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Decision model for closed loop supply chain with uncertain demand and price-dependent returns

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Abstract: A single product closed-loop supply chain that satisfies an uncertain market demand with original and remanufactured products is considered. The yield of the recovery process is random and depends on the acquisition price offered for the end-of-life products. In such a stochastic setting, a firm needs to make production and procurement decisions so that the total expected profit is maximized. Both centralized and decentralized models are established depending on the party collecting the returns. The optimal acquisition price and production quantities of original and remanufactured products are determined for the firm. The contracts to coordinate the decentralized systems are chosen and the optimal contract parameters are determined. A computational experiment is given to show the effects of recovery parameters on the system performance. Results show that the recovery parameters have a high impact on the profitability of the manufacturer in the centralized model and on that of the collection agency in the decentralized model.

Key words: closed-loop supply chain; demand uncertainty; production decision

Remanufacturing refers to the business of refurbishing returned products and then sending them back to market "as new". It is considered a part of a closed-loop supply chain, along with operations such as acquisition/collection, testing/grading, repairing, manufacturing, and redistribution[1]. Remanufacturing has received growing attention in recent years due to increased environmental concerns and limited availability of natural resources. Companies engage in recycling and remanufacturing processes either because of legislations in their countries or for economic reasons.

There is a growing amount of literature on remanufacturing and closed-loop supply chains. Savaskan et al.[2] addressed the problem of reverse channel structure for the collection of used products and claimed that remanufacturers can design closed loop supply chains so as to enhance their profits. Guide et al.[3] compared the performances of several static scheduling rules in a basic remanufacturing shop considering the disassembly and reassembly of products. Mitra and Webster[4] examined the effects of government subsidies as a means of promoting remanufacturing activity. Tang et al.[5] examined the process lead time, which can be used to determine the planned lead time in production planning and control of remanufacturing. Majumder and Groenevelt[6] studied a two-period model with one OEM and one independent remanufacturer where the companies compete in the sales market and in the procurement process. They aimed to find the optimal prices and production quantities in a deterministic setting. Ferrer and Swaminathan[7] extended the work to multi-period models and investigated the effects of various parameters on the system. Webster and Mitra[8] developed a general two-period model to investigate questions of interest to policy-makers in government and managers in industry. Ferrer[9] dealt with the problem of how managers make product disassembly and procurement decisions when faced with limited information on remanufacturing yields or a potential long supplier lead time. Bakal and Akcali[10] found the optimal prices for used parts and remanufactured parts and studied the effects of mean and variance of yield rate on profit by numerical analysis. Kaya[11] considered the optimal incentive determination problem in the manufacturing/remanufacturing industry using stochastic demand functions. Karakayali et al.[12] developed models to determine the optimal acquisition price of the end-of-life products and the selling price of the remanufactured parts in centralized channels as well as remanufacturer-driven and collector-driven decentralized channels.

Unlike those models considering the deterministic nature of returned products, we address the more realistic issue of production and pricing by modeling both the demand and return side as random. We analyze both centralized and decentralized models to determine the optimal acquisition price and the optimal production quantities in such a stochastic setting. Then we proceed to coordinate the decentralized model through a two-part tariff contract.

1 Assumptions and Notations

Here we just consider a single product supply chain, and the manufacturer has two alternatives for fulfilling the demand either producing products by new raw materials and components, or remanufacturing old products and bringing them back to "as new" conditions. In the recycle process, the manufacturer decides a price for the used products and collects some used products from the customer. Then the recycled products will be stocked in the recoverable inventory or be sent to remanufacturing directly. The returned quantity is assumed to be uncertain and price-dependent, and the manufacturer can control the expected quantity of returned products by adjusting the recycle price. The objective of supply planning is to control the brand-new products production and the recycled products remanufacturing process to guarantee a required service level and to maximize the total expected profit.

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In order to facilitate the study, the following assumptions are postulated.

