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Energy decay for a class of nonlinear wave equations with a critical potential type of damping

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Abstract: The Cauchy problem for the nonlinear wave equation with a critical potential type of damping coefficient $(1+|x|)^{-1}$ and a nonlinearity $|u|^{p-1}u$ is studied. The total energy decay estimates of the global solutions are obtained by using multiplier techniques to establish identity $\frac{\mathrm{d}}{\mathrm{d}t}E(t)+F(t)=0$ and skillfully selecting $f(t)$, $g(t)$, $h(t)$ when the initial data have a compact support. Using the similar method, the Cauchy problem for the nonlinear wave equation with a critical potential type of damping coefficient $(1+|x|+t)^{-1}$ and a nonlinearity $|u|^{p-1}u$ is studied, similar solutions are obtained when the initial data have a compact support.

Key words: nonlinear wave equation; energy decay; critical potential type; nonlinear damping; self-conjugate operator

In this paper, we study the energy decay for the solution to the following Cauchy problem for the nonlinear wave equations

$$u_{tt}+c^2Au+a(x)u_t-\mu\Delta u_t+u|u|^{p-1}=0 \qquad (t,x)\in(0,\infty)\times\mathbf{R}^n \tag{1}$$

$$u(0,x)=u_0(x),\ u_t(0,x)=u_1(x) \qquad x\in\mathbf{R}^n \tag{2}$$

where $1<p<\infty\ (n\leqslant 2k)$, $1<p\leqslant\frac{n+2k}{n-2k}(n>2k)$ and c, $\mu>0$ are constants. A is a self-conjugate operator and $A\geqslant 0$, i. e. $\langle Au,u\rangle\geqslant 0$, for any $u\in D(A)=H^k$. We impose several assumptions on the initial data:

$$u_0\in H^k(\mathbf{R}^n),\ u_1\in L^2(\mathbf{R}^n)$$
$$\operatorname{supp} u_0\cup\operatorname{supp} u_1\subset\{x:\ |x|\leqslant\mathbf{R}\}$$

for some $\mathbf{R}>0$. By the method in Ref. [1], it is well known that if $a(x)\in C(\mathbf{R}^n)$, problems (1) and (2) admit a weak solution $u(t,x)$ in the class $u\in C([0,\infty);H^k(\mathbf{R}^n))\cap C^1([0,\infty);L^2(\mathbf{R}^n))$ and enjoy a finite propagation speed property:

$$u(t,x)=0 \qquad |x|>ct+\mathbf{R} \tag{3}$$

Now, let us mention several previously related results concerning the decay estimates of the total energy for the wave equations with a potential type of the damping coefficient. Mochizuki et al. [2-4] studied the Cauchy problem of the linear wave equation

$$u_{tt}-\Delta u+a(x,t)u_t=0$$

From their results, we find that $a(t,x)=O(|x|^{-1})$ (as $|x|\to+\infty$) is critical from the viewpoint of the energy decay. On the other hand, for the semilinear problem, recently, Todorova and Yordanov[5] derived the (almost) optimal decay estimates of the total energy and the other quantities of the solution to the Cauchy problem

$$u_{tt}-c^2\Delta u+a(x)u_t+|u|^{p-1}u=0 \qquad (t,x)\in(0,\infty)\times\mathbf{R}^n \tag{4}$$

$$u(0,x)=u_0(x),\ u_t(0,x)=u_1(x) \qquad x\in\mathbf{R}^n \tag{5}$$

when $u_0(x)$, $u_1(x)$ have a compact support and the potential $a(x)$ satisfies

$$\frac{b_1}{(1+|x|)^{\gamma}}\leqslant a(x)\leqslant\frac{b_2}{(1+|x|)^{\gamma}} \qquad \gamma\in[0,1)$$

where b_1 and b_2 are positive constants. It is essential that $\gamma<1$. In Ref. [6], the authors also studied the Cauchy problem for the linear equation

$$u_{tt}-\Delta u+a(x)u_t=0 \tag{6}$$

However, in Refs. [3,5], the authors did not give the decay rate of the total energy for the solution of Eqs. (4) and (5) for the critical case $\gamma=1$. Ikehata[7] pointed out that (at least) in the case when $\gamma\in[0,1)$, Eq. (6) has a kind of parabolic structure from the viewpoint of the energy decay. When the potential depends only on t, Reissig et al. [8-11] studied some decay (and non-decay) properties of solutions to the linear wave equation with time-dependent dissipation $a(t)$ in place of $a(x)$. Their methods, however, cannot be applied to problems (1) and (2) with a potential depending on the x-variable. In Ref. [12], Ikehata and Inoue studied the decay rate of the total energy of the solution of problems (4) and (5) with the critical damping $\gamma=1$ under some assumptions for $a(x)$ and c.

