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Duality theorem for smash coproduct over quantum groupoids

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Abstract: The duality theorem of generalized weak smash coproducts of weak module coalgebras and comodule coalgebras over quantum groupoids is studied. Let H be a weak Hopf algebra, C a left weak H-comodule coalgebra and D a left weak H-module coalgebra. First, a weak generalized smash coproduct $C\times_H^1 D$ over quantum groupoids is defined and the module and comodule structures on it are constructed. The weak generalized right smash coproduct $C\times_L^r D$ is similar. Then some isomorphisms between them are obtained. Secondly, by introducing some concepts of a weak convolution invertible element, a weak co-inner coaction and a strongly relative co-inner coaction, a sufficient condition for $\underline{C\times_H^r D}$ to be isomorphic to $C\otimes_v D$ is obtained, where $v\in\overline{\mathrm{WC}(C,H)}$ and the coaction of H on D is right strongly relative co-inner. Finally, the duality theorem for a generalized smash coproduct over quantum groupoids, $(C\times_H^1 H)\times_{H^*}^1 H^*\cong C\otimes_v(H\times_{H^*}^1 H^*)$, is obtained.

Key words: weak Hopf algebras (quantum groupoids); weak generalized smash coproducts; weak module coalgebras; weak comodule coalgebras; weak bimodule coalgebras; duality theorem

Quantum groupoids (weak Hopf algebras), which are generalizations of ordinary Hopf algebras, were defined by Böhm et al. [1] The main motivation for studying weak Hopf algebras comes from quantum field theories and operator algebras. Another motivation to study quantum groupoids comes from the fact that their representation theories provide examples of monoidal categories that can be used for constructing invariants of links and 3-manifolds[2]. Many results of the classical Hopf algebra theory can be generalized to weak Hopf algebras, and the structure of a weak Hopf algebra is much more complicated than that of a Hopf algebra[3-5].

In the classical Hopf algebra theory, Molnar[6] proposed an ordinary smash coproduct $C\times H$ (semi-direct product), where H is a Hopf algebra and C is a left H-comodule coalgebra. Wang[7] extended the smash coproduct to the left (right) smash coproduct $C\times_H^1 D$ ($C\times_L^r D$), and showed the duality theorem for Hopf comodule coalgebras.

The main motivation of the present paper is to generalize the smash coproduct in Ref. [7] to the weak conditions, where C is a left weak H-comodule coalgebra and D is a left weak H-module coalgebra, and then to show the duality theorem that holds for such a weak generalized smash coproduct over quantum groupiods.

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1 Preliminaries

Let k be a fixed field and we work over k. Throughout this paper, we use Sweedler's notation[8] for a comultiplication over a coalgebra C, writing $\Delta(c)=c_1\otimes c_2$ for all $c\in C$. In this section, we recall some basic notions for coalgebras related to weak Hopf algebras[9].

Let H and L be weak Hopf algebras. Then we recall that a coalgebra C is a left (right) weak H-comodule coalgebra if C is a left (right) H-comodule with comodule structure $\rho^1: C\to H\otimes C$, ($\rho^r: C\to C\otimes H$), satisfying $c_{(-1)}\otimes c_{01}\otimes c_{02}=c_{1(-1)}c_{2(-1)}\otimes c_{10}\otimes c_{20}$ and $c_{(-1)}\varepsilon(c_0)=\varepsilon_t(c_{(-1)})\varepsilon(c_0)$, ($c_{01}\otimes c_{02}\otimes c_{(1)}=c_{10}\otimes c_{20}\otimes c_{1(1)}c_{2(1)}$ and $\varepsilon(c_0)c_{(1)}=\varepsilon(c_0)\varepsilon_s(c_{(1)})$), for all $c\in C$. Furthermore, a coalgebra C is called a weak H-L-bicomodule coalgebra if C is an H-L-bicomodule, at the same time C is both a left weak H-comodule coalgebra and a right weak L-comodule coalgebra.

