ratio for the passive layer and the piezoelectric layer is not around 1.0 which is usually adopted by some previous researchers. Since there exists a size effect of the micro beam deflection, the optimal value of this thickness ratio should be greater than 1.0 in micro scale.

Key words: nonlinear static characteristic; piezoelectric unimorph micro actuator; couple stress theory; Rayleigh-Ritz method

Piezoactuation is one of the most common mechanisms for actuation and sensing in micro electromechanical systems devices. A piezoelectric thin film can be deposited on an elastic membrane, and by applying an electric field to the film, the resulting strain (due to the direct piezoelectric effect) may cause the membrane to bend. The piezoelectric bending actuator is one of the most common applications of the piezoelectric material, and has been widely used in fields such as precision position control, noise control, acoustic and pressure sensing, etc[1−2].

However in practical applications, a strong electric field is often applied to the piezoelectric bending actuators to achieve sufficient large displacement or force, and the performance of the actuators shows nonlinearities under strong applied electric fields[3−7]. To optimize the actuator performance, detailed analysis of the actuator bending mechanism, especially under a strong electric field, is necessary.

Also, the size dependence of the deformation behavior of a structure in micro scale has been experimentally observed in metals and polymers [8−11]. The behavior cannot be explained by the conventional theories of mechanics, and the couple stress theory [12−13] are used to explain the size effect of the deformation behaviors. Recently along with the development of the micro electro mechanical system (MEMS), the strain gradient theory has been developed greatly[14−16]. Since the thickness of the piezoelectric bending actuator considered in this paper is in micro scale, the size dependence of its deformation behavior should be involved, and till now no publications have considered the size effect in the research on piezoelectric micro actuators.

This paper aims at the precise static deformation behaviors of a piezoelectric unimorph bending micro actuator which is under strong applied electric fields. The nonlinearities of the deformation of the piezoelectric micro actuator together with the size effect of the deformation behavior are both considered simultaneously. The simulation results are obtained and discussed, and the results may provide a valuable reference for the design of piezoelectric bending micro actuators.

1 Theoretical Analysis

Fig. 1 shows the configuration of a piezoelectric unimorph cantilever micro actuator. The cantilever beam consists of a piezoelectric material layer, a passive (elastic) layer and two metal electrodes. Usually the thickness of the electrode layers is far smaller than that of the piezoelectric layer or the passive layer. t_i is the thickness of the i-th layer ($i = 1, 2, 3, 4$) and it is shown in Fig. 2. The x-y plane is taken to be the neutral plane of the beam and the z-axis is taken to be positive upward from the neutral plane. The i-th layer is located between the planes $z = z_{i-1}$ and $z = z_i$ in the thickness direction, which is also shown in Fig. 2. Assume that the layers are perfectly bonded together, and the piezoelectric layer is polarized with respect to the z-axis. When the beam is driven by externally electric field E_3 (E_z) applied along the thickness (polar axis) direction ($E_x = E_y = 0$), a bending deformation is induced due to the constraining effect of each layer.

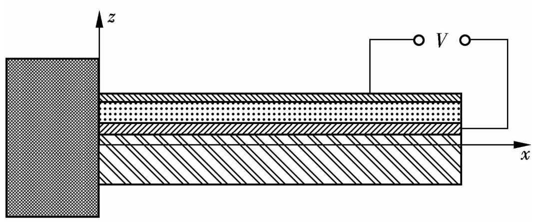

Fig. 1 Piezoelectric multilayer cantilever beam

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First, we need to determine the position of the neutral plane of the multilayer beam. According to Fig. 2, the coordinate of the neutral plane along the z-axis can be written as[2]

$$z_0 = -\frac{1}{2}\frac{\sum_{n=1}(h_n^2 - h_{n-1}^2)/\mathrm{se}(n)}{\sum_{n=1} t_n/\mathrm{se}(n)}$$

where $h_n = \sum_{i=1}^{n} t_i$ and $\mathrm{se}(n)$ is the relevant compliance term of the n-th later. Assume that the mechanical property is isotropic, then $\mathrm{se}(n)$ can be considered as $\mathrm{se}(n) = s_{11,n}^E$. Here $s_{11,n}^E$ is the compliance coefficient of the n-th layer under a constant electric field E.

