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Robust UKF algorithm in SINS initial alignment

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Abstract: In the traditional unscented Kalman filter (UKF), accuracy and robustness decline when uncertain disturbances exist in the practical system. To deal with the problem, a robust UKF algorithm based on an H-infinity norm is proposed. In Krein space, a robust element is added in the simplified UKF so as to improve the algorithm. The filtering gain is adjusted by the robust element and in this way the performance of the robustness of the filtering algorithm is promoted. In the initial alignment process of the large heading misalignment angle of the strapdown inertial navigation system (SINS), comparative studies are conducted on the robust UKF and the simplified UKF. The simulation results illustrate that compared with the simplified UKF, the robust UKF is more accurate, and the estimation error of heading misalignment decreases from 16.9′ to 4.3′. In short, the robust UKF can reduce the sensitivity to the system disturbances resulting in better performance.

Key words: unscented Kalman filter (UKF); robustness; Krein space; initial alignment; large heading misalignment angle

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Initial alignment is a key technology in the strapdown inertial navigation system (SINS). With developing demands on the SINS, the linear error model based on the small misalignment angle can hardly accurately describe the system error propagation properties. The nonlinear error models based on the large misalignment angle constraint emerge continually, of which the model with a large heading misalignment angle is the most representative one [1–2].

The filtering estimation is of significant importance in the initial alignment process. The UKF is a new nonlinear filtering algorithm that has emerged in recent years. It adopts a deterministic sampling approach and uses a set of weighted sigma-points to approximate the mean and variance of the system state vector by unscented transformation (UT). This algorithm can theoretically capture at least the second-order Taylor series expansion of any nonlinear equation, and reduce the linearization error of the traditional extended Kalman filter (EKF). Meanwhile, the UKF uses the same recursive algorithm frame of “predict-correct” as the standard Kalman filter (KF). Therefore, the UKF is a research hotspot due to its advantages of high precision and simple structure [3–9]. In the SINS initial alignment with a large heading misalignment angle, a simplified UKF algorithm named the Rao-Blackwellised additive unscented Kalman filter (RBAUKF) [2,4] is specifically designed for the dynamic system with additive noise, the nonlinear state equation and the linear measurement equation. The simplified UKF does not need to augment the number of sigma-points, and it solves the prediction of the nonlinear state as well as its variance by UT, while its other steps are the same as those of the standard KF. Its computational complexity is decreased, and the accuracy is similar to that of the standard UKF.

The UKF is an extension of the standard KF, so it is still a minimum variance estimation, which requires the mathematical model and the statistical properties of the noise signals to be accurately available. In practical application, however, the uncertainties of the model and the lack of statistical information on the noise signals lead to the decrease in accuracy and stability of the UKF. In Ref. [7], on the basis of the maximum posterior estimation theory and the exponentially weighted method, the statistical properties of the unknown time-varying noises are real-timely estimated and corrected. In Ref. [8], the process noise is estimated online by filter innovation and a residual orthogonal method. In Ref. [9], the measurement update of the UKF is converted to the Huber estimate method in solving the linear regressive problem, from which a filtering algorithm on L_1 and L_2 mixed norm minimum estimation is derived. The improved UKF algorithms mentioned above are of a certain degree of robustness and adaptability when the measurement noise is polluted by Gaussian white noise. In this paper, the H-infinity norm is introduced on the basis of the RBAUKF, and a robust UKF algorithm of H-infinity suboptimal estimation is derived. The algorithm makes no assumption on the nature of the disturbance and adds the robust element in order to acquire satisfying robustness. Finally, the simulation test of SINS initial alignment with a large heading misalignment angle is processed. The results illustrate that the robust UKF is of higher robustness and precision than the standard simplified UKF.