A coalgebra C is called a left (right) weak H-module coalgebra if C is a left (right) H-module via $h\otimes c\mapsto h\rightarrow c$, ($c\otimes h\mapsto c\leftarrow h$), satisfying $\Delta(h\rightarrow c)=h_1\rightarrow c_1\otimes h_2\rightarrow c_2$ and $\varepsilon_s(h)\rightarrow c=\varepsilon(h\rightarrow c_2)c_1$, ($\Delta(c\leftarrow h)=c_1\leftarrow h_1\otimes c_2\leftarrow h_2$ and $c\leftarrow\varepsilon_t(h)=\varepsilon(c_1\leftarrow h)c_2$). Moreover, C is called a weak H-L-bimodule coalgebra if C is not only an H-L-bimodule but also a left weak H-module coalgebra and a right weak L-module coalgebra.

Definition 1 We call C a left (right) weak L-H-dimodule coalgebra if C is not only a left (right) weak L-comodule coalgebra but also a left (right) weak H-module coalgebra such that the following condition $\rho^1(h\rightarrow c)=c_{(-1)}\otimes h\rightarrow c_0$, ($\rho^r(c\leftarrow h)=c_0\leftarrow h\otimes c_{(1)}$) holds, for all $h\in H$, $c\in C$.

If C is a coalgebra and A is an algebra, then $\mathrm{Hom}(C,A)$ is an algebra with the multiplication $*$ and a unit $\mu_A\varepsilon_C$, where $*$ is defined by

$$(f*g)(c)=f(c_1)g(c_2)\qquad f,\ g\in\mathrm{Hom}(C,A);\ c\in C$$

and we call the result algebra $\mathrm{Hom}(C,A)$ a convolution algebra, denoted by $\mathrm{Conv}(C,A)$.

2 Generalized Smash Coproducts

Let C be a left weak H-comodule coalgebra and D a left weak H-module coalgebra. Then we form a weak generalized smash coproduct denoted by $C\times_H^1 D$; the lower H means that the weak Hopf algebra in question is H, and the upper l means that the comodule and module structures are on the left.

Definition 2 The coalgebra $C\times_H^1 D$ is defined on the tensor product $C\otimes_t D$, where D is a left H_t-module via its left H-action, and C is a right H_t-module via $c\leftarrow x=$

$\varepsilon(c_{(-1)}S^{-1}(x))c_0$, for all $x\in H_t$, $c\in C$. The comultiplication and counit are given by

$$\Delta(c\times_H^{\mathrm{l}} d)=c_1\times_H^{\mathrm{l}} c_{2(-1)}\rightharpoonup d_1\otimes c_{20}\times_H^{\mathrm{l}} d_2$$
$$\varepsilon(c\times_H^{\mathrm{l}} d)=\varepsilon_C(c)\varepsilon_D(d)$$

for all $c\in C, d\in D$.

Similarly, suppose that D is a right weak L-comodule coalgebra, and C is a right weak L-module coalgebra. We form a weak generalized right smash coproduct denoted by $C\times_L^{\mathrm{r}} D$. The coalgebra $C\times_L^{\mathrm{r}} D$ is defined on the tensor product $C\otimes_{\mathrm{s}} D$, where D is a left L_{s}-module via $x\rightharpoonup d=\varepsilon(S^{-1}(x)d_{(1)})d_0$, and C is a right L_{s}-module via its right L-action. The comultiplication and counit are given by

$$\Delta(c\times_L^{\mathrm{r}} d)=c_1\times_L^{\mathrm{r}} d_{10}\otimes c_2\leftharpoonup d_{1(1)}\times_L^{\mathrm{r}} d_2$$
$$\varepsilon(c\times_L^{\mathrm{r}} d)=\varepsilon_C(c)\varepsilon_D(d)$$

for all $c\in C$, $d\in D$.

Example 1 Suppose that H is a weak Hopf algebra and H^* is its dual. Then H is a right weak H^*-comodule coalgebra via $\langle\rho^{\mathrm{r}}(h), f\otimes g\rangle=\langle gh, f\rangle$, where $g,h\in H; f\in H^*$. If C is a left weak H-comodule coalgebra, then C is a right weak H^*-module coalgebra from example 2. So we may form the right smash coproduct $C\times_{H^*}^{\mathrm{r}} H$.

Lemma 1 Suppose that D is a weak H-L-bicomodule coalgebra and E is a left weak H-module coalgebra. Then $D\times_H^{\mathrm{l}} E$ is a right weak L-comodule coalgebra with L-coaction induced by the right L-coaction on D, i. e.,

$$(d\times_H^{\mathrm{l}} e)_0\otimes(d\times_H^{\mathrm{l}} e)_{(1)}=d_0\times_H^{\mathrm{l}} e\otimes d_{(1)}$$

Similarly, we have the following lemma.