Assumption 1 In our study, we consider a manufacturer that produces both original products using virgin materials and remanufactured products by collecting used products. We assume that the customers have the same valuations for the original products produced from virgin materials and the products produced by using the returns. Thus, these two types of products have the same price and demand in the market and customers are indifferent with regard to them.

Assumption 2 We assume that the unit cost of producing the end products form original materials is c_{m}, and the total cost of extracting the useful parts from the returns and producing the end products using these parts in the manufacturing process is c_{r}. We assume that $c_{\mathrm{r}} < c_{\mathrm{m}}$; otherwise, the company has no benefit in collecting returns and only uses original materials in their production process.

Assumption 3 Since the customers are indifferent with regard to these two types of products, we denote the market price for both types with p and the density and cumulative distribution function for the total demand of the original and remanufactured products with $f(x)$ and $F(x)$, respectively.

Assumption 4 The manufacturer offers to pay a certain amount of money p_{r} to collect the right amount of used materials from the market. The amount of returns collected from the market is denoted by $R(p_{\mathrm{r}})$ which is a function of p_{r}. We assume that the amount of returns is a linear function of p_{r} where $R(p_{\mathrm{r}}) = a + bp_{\mathrm{r}}$.

2 Production Decision Models

2.1 Centralized model

In the centralized setting, the manufacturer collects the used products directly from the customers and manages all the collection and the manufacturing processes. The manufacturer's problem is

$$\max \pi = \int_0^{q_{\mathrm{r}}+q_{\mathrm{m}}} pxf(x)\,\mathrm{d}x + \int_{q_{\mathrm{r}}+q_{\mathrm{m}}}^{\infty} p(q_{\mathrm{r}} + q_{\mathrm{m}})f(x)\,\mathrm{d}x - c_{\mathrm{m}}q_{\mathrm{m}} - c_{\mathrm{r}}q_{\mathrm{r}} - p_{\mathrm{r}}R(p_{\mathrm{r}})$$

where p is the market price of products; c_{r} is the remanufactured cost for each unit; q_{r} is the quantity of remanufacturing products; p_{r} is the certain amount of money that the manufacturer offers to pay to collect the right amount of used materials from the market; $R(p_{\mathrm{r}})$ denotes the amount of returns collected from the market, and we assume that it is a linear function of p_{r} where $R(p_{\mathrm{r}}) = a + bp_{\mathrm{r}}$. The market demand X is a random variable; its probability density function is $f(x)$ and its distribution function is $F(x)$.

We can easily find that either $R(p_{\mathrm{r}}) = q_{\mathrm{r}}$ or $p_{\mathrm{r}} = 0$ in the optimal solution. Otherwise, the manufacturer can increase his profit by decreasing p_{r} which decreases $R(p_{\mathrm{r}})$ until $R(p_{\mathrm{r}}) = q_{\mathrm{r}}$ or $p_{\mathrm{r}} = 0$.

First, we assume that $p_{\mathrm{r}} > 0$ and $R(p_{\mathrm{r}}) = q_{\mathrm{r}}$ in the optimal solution. Then, the manufacturer's problem becomes

$$\max \pi = \int_0^{q_{\mathrm{r}}+q_{\mathrm{m}}} pxf(x)\,\mathrm{d}x + p(q_{\mathrm{r}} + q_{\mathrm{m}})\overline{F}(q_{\mathrm{r}} + q_{\mathrm{m}}) - c_{\mathrm{m}}q_{\mathrm{m}} - c_{\mathrm{r}}q_{\mathrm{r}} - p_{\mathrm{r}}q_{\mathrm{r}}$$

Considering the first and second order derivatives w. r. t. q_{r} and q_{m}, we observe that the objective function is concave w. r. t. q_{r} and q_{m}, and thus equating the first order derivative to 0 will give us the optimal values of p_{r} and q_{r} if the resulting $p_{\mathrm{r}} \geqslant 0$.