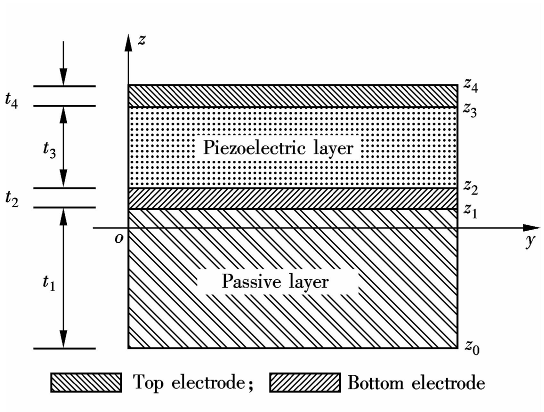

Fig. 2 Cross-section of the cantilever beam

1.1 Couple stress theory

The mechanical properties of micro structures are different from those of macro structures. The deformation behavior of structures in micro scale is dependent on the size of the structures, and this has been observed by a number of experiments [8–11]. The size-dependent behavior cannot be explained by the conventional theories of mechanics, and the couple stress theory is used to explain the size dependence of the deformation behavior. In this paper the thickness of the transducer evaluated is in several microns; hence, the size effect must be considered.

In the couple stress theory, a couple stress tensor is involved excepting the Cauchy stress tensor. The force and couple vector per unit area transmitted through the surface of a continuum can be written as

$$\boldsymbol{F} = \boldsymbol{n}\cdot\boldsymbol{t}, \qquad \boldsymbol{M} = \boldsymbol{n}\cdot\boldsymbol{m} \tag{1}$$

where $\boldsymbol{n}$ denotes the external normal vector; $\boldsymbol{t}$ and $\boldsymbol{m}$ are the Cauchy stress tensor and the couple stress tensor. When the couple stress tensor is involved in the equilibrium, the Cauchy stress tensor is no longer symmetric, and it can be decomposed as

$$\boldsymbol{t} = \boldsymbol{\sigma} + \boldsymbol{\tau} \tag{2}$$

where $\boldsymbol{\sigma} = (\boldsymbol{t} + \boldsymbol{t}^{\mathrm{T}})/2$ and $\boldsymbol{\tau} = (\boldsymbol{t} - \boldsymbol{t}^{\mathrm{T}})/2$ are the symmetric and antisymmetric parts of the Cauchy stress tensor, respectively. When body force and body couples are not considered, the equilibrium equations of the deformable body are

$$\nabla\cdot\boldsymbol{t} = \boldsymbol{0} \tag{3a}$$

$$\nabla\cdot\boldsymbol{m} - \boldsymbol{e}:\boldsymbol{t} = \boldsymbol{0} \tag{3b}$$

Equilibrium equation (3b) indicates that the stress tensor $\boldsymbol{t}$ generates an equivalent body couple $-\boldsymbol{e}:\boldsymbol{t}$ to maintain the equilibrium of the continuum. The increment of displacement can be written as

$$\mathrm{d}\boldsymbol{u} = \boldsymbol{u}\nabla\cdot\mathrm{d}\boldsymbol{x} \tag{4}$$

where $\boldsymbol{x}$ is the position vector of a material in the continuum, and $\boldsymbol{u}\nabla$ is the right gradient of the displacement vector. We can decompose $\boldsymbol{u}\nabla$ as the symmetric part $\boldsymbol{\varepsilon}$ and the antisymmetric part $\boldsymbol{\omega}$,

$$\boldsymbol{u}\nabla = \boldsymbol{\varepsilon} + \boldsymbol{\omega} \tag{5}$$

where $\boldsymbol{\varepsilon} = (\boldsymbol{u}\nabla + \nabla\boldsymbol{u})/2$ and $\boldsymbol{\omega} = (\boldsymbol{u}\nabla - \nabla\boldsymbol{u})/2$. Define the rotation vector as

$$\boldsymbol{\theta} = -\boldsymbol{e}:\frac{\boldsymbol{\omega}}{2} \tag{6}$$

Then the rotation gradient $\boldsymbol{\chi} = \boldsymbol{\theta}\nabla$ can be rewritten as

$$\boldsymbol{\chi} = -\boldsymbol{e}:\frac{\boldsymbol{u}\nabla - \nabla\boldsymbol{u}}{2} = -(\boldsymbol{u}\times\nabla)\frac{\nabla}{2} \tag{7}$$
$$\chi_{ij} = \frac{e_{its}u_{s,t,j}}{2} = e_{its}\varepsilon_{j,s,t}$$