Lemma 2 Suppose that D is a weak H-L-bicomodule coalgebra and C is a right weak L-module coalgebra. Then $C\times_L^{\mathrm{r}} D$ is a left weak H-comodule coalgebra with H-coaction induced by the left H-coaction on D, i. e.,

$$(c\times_L^{\mathrm{r}} d)_{(-1)}\otimes(c\times_L^{\mathrm{r}} d)_0=d_{(-1)}\otimes c\times_L^{\mathrm{r}} d_0$$

By lemma 1 and lemma 2, the following theorem is easy to obtain.

Theorem 1 Suppose that C is a right weak L-module coalgebra, D is a weak H-L-bicomodule coalgebra and E is a left weak H-module coalgebra. Then the map taking $(c\times_L^{\mathrm{r}} d)\times_H^{\mathrm{l}} e$ to $c\times_L^{\mathrm{r}}(d\times_H^{\mathrm{l}} e)$ is a natural isomorphism from $(C\times_L^{\mathrm{r}} D)\times_H^{\mathrm{l}} E$ to $C\times_L^{\mathrm{r}}(D\times_H^{\mathrm{l}} E)$, where the smash coproducts $C\times_L^{\mathrm{r}} D$ and $D\times_H^{\mathrm{l}} E$ have the left H-comodule and right L-comodule structures defined in lemma 2 and lemma 1, respectively.

Proposition 1 ① Suppose that E is a left weak L-module coalgebra and D is a left weak L-H-dimodule coalgebra. Then $D\times_L^{\mathrm{l}} E$ is a left weak H-module coalgebra under the left H-action induced by that on D, i. e.,

$$h\rightharpoonup(d\times_L^{\mathrm{l}} e)=h\rightharpoonup d\times_L^{\mathrm{l}} e$$

② Suppose that D is a left weak L-H-dimodule coalgebra and C is a left weak H-comodule coalgebra. Then $C\times_H^{\mathrm{l}} D$ is a left weak L-comodule coalgebra under the left L-coaction induced by D, i. e.,

$$(c\times_H^{\mathrm{l}} d)_{(-1)}\otimes(c\times_H^{\mathrm{l}} d)_0=d_{(-1)}\otimes c\times_H^{\mathrm{l}} d_0$$

It follows immediately from proposition 1 that theorem 2 is obtained.

Theorem 2 Suppose that C is a left weak H-comodule coalgebra, D is a left weak L-H-dimodule coalgebra and E is a left weak L-module coalgebra. Then the map taking $(c\times_H^{\mathrm{l}} d)\times_L^{\mathrm{l}} e$ to $c\times_H^{\mathrm{l}}(d\times_L^{\mathrm{l}} e)$ is a natural isomorphism from $(C\times_H^{\mathrm{l}} D)\times_L^{\mathrm{l}} E$ to $C\times_H^{\mathrm{l}}(D\times_L^{\mathrm{l}} E)$, where the smash coproducts $C\times_H^{\mathrm{l}} D$ and $D\times_L^{\mathrm{l}} E$ have the left L-comodule and left H-module structures defined in proposition 1 ② and ①, respectively.

Remark We can obtain a right version of theorem 2 for the weak generalized right smash coproduct and the dual setting of theorem 1. We omit them.

3 Duality Theorem

In this section, as an application of our theory, we prove the duality theorem for the generalized smash coproduct over quantum groupoids.

The proof of the following proposition is straightforward.

Proposition 2 Suppose that C is a left weak H-comodule coalgebra and D is a left weak H-module coalgebra, and furthermore that C is also a right weak L-module coalgebra and D is a right weak L-comodule coalgebra such that for all $c\in C, d\in D$,

$$d_0\otimes c\leftharpoonup d_{(1)}=c_{(-1)}\rightharpoonup d\otimes c_0$$

Then there is a natural coalgebra isomorphism from $C\times_H^{\mathrm{l}} D$ to $C\times_L^{\mathrm{r}} D$ defined by mapping $c\times_H^{\mathrm{l}} d$ to $c\times_L^{\mathrm{r}} d$.