Motivated by the ideas of Ref. [12], we investigate the decay rate of the total energy for Cauchy problems (1) and (2) with the critical damping $\gamma=1$. Denote

$$\int_{\mathbf{R}^n}|A^{1/2}u(t,x)|^2\mathrm{d}x=\langle Au(t,x),u(t,x)\rangle$$

In this paper, we make the following assumptions:

① $1-\frac{b_0}{c}<\delta<1$;

② $1-\frac{b_0}{c+1}<\delta<1$;

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③ $0 \leqslant \delta < 1$.

where b_0 is given in Eq. (7).

Theorem 1 Assume that $a(x) \in C(\mathbf{R}^n)$ satisfies

$$a(x) \geqslant \frac{b_0}{|x| + 1} \qquad b_0 > 0 \tag{7}$$

Then the weak solution of Eqs. (1) and (2) satisfies

$$\int_{\mathbf{R}^n} (u_t(t,x)^2 + c^2 |A^{1/2}u(t,x)|^2)\,\mathrm{d}x = O(t^{-1+\delta}) \qquad t \to \infty$$

$$\int_{\mathbf{R}^n} |u(t,x)|^{p+1}\mathrm{d}x = O(t^{-1+\delta}) \qquad t \to \infty$$

for any δ satisfying ① in the case when $0 < b_0 \leqslant c$, and ③ when $c < b_0$. In addition, if $n > 2$, $q = 2n/(n - 2k + 2)$, we obtain

$$\|\nabla u\|^2_{L^q(\mathbf{R}^n)} = O(t^{-1+\delta}) \qquad t \to \infty$$

We can also deal with the Cauchy problem

$$u_{tt} + c^2 Au + a(t,x)u_t - \Delta u_t + u|u|^{p-1} = 0$$
$$(t,x) \in (0,\infty) \times \mathbf{R}^n \tag{8}$$

$$u(0,x) = u_0(x),\ u_t(0,x) = u_1(x) \qquad x \in \mathbf{R}^n \tag{9}$$

In a similar way, we obtain the following result.

Theorem 2 Assume that $a(t,x) \in C([0,\infty) \times \mathbf{R}^n)$ with $a_t(t,x) \in C([0,\infty) \times \mathbf{R}^n)$ satisfying

$$a(t,x) \geqslant \frac{b_0}{t + |x| + 1} \qquad b_0 > 0;\ a_t(t,x) \leqslant 0 \tag{10}$$

Then the weak solution $u \in C([0,\infty); H^k(\mathbf{R}^n)) \cap C^1([0,\infty); L^2(\mathbf{R}^n))$ of Eqs. (8) and (9) satisfies

$$\int_{\mathbf{R}^n} (u_t(t,x)^2 + c^2 |A^{1/2}u(t,x)|^2)\,\mathrm{d}x = O(t^{-1+\delta}) \qquad t \to \infty$$

$$\int_{\mathbf{R}^n} |u(t,x)|^{p+1}\mathrm{d}x = O(t^{-1+\delta}) \qquad t \to \infty$$

for any δ satisfying ② in the case when $0 < b_0 \leqslant c + 1$, and ③ when $c + 1 < b_0$. In addition, if $n > 2$, $q = 2n/(n - 2k + 2)$, we obtain $\|\nabla u\|^2_{L^q(\mathbf{R}^n)} = O(t^{-1+\delta})$, $t \to \infty$.