1 Simplified UKF Algorithm

Consider the following system model:

$$\boldsymbol{X}_k = f(\boldsymbol{X}_{k-1}) + \boldsymbol{\Gamma}_{k-1}\boldsymbol{W}_{k-1}, \quad \boldsymbol{Y}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{V}_k \tag{1}$$

where $\boldsymbol{X}_k$ is the system state vector; $f(\cdot)$ is a nonlinear function; $\boldsymbol{\Gamma}_k$ is the system process noise matrix; $\boldsymbol{Y}_k$ is the the measurement vector; $\boldsymbol{H}_k$ is the measurement matrix; $\boldsymbol{W}_k$ is the system process noise vector; and $\boldsymbol{V}_k$ is the measurement noise vector. Assume that $E[\boldsymbol{V}_k]=0$, $E[\boldsymbol{W}_k]=0$, $E[\boldsymbol{V}_k\boldsymbol{V}_j^{\mathrm{T}}] = \boldsymbol{R}_k\delta_{kj}$, $E[\boldsymbol{W}_k\boldsymbol{W}_j^{\mathrm{T}}] = \boldsymbol{Q}_k\delta_{kj}$, $E[\boldsymbol{V}_k\boldsymbol{W}_j^{\mathrm{T}}]=0$, $\boldsymbol{Q}_k \geqslant 0$, $\boldsymbol{R}_k > 0$.

According to the additive noise model as Eq.(1), the state equation is nonlinear, and the measurement equation is linear. Then the simplified UKF is as follows[2,4]:

1) Initialization

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$$\hat{X}_0 = E[X_0], \quad P_0 = E[(X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^{\mathrm{T}}]$$

2) Time update

$$\chi_0 = X_{k-1}$$
$$\chi_i = [X_{k-1}]_n + \sqrt{(n+\lambda P_{k-1}} \qquad i = 1, 2, \cdots, n$$
$$\chi_i = [X_{k-1}]_n - \sqrt{(n+\lambda)P_{k-1}} \qquad i = n+1, \cdots, 2n$$
$$\chi_{k|k-1} = f(\chi_{k-1})$$
$$\chi_{k|k-1} = \sum_{i=0}^{2n} W_i^{\mathrm{m}} \chi_{i,k|k-1}$$
$$P_{xx,k|k-1} = \sum_{i=0}^{2n} W_i^{\mathrm{c}} (\chi_{i,k|k-1} - X_{k|k-1}) \cdot (\chi_{i,k|k-1} - X_{k|k-1})^{\mathrm{T}} + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^{\mathrm{T}}$$
$$W_0^{\mathrm{m}} = \frac{\lambda}{n+\lambda}$$
$$W_0^{\mathrm{c}} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$$
$$W_i^{\mathrm{m}} = W_i^{\mathrm{c}} = \frac{1}{2(n+\lambda)} \qquad i = 1, 2, \ldots, 2n$$
$$\lambda = \alpha^2(n+\kappa) - n$$

where W^{m} is the weight of the first-order statistical property, and W^{c} is the weight of the second-order statistical property. λ, α, β and κ are the scaling parameters.

3) Measurement update

$$K_k = P_{xx,k|k-1} H_k^{\mathrm{T}} (H_k P_{xx,k|k-1} H_k^{\mathrm{T}} + R_k)^{-1}$$
$$X_k = X_{k|k-1} + K_k (Y_k - H_k X_{k|k-1})$$
$$P_k = (I - K_k H_k) P_{xx,k|k-1}$$

In the recursive calculation of the simplified UKF, the sigma-points are only updated to compute the state and predict the variance, and the other steps are updated with the KF. Thus, the computation is reduced.

2 Robust UKF Algorithm

Consider the system model as Eq. (1), and make no assumption on the statistical properties of W_k and V_k. In general, some arbitrary linear combination of the state needs to be estimated,

$$Z_k = L_k X_k$$

where L_k is a given matrix.

Z_k is extended to the measurement vector V_k; thus the Krein-space system is as follows:

$$X_k = f(X_{k-1}) + \Gamma_{k-1} W_{k-1}, \qquad \begin{bmatrix} Y_k \\ Z_k \end{bmatrix} = \begin{bmatrix} H_k \\ L_k \end{bmatrix} X_k + V'_k \tag{2}$$

W_k and V'_k are regarded as finite l_2 energy noises, and we make no assumptions on their nature. Note that X_0, W_k and V'_k satisfy[10]

$$\left\langle \begin{bmatrix} X_0 \\ W_j \\ V'_j \end{bmatrix}, \begin{bmatrix} X_0 \\ W_k \\ V'_k \end{bmatrix} \right\rangle = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & I\delta_{jk} & 0 \\ 0 & 0 & R'\delta_{jk} \end{bmatrix}$$

where $R' = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$.