$$\frac{\partial \pi}{\partial q_{\mathrm{r}}} = p - pF(q_{\mathrm{r}} + q_{\mathrm{m}}) - c_{\mathrm{r}} - \frac{2q_{\mathrm{r}} - a}{b}$$

$$\frac{\partial \pi}{\partial q_{\mathrm{m}}} = p - pF(q_{\mathrm{r}} + q_{\mathrm{m}}) - c_{\mathrm{m}}$$

Solving the above two equations simultaneously, we can obtain

$$c_{\mathrm{m}} = c_{\mathrm{r}} + \frac{2q_{\mathrm{r}} - a}{b}$$

From this relation, we can find that

$$q_{\mathrm{r}}^* = \frac{b(c_{\mathrm{m}} - c_{\mathrm{r}}) + a}{2}$$

From $R(p_{\mathrm{r}}) = q_{\mathrm{r}}$, we can obtain $p_{\mathrm{r}}^* = \dfrac{q_{\mathrm{r}}^* - a}{b}$, and then $q_{\mathrm{m}}^* = F^{-1}\left(\dfrac{p - c_{\mathrm{m}}}{p}\right) - q_{\mathrm{r}}^*$.

However, if $F^{-1}\left(\dfrac{p - c_{\mathrm{m}}}{p}\right) < q_{\mathrm{r}}^*$, then $q_{\mathrm{m}}^* = 0$ is the optimal solution. The manufacturer's problem becomes

$$\max \pi = \int_0^{q_{\mathrm{r}}} pxf(x)\,\mathrm{d}x + p(q_{\mathrm{r}})\overline{F}(q_{\mathrm{r}}) - (c_{\mathrm{r}} + p_{\mathrm{r}})q_{\mathrm{r}}$$

We observe that the objective function is concave w. r. t. q_{r}. Thus, equating the first order derivative to 0 will give us the optimal values of p_{r} and q_{r} if $p_{\mathrm{r}} \geqslant 0$.

$$pF(q_{\mathrm{r}}) = p - c_{\mathrm{r}} - \frac{2q_{\mathrm{r}} - a}{b}$$

From the above analysis we can obtain the optimal value of q_{r} if $q_{\mathrm{m}}^* = 0$.

If the above conditions are not satisfied and the resulting incentive values in the above cases are less than 0, then $p_{\mathrm{r}} = 0$ in the optimal solution. In that case, the manufacturer produces all the products using the used materials since $c_{\mathrm{r}} < c_{\mathrm{m}}$, $q_{\mathrm{r}}^* = F^{-1}\left(\dfrac{p - c_{\mathrm{r}}}{c_{\mathrm{r}}}\right)$ and $q_{\mathrm{m}}^* = 0$. However, for $p_{\mathrm{r}} = 0$, he can produce $R(0)$ at most. Thus, $q_{\mathrm{r}}^* = \min\left\{F_{\mathrm{r}}^{-1}\left(\dfrac{p - c_{\mathrm{r}}}{c_{\mathrm{r}}}\right), R(0)\right\}$ will be the optimal production quantity.

2.2 Decentralized model

In this section, we consider the decentralized model. In the decentralized setting, a third party collection agency collects the used products from the market and sells them to the manufacturer at a wholesale price. The collection agency decides on the acquisition price and the manufacturer decides on the production amount. The profit functions for both companies are

$$\pi_r = \int_0^{q_r+q_m} pxf(x)\,dx + \int_{q_r+q_m}^{\infty} p(q_r + q_m)f(x)\,dx - c_m q_m - c_r q_r - wq_r$$

$$\pi_1 = wq_r - p_r R(p_r)$$

where w is the unit wholesale price that the collection agency sets for the used products in the decentralized model.

We solve the manufacturer's problem first. The manufacturer will not buy any used materials from the collection agency if $w > c_m - c_r$. Thus, in the optimal solution $w \leqslant c_m - c_r$.