Lemma 1 Let u be the weak solution to Eqs. (1) and (2). Then it is true that

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t) + F(t) = 0$$

where

$$E(t) = \frac{1}{2}\int_{\mathbf{R}^n} \Big[f(u_t^2 + c^2|A^{1/2}u|^2) + 2guu_t + (ag - g_t)u^2 + \mu g|\nabla u|^2 + \frac{2f}{p+1}|u|^{p+1} \Big]\mathrm{d}x$$

$$F(t) = \frac{1}{2}\int_{\mathbf{R}^n}(2af - f_t - 2g)u_t^2\mathrm{d}x + \frac{c^2}{2}\int_{\mathbf{R}^n}(2g - f_t)|A^{1/2}u|^2\mathrm{d}x + \frac{1}{2}\int_{\mathbf{R}^n}(g_{tt} - ag_t)u^2\mathrm{d}x + \int_{\mathbf{R}^n}\Big(g - \frac{f_t}{p+1}\Big)|u|^{p+1}\mathrm{d}x + \mu\int_{\mathbf{R}^n} f|\nabla u_t|^2\mathrm{d}x - \frac{\mu}{2}\int_{\mathbf{R}^n} g_t|\nabla u|^2\mathrm{d}x$$

Proof By an approximation, we may assume that the solution u is sufficiently smooth and vanishes for a large x.

Now we multiply both sides of Eq. (1) by $fu_t + gu$ and integrate the resulting equation over $\mathbf{R}^n$. First, multiplying Eq. (1) by fu_t and integrating the resulting equation over $\mathbf{R}^n$, we obtain

$$\int_{\mathbf{R}^n} \Big[\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{fu_t^2}{2}\Big) - \frac{f_tu_t^2}{2} - c^2\frac{f_t|A^{1/2}u|^2}{2} + afu_t^2 + \frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{f}{p+1}|u|^{p+1}\Big) - \frac{f_t}{p+1}|u|^{p+1} - \mu fu_t\Delta u_t \Big]\mathrm{d}x + \frac{\mathrm{d}}{\mathrm{d}t}(c^2 f\langle Au, u\rangle) - \frac{c^2 f_t\langle Au, u\rangle}{2} = 0 \tag{11}$$

Similarly, multiplying Eq. (1) by gu and integrating yield

$$\int_{\mathbf{R}^n} \Big[(guu_t)_t - \frac{1}{2}(\mu g_t u^2)_t + \frac{1}{2}g_{tt}u^2 - gu_t^2 + \Big(a\frac{gu^2}{2} + \frac{\mu g|\nabla u|^2}{2}\Big)_t - \frac{g_tu^2}{2}a - \frac{g_tu^2}{2}a - \frac{g_t|\nabla u|^2}{2} + g|u|^{p+1} \Big]\mathrm{d}x + c^2 g\langle Au, u\rangle = 0 \tag{12}$$

Then, adding Eq. (11) to Eq. (12), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t) + F(t) = 0$$

Lemma 2 Assume that the smooth functions $f(t)$, $g(t)$, $h(t) > 0$ satisfy the following conditions, for $x \in \mathbf{R}^n$ and $t \geqslant t_0 > 0$,

1) $2af - f_t - 2g \geqslant 0$;
2) $2g - f_t \geqslant 0$;
3) $g_{tt} - ag_t \geqslant 0$;
4) $g - \dfrac{f_t}{p+1} \geqslant 0$;
5) $g_t \leqslant 0$;
6) $ga(x) - g_t - h(t)^{-1}g \geqslant 0$.

If u is the weak solution to Eqs. (1) and (2), then

$$\frac{1}{2}\int_{\mathbf{R}^n}(f - hg)(u_t^2 - c^2|A^{1/2}u|^2\mathrm{d}x + \mu\int_{\mathbf{R}^n} g|\nabla u|^2)\mathrm{d}x + \frac{1}{p+1}\int_{\mathbf{R}^n} f|u|^{p+1}\mathrm{d}x \leqslant E(t_0) \qquad t \geqslant t_0 > 0$$

Proof By conditions 1) to 6) and lemma 1, we have $F(t) > 0$ and $E'(t) < 0$. This gives $E(t) \leqslant E(t_0)$ for $t \geqslant t_0$. Since $h(t) > 0$, it follows that

$$|2guu_t| \leqslant h(t)gu_t^2 + h(t)^{-1}gu^2$$

Thus, we obtain

$$E(t) \geqslant \frac{1}{2}\int_{\mathbf{R}^n} \Big[(f - h(t)g)(u_t^2 + c^2|A^{1/2}u|^2)\mathrm{d}x + (ga - g_t - h(t)^{-1}g)u^2 + \mu g|\nabla u|^2 \Big]\mathrm{d}x + \frac{1}{p+1}\int_{\mathbf{R}^n} f|u|^{p+1}\mathrm{d}x$$

From condition 6), we obtain the desired estimate.