The H-infinity algorithm is more robust and less sensitive to parameter variations. The Riccati equation[10] needs to be altered so that it can be applied to the robust UKF algorithm in the nonlinear system as Eq.(2). $P_{xx,k|k-1}$ in the nonlinear system is obtained through UT in the simplified UKF, and it is similar to $P_{k|k-1}$ in the linear system[10]. As a result, the Riccati equation in the robust UKF is as

$$P_{xx,k} = \left(I - P_{xx,k|k-1} [H_k^{\mathrm{T}} \quad L_k^{\mathrm{T}}] R_{e,k}^{-1} \begin{bmatrix} H_k \\ L_k \end{bmatrix} \right) P_{xx,k|k-1} \tag{3}$$

with $R_{e,k} = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H_k \\ L_k \end{bmatrix} P_{xx,k|k-1} [H_k^{\mathrm{T}} \quad L_k^{\mathrm{T}}]$.

Defining $K_{\infty,k} = P_{xx,k|k-1} [H_k^{\mathrm{T}} \quad L_k^{\mathrm{T}}] R_{e,k}^{-1}$, Eq.(3) is rewritten as

$$P_{xx,k} = \left(I - K_{\infty,k} \begin{bmatrix} H_k \\ L_k \end{bmatrix} \right) P_{xx,k|k-1} \tag{4}$$

Now $K_{\infty,k}$ can be written as

$$K_{\infty,k} = [P_{xx,k|k-1} H_k^{\mathrm{T}} \quad P_{xx,k|k-1} L_k^{\mathrm{T}}] \cdot \begin{bmatrix} I + H_k P_{xx,k|k-1} H_k^{\mathrm{T}} & H_k P_{xx,k|k-1} L_k^{\mathrm{T}} \\ L_k P_{xx,k|k-1} H_k^{\mathrm{T}} & -\gamma^2 I + L_k P_{xx,k|k-1} L_k^{\mathrm{T}} \end{bmatrix}^{-1} \tag{5}$$

Theorem 1[11] Partitioned matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, where A_{11} is an invertible square matrix. Then the inverse matrix A^{-1} is as

$$A^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

where

$$B_{22} = (A_{22} - A_{21} A_{11}^{-1} A_{12})^{-1}, \quad B_{12} = -A_{11}^{-1} A_{12} B_{22}$$
$$B_{21} = -B_{22} A_{21} A_{11}^{-1}, \quad B_{11} = A_{11}^{-1} - B_{12} A_{21} A_{11}^{-1}$$

Define

$$A_{11} = I + H_k P_{xx,k|k-1} H_k^{\mathrm{T}}, \; A_{12} = H_k P_{xx,k|k-1} L_k^{\mathrm{T}}$$
$$A_{21} = L_k P_{xx,k|k-1} H_k^{\mathrm{T}}, \; A_{22} = -\gamma^2 I + L_k P_{xx,k|k-1} L_k^{\mathrm{T}}$$

According to Theorem 1, Eq. (5) can be simplified as

$$K_{\infty,k} = [P_{xx,k|k-1} H_k^{\mathrm{T}} \quad P_{xx,k|k-1} L_k^{\mathrm{T}}] \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = [P_{xx,k|k-1} H_k^{\mathrm{T}} B_{11} + P_{xx,k|k-1} L_k^{\mathrm{T}} B_{21} \quad P_{xx,k|k-1} H_k^{\mathrm{T}} B_{12} + P_{xx,k|k-1} L_k^{\mathrm{T}} B_{22}] \tag{6}$$

Defining $K_{\infty,k} = [K_Y \quad K_Z]$, K_Y is simplified as

$$K_Y = P_{xx,k|k-1} H_k^{\mathrm{T}} B_{11} + P_{xx,k|k-1} L_k^{\mathrm{T}} B_{21} = P_{xx,k|k-1} H_k^{\mathrm{T}} (I + H_k P_{xx,k|k-1} H_k^{\mathrm{T}})^{-1} + P_{xx,k|k-1} (H_k^{\mathrm{T}} A_{11}^{-1} A_{12} - L_k^{\mathrm{T}}) \cdot (A_{22} - A_{21} A_{11}^{-1} A_{12})^{-1} L_k P_{xx,k|k-1} H_k^{\mathrm{T}} (I + H_k P_{xx,k|k-1} H_k^{\mathrm{T}})^{-1} \tag{7}$$