Hence $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$ are two variables which can be used to describe the deformation of the continuum. $\boldsymbol{\chi}$ is a non-symmetric tensor, and its trace χ_{kk} is equal to zero. The work done by the external force and couples is

$$\begin{aligned}
&\int_S (\boldsymbol{F}\cdot\boldsymbol{u} + \boldsymbol{M\theta})\,\mathrm{d}s = \\
&\quad\int_S (\boldsymbol{n}\cdot\boldsymbol{t}\cdot\boldsymbol{u} + \boldsymbol{n}\cdot\boldsymbol{m}\cdot\boldsymbol{\theta})\,\mathrm{d}s = \\
&\quad\int_V \nabla\cdot(\boldsymbol{t}\cdot\boldsymbol{u} + \boldsymbol{m}\cdot\boldsymbol{\theta})\,\mathrm{d}v = \\
&\quad\int_V (\boldsymbol{t}:\nabla\boldsymbol{u} + \nabla\cdot\boldsymbol{t}\cdot\boldsymbol{u} + \boldsymbol{m}:\nabla\boldsymbol{\theta} + \nabla\cdot\boldsymbol{m}\cdot\boldsymbol{\theta})\,\mathrm{d}v
\end{aligned}$$

Substituting $\nabla\cdot\boldsymbol{t} = \boldsymbol{0}$ and $\nabla\cdot\boldsymbol{m}\cdot\boldsymbol{\theta} = -\boldsymbol{\tau}:\nabla\boldsymbol{u}$ into the above equation yields

$$\begin{aligned}
&\int_S (\boldsymbol{F}\cdot\boldsymbol{u} + \boldsymbol{M}\cdot\boldsymbol{\theta})\,\mathrm{d}s = \\
&\quad\int_V (\boldsymbol{t}:\nabla\boldsymbol{u} - \boldsymbol{\tau}:\nabla\boldsymbol{u} + \boldsymbol{m}:\boldsymbol{\theta})\,\mathrm{d}v = \\
&\quad\int_V (\boldsymbol{\sigma}:\boldsymbol{\varepsilon} + \boldsymbol{m}^{\mathrm{T}}:\boldsymbol{\chi})\,\mathrm{d}v
\end{aligned} \tag{8}$$

It can be seen from Eq. (8) that $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$ are conjugated to σ and $\boldsymbol{m}^{\mathrm{T}}$, respectively. Denote that w is the density of the strain energy, thus we have

$$\sigma = \frac{\partial w}{\partial\boldsymbol{\varepsilon}}, \qquad \boldsymbol{m}^{\mathrm{T}} = \frac{\partial w}{\partial\boldsymbol{\chi}} \tag{9}$$

For the elastic deformation of the continuum, we have

$$w = \frac{\lambda\varepsilon_{kk}^2}{2} + \mu(\boldsymbol{\varepsilon}:\boldsymbol{\varepsilon} + l^2\boldsymbol{\chi}:\boldsymbol{\chi}) \tag{10}$$

where λ and μ are Lame's constants, and l is the length scale parameter of the continuum.

1.2 Constitutive equations for piezoelectric layer

Since the piezoelectric layer is in micro scale and the electric field may be very strong, the strain caused by electrostriction which is directly proportional to the quadratic of the electric field must be considered in the constitutive equation of the piezoelectric layer. Here the mechanical stress $\boldsymbol{T}$ and electric field $\boldsymbol{E}$ are chosen as independent variables, and a set of nonlinear constitutive equations that can describe the behavior of piezoelectric materials under a strong electric field can be written as[3-4]

$$\boldsymbol{S} = \boldsymbol{s}^E \boldsymbol{T} + \boldsymbol{dE} + \alpha \boldsymbol{E}^2 = \boldsymbol{S}^{(T)} + \boldsymbol{S}^{(E)} \tag{11a}$$

$$\boldsymbol{D} = \boldsymbol{dT} + \boldsymbol{\varepsilon}^T \boldsymbol{E} \tag{11b}$$