Now, we choose the functions $f(t)$, $g(t)$ and $h(t)$ as follows:

$$f(t)=(1+t)^{1-\delta},\ g(t)=\frac{1-\delta}{2}(1+t)^{-\delta},\ h(t)=t+1 \tag{13}$$

where δ is a positive constant.

Lemma 3 Let f, g and h be defined by (13). Then conditions 1) to 6) hold true for sufficiently large $t\geqslant t_0\gg 1$.

Proof By a direct calculation, we obtain

$$f_t=(1-\delta)(1+t)^{-\delta},\ g_t=-\delta\frac{1-\delta}{2}(1+t)^{-1-\delta}$$

$$g_{tt}=\delta(1+\delta)\frac{1-\delta}{2}(1+t)^{-2-\delta}$$

Obviously, $g_t(t)\leqslant 0$ and $g_{tt}(t)\geqslant 0$. This implies that conditions 3) and 5) hold.

Step 1 Using (3) and (7), we obtain

$$\begin{aligned}2a(x)f-f_t-2g&=2(1+t)^{-\delta}((1+t)a-1+\delta)\geqslant\\ &2(1+t)^{-\delta}\left(b_0\frac{1+t}{|x|+1}-1+\delta\right)\geqslant\\ &2(1+t)^{-\delta}\left(b_0\frac{1+t}{ct+R+1}-1+\delta\right)>0\end{aligned}$$

for all x with sufficiently large $t\geqslant t_0\gg 1$. Here we use assumptions ① and/or ③ which imply

$$\lim_{t\to+\infty}\left(b_0\frac{1+t}{ct+R+1}+\delta-1\right)=\frac{b_0}{c}+\delta-1>0$$

Step 2 $2g-f_t=(1-\delta)(1+t)^{-\delta}-(1-\delta)(1+t)^{-\delta}=0$.

Step 3 From $p>1$, we obtain

$$g-\frac{f_t}{p+1}=\frac{1-\delta}{2}(1+t)^{1-\delta}-\frac{1-\delta}{p+1}(1+t)^{1-\delta}>0$$

Step 4 In a similar way, we obtain

$$\begin{aligned}a(x)g-g_t-(1+t)^{-1}g&=\frac{1-\delta}{2}(1+t)^{-1-\delta}(a(1+t)-1+\delta)\geqslant\\ &\frac{1-\delta}{2}(1+t)^{-1-\delta}\left(b_0\frac{1+t}{|x|+1}-1+\delta\right)\geqslant\\ &\frac{1-\delta}{2}(1+t)^{-1-\delta}\left(b_0\frac{1+t}{ct+R+1}-1+\delta\right)>0\end{aligned}$$

for all x and large $t\geqslant t_0\gg 1$. Here we use assumptions ① and/or ③ similarly. It completes the proof of lemma 3.

Proof of Theorem 1 By lemmas 2 and 3, since

$$f(t)-h(t)g(t)=\frac{1+\delta}{2}(1+t)^{1-\delta}$$

for sufficiently large $t\geqslant t_0\gg 1$, we obtain

$$\int_{\mathbf{R}^n}(u_t^2+c^2|A^{1/2}u|^2)\,\mathrm{d}x\leqslant\frac{4}{1+\delta}E(t_0)(t+1)^{-1+\delta}\qquad t\to\infty$$

$$\int_{\mathbf{R}^n}|u|^{p+1}\mathrm{d}x\leqslant(p+1)E(t_0)(t+1)^{-1+\delta}\qquad t\to\infty$$