Substituting $K_k = P_{xx,k|k-1} H_k^{\mathrm{T}} (I + H_k P_{xx,k|k-1} H_k^{\mathrm{T}})^{-1}$ into Eq. (7), Eq.(7) can be rewritten as

$$
\begin{aligned}
\boldsymbol{K}_Y = & \boldsymbol{K}_k + \boldsymbol{P}_{xx,k|k-1}(\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{A}_{11}^{-1}\boldsymbol{A}_{12} - \boldsymbol{L}_k^{\mathrm{T}}) \cdot \\
& (\boldsymbol{A}_{22} - \boldsymbol{A}_{21}\boldsymbol{A}_{11}^{-1}\boldsymbol{A}_{12})^{-1}\boldsymbol{L}_k\boldsymbol{K}_k = \\
& \{\boldsymbol{I} + (\boldsymbol{K}_k\boldsymbol{H}_k - \boldsymbol{I})\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}[-\gamma^2\boldsymbol{I} + \\
& \boldsymbol{L}_k(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}]^{-1}\boldsymbol{L}_k\}\boldsymbol{K}_k
\end{aligned} \tag{8}
$$

Eq. (8) is equivalent to

$$
\boldsymbol{K}_Y = (\boldsymbol{I} - \boldsymbol{M}_k)\boldsymbol{K}_k \tag{9}
$$

with $\boldsymbol{M}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}[-\gamma^2\boldsymbol{I} + \boldsymbol{L}_k(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k) \cdot \boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}]^{-1}\boldsymbol{L}_{\mathrm{k}}$.

Similarly, $\boldsymbol{K}_Z$ can be simplified as

$$
\boldsymbol{K}_Z = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}[-\gamma^2\boldsymbol{I} + \boldsymbol{L}_k(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}]^{-1} \tag{10}
$$

Substituting the simplified $\boldsymbol{K}_{\infty,k} = [\boldsymbol{K}_Y \quad \boldsymbol{K}_Z]$ into Eq. (4), we obtain

$$
\boldsymbol{P}_k = \boldsymbol{P}_{xx,k|k-1} - (\boldsymbol{I} - \boldsymbol{M}_k)\boldsymbol{K}_k\boldsymbol{H}_k\boldsymbol{P}_{xx,k|k-1} - \boldsymbol{M}_k\boldsymbol{P}_{xx,k|k-1} \tag{11}
$$

Finally, the robust UKF algorithm is as follows:

1) Initialization

$$
\hat{\boldsymbol{X}}_0 = E[\boldsymbol{X}_0], \ \boldsymbol{P}_0 = E[(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_0)(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_0)^{\mathrm{T}}]
$$

2) Time update

$$
\begin{aligned}
& \boldsymbol{\chi}_0 = \boldsymbol{X}_{k-1} \\
& \boldsymbol{\chi}_i = [\boldsymbol{X}_{k-1}]_n + \sqrt{(n+\lambda\boldsymbol{P}_{k-1}} \qquad i = 1, 2, \cdots, n \\
& \boldsymbol{\chi}_i = [\boldsymbol{X}_{k-1}]_n - \sqrt{(n+\lambda)\boldsymbol{P}_{k-1}} \qquad i = n+1, \cdots, 2n \\
& \boldsymbol{\chi}_{k|k-1} = f(\boldsymbol{\chi}_{k-1}) \\
& \boldsymbol{\chi}_{k|k-1} = \sum_{i=0}^{2n} W_i^{\mathrm{m}}\boldsymbol{\chi}_{i,k|k-1} \\
& \boldsymbol{P}_{xx,k|k-1} = \sum_{i=0}^{2n} W_i^{\mathrm{c}}(\boldsymbol{\chi}_{i,k|k-1} - \boldsymbol{X}_{k|k-1})(\boldsymbol{\chi}_{i,k|k-1} - \boldsymbol{X}_{k|k-1})^{\mathrm{T}} + \\
& \qquad \boldsymbol{\Gamma}_{k-1}\boldsymbol{\Gamma}_{k-1}^{\mathrm{T}}
\end{aligned}
$$