If $w < c_m - c_r$, the manufacturer will not buy any new materials any more, so $q_m^* = 0$.

Since π_1 is concave w. r. t. q_r, equating $\frac{\partial \pi_1}{\partial q_r}$ to 0, we can obtain $q_r^* = F^{-1}\left(\frac{p - c_r - w}{p}\right)$. Then the problem is to maximize π_1, that is

$$\pi_1 = \left(p - c_r - pF(q_r) - \frac{q_r - a}{b}\right) q_r$$

Equating the first order derivative to 0, we obtain the following equation which has the solution q_r^*:

$$\frac{\partial \pi_1}{\partial q_r} = p - c_r - pF(q_r) - \frac{q_r - a}{b} - pq_r f(q_r) - \frac{q_r}{b} = 0$$

From $q_r^* = F^{-1}\left(\frac{p - c_r - w}{p}\right)$, we can obtain $w^* = p - c_r - pF(q_r^*)$ and $p_r^* = \frac{q_r^* - a}{b}$.

If $w = c_m - c_r$, we can find $q_r^* = \min\left\{F^{-1}\left(\frac{p - c_m}{c_m}\right), R(p_r)\right\}$ and $q_m^* = F^{-1}\left(\frac{p - c_m}{p}\right) - q_r^*$.

We can obtain p_r^* from the equation $(c_m - c_r - p_r)b - R(p_r) = 0$ if $q_m^* > 0$. However, if $q_r^* > F^{-1}\left(\frac{p - c_m}{p}\right)$, then $p_r^* = R^{-1}\left(F^{-1}\left(\frac{p - c_m}{p}\right)\right)$.

If none of the above conditions is satisfied, then $p_r^* = 0$ will be the optimal incentive value. In this case, q_r^* is the value that satisfies the following relationship:

$$p - c_r - pF(q_r^*) - pq_r f(q_r^*) = 0$$

Accordingly, the optimal value of the wholesale price is $w^* = p - c_r - pF(q_r^*)$.

3 Coordination Through Contracts

From the above analysis, we observe that in the decentralized models, a lower incentive value is offered and lower amounts of products are remanufactured as opposed to centralized models where lower total system profits are obtained. In this section we search for contracts to coordinate the decentralized system to eliminate the efficiency losses and to achieve the centralized model results.

First, we consider the case $p_r^c > 0$. In this case, the optimal production quantity is given by the equation $c_m = c_r + \frac{2q_r - a}{b}$ and the optimal value of the incentive to offer is $p_r^* = \frac{q_r^* - a}{b}$.

In the decentralized model, for a given w, the optimal production quantity is equal to $q_r^c = F^{-1}\left(\frac{p - c_r - w}{p}\right)$. Thus $w = p - c_r - pF(q_r^c)$ will result in the same production quantity as in the centralized model.

Then, we consider the case $p_r^c = 0$. In this case, the optimal production quantity is $q_r^c = \min\left\{F^{-1}\left(\frac{p - c_m}{p}\right), Q(0)\right\}$ for the centralized model. If $w = 0$ is offered in the contract, the collection agency offers $p_r^* = p_r^c = 0$ as the incentive since the collection agency has no benefit in collecting any used products. Also, the optimal production quantity under this contract will be equal to q_r^c.

From the above analysis, a linear contract with transfer payments can coordinate the supply chain.

$$(w, A) = \begin{cases} (p - c_r - pF(q_r^c), \pi_1 - \pi_0) & \text{if } p_r^c > 0 \\ (0, \pi_0) & \text{if } p_r^c = 0 \end{cases}$$

where p_r^c and q_r^c are the optimal incentive value and the quantity of remanufactured products produced in the centralized model, respectively; π_1 is the profit of the collection agency with w, and π_0 is the reservation profit of the collection agency.