From the imbedding theorem $\dot H_2^k(\mathbf{R}^n)\subset\dot H_q^1(\mathbf{R}^n)$, for $q=2n/(n-2k+2)$ (see Ref. [13], here $\dot H_2^k(\mathbf{R}^n)$, $\dot H_q^1(\mathbf{R}^n)$ are the homogeneous Sobolev spaces), we have

$$\|\nabla u\|_{L^q(\mathbf{R}^n)}\leqslant C\|A^{1/2}u\|_{L^2(\mathbf{R}^n)}$$

Hence, we obtain

$$\|\nabla u\|^2_{L^q(\mathbf{R}^n)}=O(t^{-1+\delta})\qquad t\to\infty$$

for $q=2n/(n-2k+2)$. This completes the proof of theorem 1.

Proof of Theorem 2 Multiplying both sides of Eq. (8) by the same multipliers as in the proof of lemma 1, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t)+H(t)=0$$

where

$$\begin{aligned}E(t)=&\frac{1}{2}\int_{\mathbf{R}^n}\Big[f(u_t^2+c^2|A^{1/2}u|^2)+2guu_t+(ag-g_t)u^2+\\ &\mu g|\nabla u|^2+\frac{2f}{p+1}|u|^{p+1}\Big]\mathrm{d}x\\ H(t)=&\frac{1}{2}\int_{\mathbf{R}^n}(2af-f_t-2g)u_t^2\mathrm{d}x+\frac{c^2}{2}\int_{\mathbf{R}^n}(2g-f_t)|A^{1/2}u|^2\mathrm{d}x+\\ &\frac{1}{2}\int_{\mathbf{R}^n}(g_{tt}-ag_t)u^2\mathrm{d}x+\int_{\mathbf{R}^n}\left(g-\frac{f_t}{p+1}\right)|u|^{p+1}\mathrm{d}x+\\ &\mu\int_{\mathbf{R}^n}f|\nabla u_t|^2\mathrm{d}x-\frac{\mu}{2}\int_{\mathbf{R}^n}g_t|\nabla u|^2\mathrm{d}x-\int_{\mathbf{R}^n}\frac{ga_t}{2}u^2\mathrm{d}x\end{aligned}$$

Because $a_t\leqslant 0$, we can drop the last term of the right hand side of $H(t)$ and obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}E(t)+F(t)\leqslant\frac{\mathrm{d}}{\mathrm{d}t}E(t)+H(t)=0$$

Similar to lemma 2, if $f(t),g(t)$, $h(t)>0$ satisfy the conditions given in lemma 2, then we obtain

$$\begin{aligned}&\frac{1}{2}\int_{\mathbf{R}^n}(f-hg)(u_t^2+c^2|A^{1/2}u|^2\mathrm{d}x+\mu\int_{\mathbf{R}^n}g|\nabla u|^2\mathrm{d}x+\\ &\frac{1}{p+1}\int_{\mathbf{R}^n}f|u|^{p+1}\mathrm{d}x\leqslant E(t_0)\qquad t\geqslant t_0>0\end{aligned}$$

Therefore, we can use the same method of the proof of theorem 1 to show theorem 2. To finish the proof, in the following we only check conditions 1) to 6) given in lemma 2 with $a(x,t)$ in place of $a(x)$. In fact, we only need to check conditions 1), 3) and 6).

By (10), (3) and ② and/or ③, we obtain

$$\begin{aligned}2a(x,t)f-f_t-2g&=2(1+t)^{-\delta}((1+t)a-1+\delta)\geqslant\\ &2(1+t)^{-\delta}\left(b_0\frac{1+t}{t+|x|+1}-1+\delta\right)\geqslant\\ &2(1+t)^{-\delta}\left(b_0\frac{1+t}{(c+1)t+R+1}-1+\delta\right)>0\end{aligned}$$

for all x and sufficiently large $t\geqslant t_0\gg 1$. Here we just use assumptions ② and/or ③ which imply

$$\lim_{t\to+\infty}\left(b_0\frac{1+t}{(c+1)t+R+1}+\delta-1\right)=\frac{b_0}{c+1}+\delta-1>0$$