3) Measurement update

$$
\begin{aligned}
& \boldsymbol{K}_k = \boldsymbol{P}_{xx,k|k-1}\boldsymbol{H}_k^{\mathrm{T}}(\boldsymbol{H}_k\boldsymbol{P}_{xx,k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{I})^{-1} \\
& \boldsymbol{X}_k = \boldsymbol{X}_{k|k-1} + \boldsymbol{K}_k(\boldsymbol{Y}_k - \boldsymbol{H}_k\boldsymbol{X}_{k|k-1}) \\
& \boldsymbol{M}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}[-\gamma^2\boldsymbol{I} + \\
& \qquad \boldsymbol{L}_{\mathrm{k}}(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{xx,k|k-1}\boldsymbol{L}_k^{\mathrm{T}}]^{-1}\boldsymbol{L}_{\mathrm{k}} \\
& \boldsymbol{P}_k = \boldsymbol{P}_{xx,k|k-1} - (\boldsymbol{I} - \boldsymbol{M}_k)\boldsymbol{K}_k\boldsymbol{H}_k\boldsymbol{P}_{xx,k|k-1} - \boldsymbol{M}_k\boldsymbol{P}_{xx,k|k-1}
\end{aligned}
$$

Comparing the simplified UKF and the robust UKF, it is found that their essential distinction is that $\boldsymbol{P}_k$ is different. The robust element is added in the robust UKF, and the given positive number γ is adjusted so as to regulate $\boldsymbol{P}_k$, and then the filtering gain $\boldsymbol{K}_k$ is adjusted. In a word, the robust UKF adjusts $\boldsymbol{P}_k$ to improve the system robustness. The schematic of the robust UKF is shown in Fig. 1.

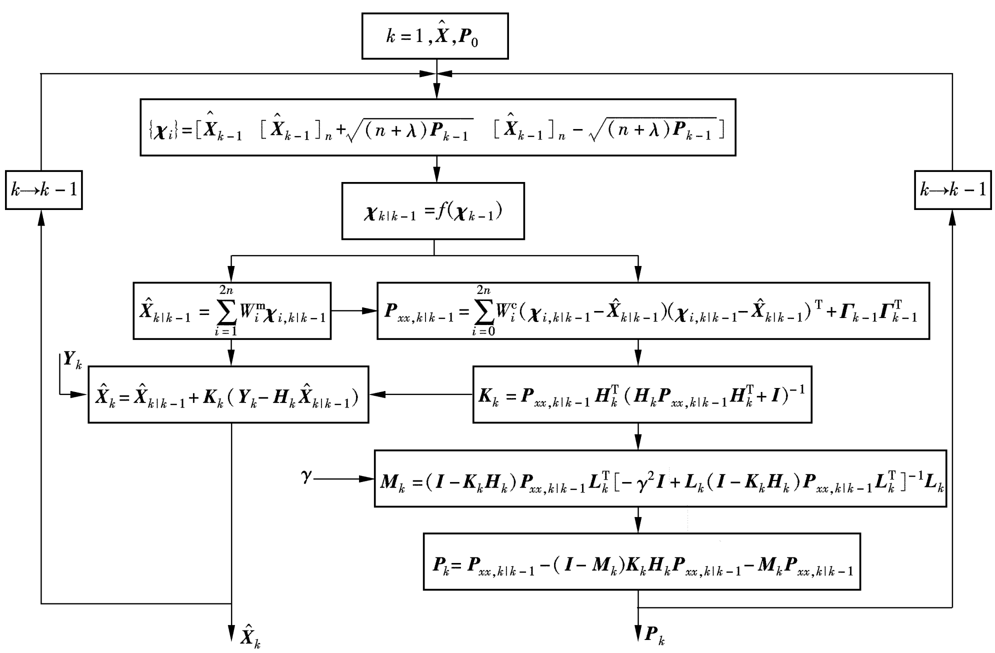

Fig. 1 Schematic of the robust UKF algorithm

3 Simulation and Results

In SINS initial alignment with a large heading misalignment angle, the navigation frame is the north-east-up (OENU) frame. The misalignment equation and the velocity-error equation can be determined as

$$
\left.
\begin{aligned}
\dot{\boldsymbol{\phi}} &= (\boldsymbol{I} - \boldsymbol{C}_n^{n'})\boldsymbol{\omega}_{in}^n + \delta\boldsymbol{\omega}_{in}^n - \boldsymbol{C}_b^n\boldsymbol{\varepsilon}^b \\
\delta\dot{\boldsymbol{V}} &= (\boldsymbol{C}_n^{n'} - \boldsymbol{I})\boldsymbol{f}^n + \delta\boldsymbol{V}^n(2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) + \\
&\quad \boldsymbol{V}^n(2\delta\boldsymbol{\omega}_{ie}^n + \delta\omega_{en}^n) + \boldsymbol{C}_b^n\nabla^b
\end{aligned}
\right\} \tag{12}
$$