4 Computational Experiments

In this section, we perform numerical studies to illustrate the applicability of our models and implement further analysis on impacts of different parameters. In all computational experiments, demands are assumed to follow the normal distribution with a mean $\mu = 100$ and a standard deviation $\sigma = 20$. Here, we set $a = 50$, $b = 5$, $p = 80$, $c_m = 50$, $c_r = 20$ for our base case. Tab. 1 gives the optimal values of the decision variables for both centralized and decentralized models of the base case. Also we conduct additional experiments by changing the values of the recovery parameters and observing their effects on our systems while keeping other parameters unchanged.

From Tab. 1, we observe that less acquisition price is offered and lower amount of remanufacturing is done in the decentralized model as opposed to the centralized model. Also we observe that as a or b increases, the collection of used products becomes less costly and more remanufacturing is done and more used products are collected. In addition, the decrease in the collection cost has a higher impact on the profitability of the manufacturer in the centralized models than in the decentralized models and the efficiency loss in the decentralized cases increases as the collection of used products becomes less costly.

In the decentralized model, we observe that a change in the recovery parameter a or b does not affect the manufacturer's profits but highly affects the collection agency's profits. So, it is much more important for the collection agency to find new ways to increase the recovery rates at a lower cost. On the other hand, the collection agency is directly affected by the change in the costs of collection or remanufacturing activities since he cannot impose any cost increase on the manufacturer.

Tab. 1 Impact of the recovery parameters on supplying decisions

Case	Centralized model				Decentralized model						$\frac{\pi^d}{\pi^c}$
	q_r^c	q_m^c	p_r^c	π^c	q_r^d	q_m^d	p_r^d	w^d	π_1^d	π_r^d	
Base case	95.0	0	9.0	4 925.0	93.6	0	8.7	30	2 923.7	1994.6	0.998
$a=0$	75.0	18.6	15.0	4 048.7	75.0	18.6	15.0	30	2 923.7	1 125.0	1.000
$a=30$	90.0	3.6	12.0	4 633.6	90.0	3.6	12.0	30	2 923.7	1 620.0	0.980
$a=100$	100.0	0	0	5 920.0	93.6	0	0	30	2923.7	2 808.0	0.968
$b=1$	50.0	43.6	0	4 423.7	50.0	43.6	0	30	2923.7	1 500.0	1.000
$b=3$	70.0	23.6	6.7	4 553.7	70.0	23.6	6.7	30	2 923.7	1 630.3	1.000
$b=6$	97.5	0	7.9	5 089.8	93.6	0	7.3	30	2 923.7	2 123.3	0.990

5 Conclusion

In this paper, we consider a closed-loop supply chain system in which the manufacturing and remanufacturing processes are simultaneously conducted. We determine the optimal value of the acquisition price and the optimal production quantities of original and remanufactured products under uncertain demand and price-dependent returns. Both centralized and decentralized models are considered. We also choose contracts to coordinate the decentralized systems and determine the optimal contract parameters. Finally, we perform a computational study to analyze the effects of recovery parameters on the system performance. We observe that the recovery parameters have a high impact for the manufacturer in the centralized model and for the collection agency in the decentralized model. We can extend our model considering that the amount of returns is a stochastic function of the acquisition price in future research.

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随机需求及回收量价格敏感的闭环供应链决策模型

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摘要:考虑一个既向市场提供全新产品又提供再制造品的单一产品供应链,在需求随机且回收量对回收价格敏感的情况下,为企业做出最优生产和回收决策,使供应链利润最大化. 依据回收主体的不同,分别建立制造商回收时的集中决策模型与第三方回收时的分散决策模型;在随机需求环境下,给出不同决策模型的最优回收价格及全新产品与再制造品的最优产量;为分散决策模型制定协调各方利益的契约并给出最优契约参数. 最后给出一个数值算例分析了回收参数对系统利润的影响. 结果显示,回收参数在集中决策模型中对制造商的利润有很大的影响,而在分散模型中对回收商有较大影响.

关键词:闭环供应链;随机需求;生产决策

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