where $\boldsymbol{S}$ is the mechanical strain vector and $\boldsymbol{T}$ the stress vector. $\boldsymbol{E}$ is the electric field vector and $\boldsymbol{D}$ the electric displacement vector. The term $\boldsymbol{s}^E$ denotes the mechanical compliant coefficient matrix at the constant electric field E. $\boldsymbol{d}$ is the matrix of the piezoelectric coefficients and α the electrostrictive coefficient matrix. $\boldsymbol{\varepsilon}^T$ is the dielectric coefficient matrix at constant mechanical stress T. $\boldsymbol{S}^{(T)}$ is the strain vector caused by the mechanical stress and $\boldsymbol{S}^{(E)}$ the strain vector caused by the electric field.

We denote z-axis the 3-axis, x- and y-axis the 1- and 2-axis, respectively. Ignoring shear stress, then we have $\boldsymbol{S} = \{S_1, S_2, S_3\}^{\mathrm{T}} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z\}^{\mathrm{T}} = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}^{\mathrm{T}}$ and $\boldsymbol{T} = \{T_1, T_2, T_3\}^{\mathrm{T}} = \{\sigma_x, \sigma_y, \sigma_z\}^{\mathrm{T}} = \{\sigma_1, \sigma_2, \sigma_3\}^{\mathrm{T}}$. For the multi-layer beam considered in this paper, since there is no external electric field along the 1- and 2-axis ($E_x = E_y = 0$), we have

$$\boldsymbol{D} = \{D_3\},\ \boldsymbol{E} = \{E_3\}$$

$$S_i = s_{ij}^E T_j + d_{i3} E_3 + \alpha_{i3} E_3^2 \qquad i,\ j = 1, 2, 3 \tag{12a}$$

$$D_3 = d_{3i} T_i + \varepsilon_{33} E_3 \qquad i = 1, 2, 3 \tag{12b}$$

1.3 Analysis of the deformation behavior

Since the properties in the x-y plane are isotropic, we have $d_{13} = d_{23}$ and $\alpha_{13} = \alpha_{23}$ for the piezoelectric layer. Hence, in the case of small strain and moderate rotation, the equation of strain, stress and displacement for the piezoelectric layer can be written as

$$\varepsilon_x = \kappa z + \varepsilon_{x0} = \varepsilon_x^{(T)} + d_{13} E_3 + \alpha_{13} E_3^2$$

where κ is the curvature of the bending beam. ε_{x0} is the extension strain of the beam due to the piezoelectric properties under the electric field,

$$\varepsilon_{x0} = \frac{t_3 (d_{31} E_3 + \alpha_{31} E_3^2) / s_{11,3}}{\sum_i t_i / s_{11,i}}$$

and $s_{11,i}$ is the compliant coefficient of the i-th layer. Hence the strain due to the mechanical stress is

$$\varepsilon_x^{(T)} = \kappa z + A \tag{13}$$

where

$$A = -\frac{\left(\frac{t_1}{s_{11,1}} + \frac{t_2}{s_{11,2}} + \frac{t_4}{s_{11,4}}\right)(d_{31} E_3 + \alpha_{31} E_3^2)}{\sum_i \frac{t_i}{s_{11,i}}}$$

For the passive layer and the electrode layers, the corresponding strain is

$$\varepsilon_x = \kappa z + \varepsilon_{x0} = \varepsilon_x^{(T)}$$

$$\varepsilon_x^{(T)} = \kappa z + B \tag{14}$$

where $B = \varepsilon_{x0}$. Considering Eq. (7), we can obtain the rotation gradient for every layer,

$$\chi_{21}^{(T)} = \kappa \tag{15a}$$

$$\chi_{11}^{(T)} = \chi_{12}^{(T)} = \chi_{13}^{(T)} = \chi_{22}^{(T)} = \chi_{23}^{(T)} = \chi_{31}^{(T)} = \chi_{32}^{(T)} = \chi_{33}^{(T)} = 0 \tag{15b}$$