Thus condition 1) is valid. Condition 3) also holds since $a(x,t)>0$, $g_t(t)<0$, $g_{tt}(t)>0$. Similar to condition 1), we obtain

$$a(x,t)g-g_t-(1+t)^{-1}g=$$

$$\frac{1-\delta}{2}(1+t)^{-1-\delta}(a(1+t)-1+\delta) \geqslant$$
$$\frac{1-\delta}{2}(1+t)^{-1-\delta}\left(b_0\frac{1+t}{t+|x|+1}-1+\delta\right) \geqslant$$
$$\frac{1-\delta}{2}(1+t)^{-1-\delta}\left(b_0\frac{1+t}{(c+1)t+R+1}-1+\delta\right) > 0$$

for all x with large $t \geqslant t_0 \gg 1$, where we use assumptions ② and/or ③ similarly. This implies that condition 6) is true. Thus, we complete the proof of theorem 2.

Remark Theorem 1 and theorem 2 are also valid for the initial and boundary value problem with the Dirichlet and/or the Neumann boundary conditions.

References

[1] Strauss W A. *Nonlinear wave equations*[M]. Providence, RI, USA: American Mathematical Society, 1989.
[2] Mochizuki K. Scattering theory for wave equations with dissipative terms[J]. *Publ Res Inst Math Sci,* 1976, **12**(1): 383 − 390.
[3] Mochizuki K, Nakazawa H. Energy decay and asymptotic behavior of solutions to the wave equations with linear dissipation[J]. *Publ Res Inst Math Sci,* 1996, **32**(3): 401 − 414.
[4] Matsumura A. Energy decay of solutions of dissipative wave equations[J]. *Proc Japan Acad,* 1977, **53**(7): 232 − 236.
[5] Todorova G, Yordanov B. Nonlinear dissipative wave equations with potential [EB/OL]. (2007) [2009-08-10]. http://www.math.utk.edu/~todorova/ty03-06-11-3-1.pdf.
[6] Todorova G, Yordanov B. Weighted L^2-estimates for dissipative wave equations with variable coefficients[J]. *J Differential Equations,* 2009, **246**(12): 4497 − 4518.
[7] Ikehata R. Some remarks on the wave equation with potential type damping coefficients[J]. *Int J Pure Appl Math,* 2005, **21**(1): 19 − 24.
[8] Reissig M. L^p-L^q decay estimates for wave equations with time-dependent coefficients[J]. *J Nonlinear Math Phys,* 2004, **11**(4): 534 − 548.
[9] Wirth J. Wave equations with time-dependent dissipation Ⅰ. Non-effective dissipation[J]. *J Differential Equations,* 2006, **222**(2): 487 − 514.
[10] Wirth J. Wave equations with time-dependent dissipation Ⅱ. Effective dissipation[J]. *J Differential Equations,* 2007, **232**(1): 74 − 103.
[11] Yamazaki T. Asymptotic behavior for abstract wave equations with decaying dissipation[J]. *Adv Differential Equations,* 2006, **11**(4): 419 − 456.
[12] Ikehata R, Inoue Y K. Total energy decay for semilinear wave equations with a critical potential type of damping[J]. *Nonlinear Anal,* 2008, **69**(4): 1396 − 1401.
[13] Bergh J, Löfström J. *Interpolation spaces*[M]. Berlin: Springer-Verlag, 1976.

一类带有临界势型阻尼的非线性波动方程的能量衰减

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摘要:研究了带有临界势型阻尼系数 $(1+|x|)^{-1}$和非线性项 $|u|^{p-1}u$ 非线性波动方程的 Cauchy 问题. 当初始函数具有紧支集时,利用乘子法建立恒等式 $\frac{\mathrm{d}}{\mathrm{d}t}E(t)+F(t)=0$ 并巧妙地选取 $f(t)$, $g(t)$, $h(t)$ 得出整体解的总能量衰减估计. 利用类似方法研究带有临界势型阻尼系数 $(1+|x|+t)^{-1}$和非线性项 $|u|^{p-1}u$ 非线性波动方程的 Cauchy 问题, 当初始函数具有紧支集时,得到相似的结果.

关键词:非线性波动方程;能量衰减;临界势型;非线性阻尼;自共轭算子
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