Example 2 Let H be a weak Hopf algebra, and let $\{f_i\}$ be an arbitrary basis of H and $\{\varphi_i\}$ its dual basis: $\langle\varphi_i, f_j\rangle=\delta_{ij}$. Let C be a left weak H-comodule coalgebra and D a left weak H-module coalgebra. Then C is a right weak H^*-module coalgebra via $c\leftharpoonup\varphi=\varphi(c_{(-1)})c_0$, and D is a right weak H^*-comodule coalgebra via $d_0\otimes d_{(1)}=\sum_i(f_i\rightharpoonup d)\otimes\varphi_i$. Thus, we have

$$d_0\otimes c\leftharpoonup d_{(1)}=\sum_i(f_i\rightharpoonup d)\otimes\varphi_i(c_{(-1)})c_0=c_{(-1)}\rightharpoonup d\otimes \mathrm{c}_0$$

It follows from proposition 2 that $C\times_H^{\mathrm{l}} D\cong C\times_{H^*}^{\mathrm{r}} D$.

Definition 3 Let H be a weak bialgebra and C a coalgebra. We define the following set:

$$\underline{\mathrm{WC}(C,H)}=\{u\in\mathrm{Conv}(C,H)\mid u*v(c)=\varepsilon_t(u(c)), v*u(c)=\varepsilon_s(u(c)), \exists\, v\in\mathrm{Hom}(C,H), \forall\, c\in C\}$$

In this case, we say that v is a weak convolution invertible element of u in $\underline{\mathrm{WC}(C,H)}$.

Similarly, we set

$$\overline{\mathrm{WC}(C,H)}=\{v\in\mathrm{Conv}(C,H)\mid u*v(c)=\bar{\varepsilon}_t(u(c)), v*u(c)=\bar{\varepsilon}_s(u(c)), \exists\, u\in\mathrm{Hom}(C,H), \forall\, c\in C\}$$

In this case, we say that u is a weak convolution invertible element of v in $\overline{\mathrm{WC}(C,H)}$.

Example 3 ① Let H be a weak Hopf algebra with an antipode S_H. Then we have $\iota_H\in\underline{\mathrm{WC}(H,H)}$; ② Let H be a weak Hopf algebra with a bijective antipode S_H. Then we have $\iota_H \in \overline{\mathrm{WC}(H^{\mathrm{op}}, H)}$, where H^{op} is a weak Hopf algebra with the anti-algebra structure.

Definition 4 ① If C is a left weak H-comodule coalgebra, then we say that the left H-coaction on C is a left weak co-inner coaction if there exists an element u in $\mathrm{WC}(C,H)$ such that

$$\rho^{\mathrm{l}}(c) = c_{(-1)} \otimes c_0 = u(c_1)v(c_3) \otimes c_2$$

for all $c \in C$. Furthermore, we say that a left weak co-inner coaction is left strongly relative co-inner if u is a coalgebra morphism.

② Suppose that C is a right weak H-comodule coalgebra. We say that the H-coaction on C is a right weak co-inner coaction if there exists an element $v \in \mathrm{WC}(C,H)$ such that

$$\rho^{\mathrm{r}}(c) = c_0 \otimes c_{(1)} = c_2 \otimes u(c_1)v(c_3)$$

for all $c \in C$. Furthermore, we say that a right weak co-inner coaction is right strongly relative co-inner if v is a coalgebra morphism.

Let C be a right weak H-module coalgebra and D a right weak H-comodule coalgebra. For $v \in \overline{\mathrm{WC}(C,H)}$, we set

$$C \otimes_v D = \{ c \otimes d \in C \otimes D \mid c \otimes d = c \leftarrow \varepsilon_{\mathrm{t}}(v(d_1)) \otimes d_2 \}$$

We now have the following theorem.

Theorem 3 Suppose that C is a right weak H-module coalgebra and D is a right weak H-comodule coalgebra such that the coaction of H on D is a right strongly relative co-inner coaction, then $C \times^{\mathrm{r}}_{H} D \cong C \otimes_v D$ as coalgebras.