In this paper the static deflection of the beam under the applied external electric field is developed using an energy method. According to Eq. (10), the elastic strain energy for every layer can be written as

$$U_{\mathrm{e},i} = \int_V \left[\frac{1}{2}\lambda_i (\varepsilon_{kk,i}^{(T)})^2 + \mu_i (\varepsilon_i^{(T)} : \varepsilon_i^{(T)} + l_i^2 \chi_i^{(T)} : \chi_i^{(T)})\right] \mathrm{d}v$$

$$i = 1, 2, 3, 4$$

$$U_{\mathrm{e}} = \sum_{i=1}^{4} U_{\mathrm{e},i} \tag{16}$$

Considering Eq. (12b), the electric potential energy for the piezoelectric layer is

$$U_{\mathrm{E}} = \int_V \frac{1}{2} \boldsymbol{D}^{\mathrm{T}} \boldsymbol{E} \mathrm{d}v = \frac{E_3 d_{31} s_{11,3} bL}{2}\left(\frac{\kappa (z_3^2 - z_2^2)}{2} + A t_3\right) + \frac{E_3^2\ \varepsilon\ t_3 bL}{2} \tag{17}$$

where b and L are the width and length of the actuator, respectively. The total potential energy is

$$U_{\text{total}} = U_{\mathrm{e}} + U_{\mathrm{E}} \tag{18}$$

In order to obtain the deflection of the beam, we use the Rayleigh-Ritz method.

$$\frac{\partial U_{\text{total}}}{\partial \kappa} = 0 \tag{19}$$

Then, we can obtain the curvature of the piezoelectric unimorph micro actuator as

$$\kappa = \frac{-A\left(\frac{\lambda_3}{2} + \mu_3\right)(z_3^2 - z_2^2) - B \sum_{i \neq 3} \left(\frac{\lambda_i}{2} + \mu_i\right)(z_i^2 - z_{i-1}^2) - E_3 d_{31} s_{11,3} \frac{z_3^2 - z_2^2}{4}}{C + D} \tag{20}$$

where $C = \frac{2}{3}\sum_i \left(\frac{\lambda_i}{2}+\mu_i\right)(z_i^2 - z_{i-1}^2)$, and $D = 2\sum_i \mu_i l_i^2 t_i$.

2 Numerical Results and Discussion

The materials of the piezoelectric layer and the passive layer for the unimorph beam are ALN and SiO_2, respectively. The metals for the top and bottom electrodes are Au and Pt, and the thicknesses of the two electrodes are both 0.1 μm. Young's moduli for ALN and SiO_2 are 395 GPa and 75 GPa, respectively. For the Au and Pt, Young's moduli are 78.3 GPa and 150 GPa, respectively. The Poisson ratios are 0.28 and 0.17 for ALN and SiO_2, and are 0.4 and 0.38 for Au and Pt, respectively. The piezoelectric coefficient $d_{13} = 2.1\times10^{-10}$, and the electrostrictive coefficient $\alpha_{13} = 1.3\times10^{-16}$. The length scale parameters for each layer are supposed to be 1.0 μm. By the use of Matlab language, the numerical results are obtained based on the above theoretical formulae.

Fig. 3 and Fig. 4 show the nonlinearities of the curvature via the applied electric field. It can be seen from Fig. 3 that the nonlinearities vary when the ratio of the piezoelectric layer and the passive layer varies, but it does not deviate much with the change of this thickness ratio. However, the nonlinearities of the curvature vary distinctly with the change of the electrostrictive coefficient when the applied electric field increases (see Fig. 4).

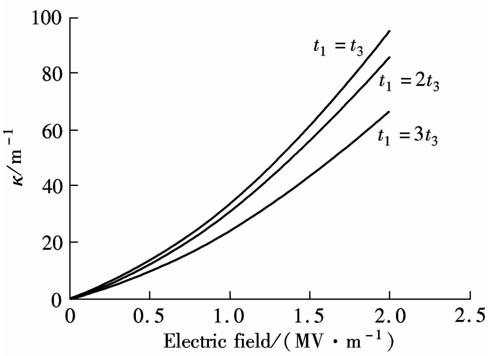

Fig. 3 Nonlinearity of curvature via electric field at various ratios of t_1 and t_3 ($t_3 = 5$ μm)

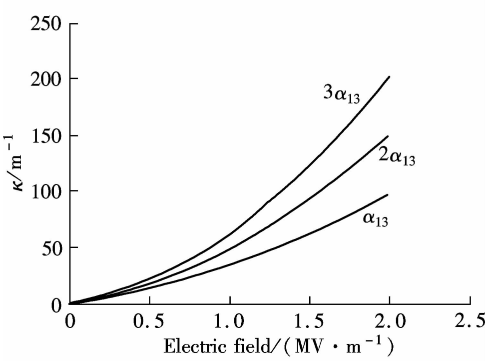

Fig. 4 Nonlinearity of curvature via electric field at various electrostrictive coefficients ($t_3 = 5$ μm, $t_1 = t_3$)

Fig. 5 and Fig. 6 show the determination of the optimal thickness ratio of the piezoelectric layer and the passive layer. From Fig. 5, we can see that the choice of this optimal ratio is dependent on the thickness of the piezoelectric layer, and this ratio will tend to be 1.0 when the piezoelectric thickness becomes macro scale. However, when the thickness of the piezoelectric layer is in micro scale, this ratio value should be greater than 1.0 (see Fig. 6). This phenomenon results from the size effect of the deformation behavior of a microstructure. Regarding the exact ratio value, it is related to the length scale parameters of the piezoelectric material and the passive layer material which need to be measured by special experiments. It is worthy pointing out that this optimal ratio is usually taken to be about 1.0[2] which can be obtained from Fig. 7. Fig. 7 is obtained using the conventional theory of mechanics and does not consider the size effect. Fortunately some actuators reported[5-7] are usually in macro scale, which does not cause any obvious deviation.

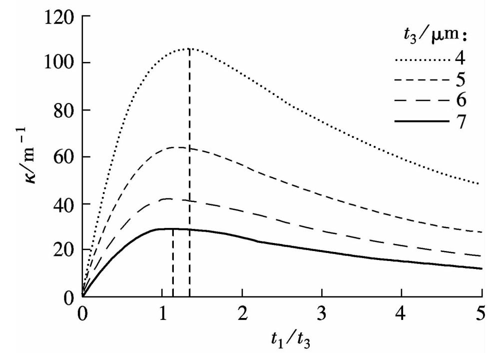

Fig. 5 Curvature of unimorph beam via ratio t_1/t_3 at various thicknesses of piezoelectric layer ($V = 7.5$ V)

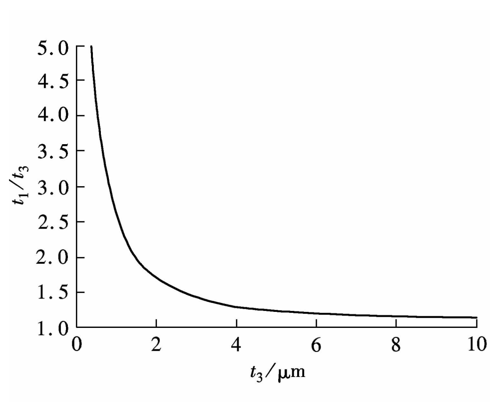

Fig. 6 The optimal ratio of piezoelectric layer and passive layer ($V = 7.5$ V)

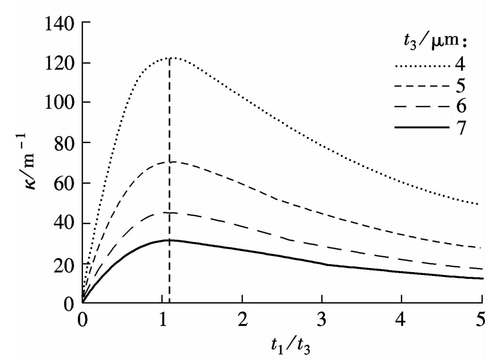

Fig. 7 Curvature of beam via ratio t_1/t_3 at different thicknesses of piezoelectric layer ($V = 7.5$ V, not considering size effect)

3 Conclusion

The static characteristics of a piezoelectric unimorph bending actuator driven by an applied electric field are evaluated in this paper. The nonlinearities of the deflection of the unimorph beam caused by the electrostriction of the piezoelectric layer under a strong electric field and the size dependence of the deformation behavior of the beam actuator are considered in the evaluation. The results show that the nonlinearities of the deformation increase along with the increase of the electric field which means that softening of the rigidity exists when a strong external electric field is applied. Moreover, the optimal value of the ratio for the passive layer and the piezoelectric layer is not around 1.0 which has usually been adopted by some previous researchers. Since there exists a size dependence behavior of the beam deformation, the optimal value of this thickness ratio should be greater than 1.0 when the actuator is in micro scale.