Proof Set Ψ: $C \otimes_v D \to C \times^{\mathrm{r}}_{H} D$, $c \otimes_v d \mapsto c \leftarrow v(d_1) \times^{\mathrm{r}}_{H} d_2$ where $v = S^{-1} \circ u$. We claim that Ψ has an inverse map Φ: $C \times^{\mathrm{r}}_{H} D \mapsto C \otimes_v D$ by $c \times^{\mathrm{r}}_{H} d \mapsto c \leftarrow u(d_1) \otimes_v d_2$, as follows:

$$\begin{aligned}
&\Psi\Phi(c \times^{\mathrm{r}}_{H} d) = c \leftarrow u(d_1)v(d_2) \times^{\mathrm{r}}_{H} d_3 = \\
&\quad c \leftarrow \overline{\varepsilon}_{\mathrm{t}}(u(d_1)) \times^{\mathrm{r}}_{H} d_2 = c \leftarrow d_{1(1)} \times^{r}_{H} d_2 \varepsilon(d_{10}) = \\
&\quad c \times^{\mathrm{r}}_{H} \overline{\varepsilon}_{\mathrm{t}}(d_{(1)}) \rightarrow d_0 = c \times^{\mathrm{r}}_{H} \varepsilon(S(d_{0(1)})\overline{\varepsilon}_{\mathrm{t}}(d_{(1)}))d_{00} = \\
&\quad c \times^{\mathrm{r}}_{H} \varepsilon(S(d_{0(1)})d_{(1)})d_{00} = c \times^{\mathrm{r}}_{H} \varepsilon(\varepsilon_{\mathrm{s}}(d_{(1)}))d_0 = \\
&\quad c \times^{\mathrm{r}}_{H} \varepsilon(d_{2(1)})\varepsilon(d_{20})d_1 = c \times^{\mathrm{r}}_{H} d
\end{aligned}$$

and

$$\begin{aligned}
\Phi\Psi(c \otimes_v d) &= c \leftarrow v(d_1)u(d_2) \otimes_v d_3 = \\
&c \leftarrow \overline{\varepsilon}_{\mathrm{s}}(u(d_1)) \otimes_v d_2 = \\
&c \leftarrow \varepsilon_{\mathrm{t}}(v(d_1)) \otimes_v d_2 = c \otimes_v d
\end{aligned}$$

Now we check that Ψ is a coalgebra map as follows:

$$\begin{aligned}
&\Delta_{C \times^{\mathrm{r}}_{H} D}\Psi(c \otimes_v d) = \\
&\quad c_1 \leftarrow v(d_{11}) \times^{\mathrm{r}}_{H} d_{210} \otimes c_2 \leftarrow v(d_{12})d_{21(1)} \times^{\mathrm{r}}_{H} d_{22} = \\
&\quad c_1 \leftarrow v(d_1) \times^{\mathrm{r}}_{H} d_{32} \otimes c_2 \leftarrow v(d_2)u(d_{31})v(d_{33}) \times^{\mathrm{r}}_{H} d_4 = \\
&\quad c_1 \leftarrow v(d_1) \times^{\mathrm{r}}_{H} d_3 \otimes c_2 \leftarrow \overline{\varepsilon}_{s}(u(d_2))v(d_4) \times^{\mathrm{r}}_{H} d_5 = \\
&\quad c_1 \leftarrow v(d_1) \times^{\mathrm{r}}_{H} d_3 \otimes c_2 \leftarrow \varepsilon_{\mathrm{t}}(v(d_2))v(d_4) \times^{\mathrm{r}}_{H} d_5 = \\
&\quad c_1 \leftarrow v(d_1) \times^{\mathrm{r}}_{H} d_3 \otimes \varepsilon(c_2 \leftarrow v(d_2))(c_3 \leftarrow v(d_4)) \times^{\mathrm{r}}_{H} d_5 = \\
&\quad c_1 \leftarrow \overline{\varepsilon}_{\mathrm{t}}(v(d_2))v(d_1) \times^{\mathrm{r}}_{H} d_3 \otimes c_2 \leftarrow v(d_4) \times^{\mathrm{r}}_{H} d_5 = \\
&\quad c_1 \leftarrow v(d_1) \times^{\mathrm{r}}_{H} d_2 \otimes c_2 \leftarrow v(d_3) \times^{\mathrm{r}}_{H} d_4 = \\
&\quad (\Psi \otimes \Psi)\Delta_{C \otimes_v D}(c \otimes_v d)
\end{aligned}$$

and

$$\begin{aligned}
\varepsilon_{C \times^{\mathrm{r}}_{H} D}\Psi(c \otimes_v d) &= \varepsilon(c \leftarrow \varepsilon_{\mathrm{t}}(v(d))) = \\
&\varepsilon(c \leftarrow \varepsilon_{\mathrm{t}}(v(d_1)) \otimes d_2) = \varepsilon_{C \otimes_v D}(c \otimes_v d)
\end{aligned}$$

This completes the proof.