References

[1] Uchino K. *Piezoelectric actuator and ultrasonic motor* [M]. Boston: Kluwer Academic, 1996.

[2] Muralt P, Ledermann N, Baborowski J. Piezoelectric micromachined ultrasonic transducers based on PZT thin film [J]. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, 2005, **52**(12): 2276 − 2288.

[3] Arafa M, Baz A. On the nonlinear behavior of piezoelectric actuator [J]. *Journal of Vibration and Control*, 2004, **10** (3): 387 − 398.

[4] Joshi S P. Nonlinear constitutive relation for piezoceramic materials [J]. *Smart Mater and Struct*, 1992, **1**(1): 80 − 83.

[5] Wang Q M, Zhang Q M, Xu B M, et al. Cross. Nonlinear piezoelectric behavior of ceramic bending mode actuators under strong electric fields [J]. *Journal of Applied Physics*, 1999, **86**(6): 3352 − 3360.

[6] Yao Linquan, Ding Rui. The dynamic analysis of piezoelectric bending actuator considering nonlinear piezoelectric effect [J]. *Acta Mechanica Sinica*, 2005, **37**(2): 183 − 189. (in Chinese)

[7] Yao L Q, Zhang J G, Lu L, et al. Nonlinear static characteristics of piezoelectric bending actuator under strong applied electric field [J]. *Sensor and Actuator A*, 2004, **115**(1): 168 − 175.

[8] Chong A C M, Yang F, Lam D C C, et al. Torsion and bending of micro-scale structures [J]. *J Mater Res*, 2001, **16**(4): 1052 − 1058.

[9] Fleck N A, Muller G M, Ashby M F, et al. Strain gradient plasticity: theory and experiment [J]. *Acta Metall Mater*, 1994, **42**(2): 475 − 487.

[10] Stolken J S, Evans A G. A microbend test method for measuring the plasticity length scale [J]. *Acta Mater*, 1998, **46** (14): 5109 − 5115.

[11] Yang F, Chong A C M, Lam D C C, et al. Couple stress based strain gradient theory for elasticity [J]. *International Journal of Solids and Structures*, 2002, **39**(10): 2731 − 2743.

[12] Mindlin R D, Tiersten H F. Effect of couple-stresses in linear elasticity [J]. *Arch Rational Mech Anal*, 1962, **11**(1): 415 − 448.

[13] Toupin R A. Elastic materials with couple-stresses [J]. *Arch Rational Mech Anal*, 1962, **11**(1): 385 − 414.

[14] Ding Jianning, Meng Yonggang, Wen Shizhu. Research of the size effect on strength of polysilicon micro-electromechanical devices [J]. *Journal of Mechanical Strength*, 2001, **23**(4): 385 − 388. (in Chinese)

[15] Hwang Kehchih, Qiu Xinming, Jiang Hanqing. Recent advances in strain gradient plasticity-I-couple stress theory and SG theory [J]. *Journal of Mechanical Strength*, 1999, **21** (2): 81 − 87. (in Chinese)

[16] Chen Shaohua, Wang Ziqiang. Advances in strain gradient theory [J]. *Advances in Mechanics*, 2003, **33**(2): 207 − 216. (in Chinese)

单层压电弯曲微执行器的非线性静态特性

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摘要:采用偶应力理论对单层压电悬臂梁式微执行器在强外加电场作用下的非线性静态特征进行了研究. 悬臂式执行器包括压电层、被动(弹性)层和 2 个电极层. 首先,采用瑞利-里兹方法分析了在强外加电场作用下由于压电层电致伸缩效应引起的执行器非线性静态特性. 其次,由于悬臂执行器的厚度在 μm 量级,存在变形尺度效应,采用偶应力理论对变形的尺度效应进行了分析. 分析结果表明:悬臂执行器的非线性随着外加电场强度的增大而增大,当外加电场很大时,出现刚度软化现象; 而且,执行器被动层和压电层厚度比的最优值不是通常采用的 1.0. 在微尺度下,由于微梁变形存在尺度效应,该厚度比的最优值应比 1.0 大.

关键词:非线性静态特性;单层压电微执行器;偶应力理论;瑞利-里兹方法

中图分类号:O343.5