Let H be a weak Hopf algebra and H^* be its dual weak Hopf algebra. We can see that H is a weak H^*-H^*-bicomodule coalgebra and a left weak H^*-H-dimodule coalgebra. The left H-module structure on H and the left H^*-module structure on H^* are given by the multiplication maps, respectively. The left H^*-comodule structure on H is given by

$$\langle \rho^{\mathrm{l}}(h), g \otimes f \rangle = \langle hg, f \rangle \qquad g, h \in H; f \in H^*$$

and the right H^*-comodule structure on H is given by

$$\langle \rho^{\mathrm{r}}(h), f \otimes g \rangle = \langle gh, f \rangle \qquad g, h \in H;\ f \in H^*$$

Now, if C is a left weak H-comodule coalgebra, by theorem 2, we obtain

$$(C \times^{\mathrm{l}}_{H} H) \times^{\mathrm{l}}_{H^*} H^* \cong C \times^{\mathrm{l}}_{H} (H \times^{\mathrm{l}}_{H^*} H^*)$$

where $C \times^{\mathrm{l}}_{H} H$ has the left H^*-comodule structure defined in proposition 1 ② and $H \times^{\mathrm{l}}_{H^*} H^*$ has the left H-module structure defined in proposition 1 ①.

By example 2, C is a right weak H^*-module coalgebra. And $H \times^{\mathrm{l}}_{H^*} H^*$ is a right weak H^*-comodule coalgebra by lemma 1. Then for all $c \in C$, $h \times^{\mathrm{l}}_{H^*} f^* \in H \times^{\mathrm{l}}_{H^*} H^*$, we obtain

$$(h \times^{\mathrm{l}}_{H^*} f^*)_0 \otimes c \leftarrow (h \times^{\mathrm{l}}_{H^*} f^*)_{(1)} = c_{(-1)} \rightarrow (h \times^{\mathrm{l}}_{H^*} f^*) \otimes c_0$$

Furthermore, by proposition 2, we obtain

$$(C \times^{\mathrm{l}}_{H} H) \times^{\mathrm{l}}_{H^*} H^* \cong C \times^{\mathrm{r}}_{H^*} (H \times^{\mathrm{l}}_{H^*} H^*)$$

Applying theorem 3, we obtain the duality theorem as follows.

Theorem 4 Let H be a weak Hopf algebra, H^* its dual weak Hopf algebra. Suppose that C is a left weak H-comodule coalgebra, and that the right coaction of H^* on $H \times^{\mathrm{l}}_{H^*} H^*$ is right strongly relative co-inner. Then

$$(C \times^{\mathrm{l}}_{H} H) \times^{\mathrm{l}}_{H^*} H^* \cong C \otimes_v (H \times^{\mathrm{l}}_{H^*} H^*)$$

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量子群胚上的 smash 余积对偶定理

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摘要:研究了量子群胚上与弱模余代数和余模余代数相关的弱广义 smash 余积的对偶定理. 设 H 是弱 Hopf 代数,C 是弱左 H 余模余代数,D 是弱左 H 模余代数. 首先,给出量子群胚上的弱广义 smash 余积 $C\times_H^{\mathrm{l}}D$ 的定义,并构造其模和余模结构. 类似考虑右广义 smash 余积 $C\times_L^{\mathrm{r}}D$. 然后得到它们之间的同构. 其次,通过引入弱卷积逆,弱余内作用和强相关余内作用的概念,得到 $C\times_H^{\mathrm{r}}D$ 和 $C\otimes_v D$ 同构的充分条件,其中 $v\in\overline{\mathrm{WC}(C,H)}$,$H$ 在 D 上的余作用是右强相关余内作用. 最后,证明了量子群胚上广义 smash 余积的对偶定理:$(C\times_H^{\mathrm{l}}H)\times_{H^*}^{\mathrm{l}}H^*\cong C\otimes_v(H\times_{H^*}^{\mathrm{l}}H^*)$.

关键词:弱 Hopf 代数(量子群胚);弱广义 smash 余积;弱模余代数;弱余模余代数;弱双模余代数;对偶定理
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