where $\boldsymbol{\phi} = [\phi_e, \phi_n, \phi_u]^{\mathrm{T}}$ is the misalignment angle vector. $\boldsymbol{\varepsilon}^b = [\varepsilon_x, \varepsilon_y, \varepsilon_z]^{\mathrm{T}}$ is the gyroscope drift, which is constituted by random constant drift and random noise. $\delta\boldsymbol{V} = [\delta V_e, \delta V_n, \delta V_u]^{\mathrm{T}}$ is the velocity-error vector, $\boldsymbol{\nabla}^b = [\nabla_x, \nabla_y, \nabla_z]^{\mathrm{T}}$ is the accelerator bias, which can be treated as constant bias and random noise. $\boldsymbol{C}_n^{n'}$ is the misalignment matrix.

$$
\boldsymbol{C}_n^{n'} = \begin{bmatrix} \cos\phi_u & \sin\phi_u & -\phi_n \\ -\sin\phi_u & \cos\phi_u & \phi_e \\ \phi_n\cos\phi_u + \phi_e\sin\phi_u & \phi_n\sin\phi_u - \phi_e\cos\phi_u & 1 \end{bmatrix} \tag{13}
$$

According to SINS error equations (12), the system state vector and the measurement vector are defined as $X = [\delta v_e, \delta v_n, \phi_e, \phi_n, \phi_u, \nabla_x, \nabla_y, \varepsilon_x, \varepsilon_y, \varepsilon_z]^{\mathrm{T}}$, $Y = [\delta v_e, \delta v_n]^{\mathrm{T}}$. The system model is illustrated as

$$\dot{X} = f(X) + W, \quad Y = HX + V \tag{14}$$

where $H = [I_{2\times 2} \quad \mathbf{0}_{2\times 8}]$.

$$f(\cdot) = \begin{cases} f_e(\cos\phi_u - 1) + f_n\sin\phi_u + \delta V_e\dfrac{V_n}{R_e}\tan L + \delta V_n\left(2\omega_{ie}\sin L + \dfrac{V_e}{R_e}\tan L\right) + \nabla_e \\ -f_e\sin\phi_u + (\cos\phi_u - 1)f_n + f_u\phi_e - 2\delta V_e\left(\omega_{ie}\sin L + \dfrac{V_e}{R_e}\tan L\right) + \nabla_n \\ -(1-\cos\phi_u)\dfrac{V_n}{R_n} - \sin\phi_u\left(\omega_{ie}\cos L + \dfrac{V_e}{R_e}\right) + \phi_n\left(\omega_{ie}\sin L + \dfrac{V_e}{R_e}\tan L\right) - \dfrac{\delta V_n}{R_n} - \varepsilon_e \\ -\dfrac{V_n}{R_n}\sin\phi_u + (1-\cos\phi_u)\left(\omega_{ie}\cos L + \dfrac{V_e}{R_e}\right) - \phi_e\left(\omega_{ie}\sin L + \dfrac{V_e}{R_e}\tan L\right) + \dfrac{\delta V_e}{R_e} - \varepsilon_n \\ (\phi_n\cos\phi_u + \phi_e S\phi_u)\dfrac{V_n}{R_n} + (-\phi_n\sin\phi_u + \phi_e\cos\phi_u)\left(\omega_{ie}\cos L + \dfrac{V_e}{R_e}\right) + \dfrac{\delta V_e}{R_e}\tan L - \varepsilon_u \\ \mathbf{0}_{5\times 1} \end{cases}$$

Discretizing Eq.(14) with the 4th-order Rubge-Kutta method, we can obtain the nonlinear discrete system model. Simulation tests are carried out by the simplified UKF and the robust UKF, respectively. The simulation condition of a ship SINS initial alignment is defined as follows: the ship is in the three-degree-freedom swaying state; the initial latitude is 32°; the initial longitude is 118°; the east velocity is 0 m/s; the north velocity is 10 m/s; the initial misalignment angles ϕ_e, ϕ_n and ϕ_u are 0.5°, 0.5° and 10.0°, respectively. The gyroscope constant drift is 0.05 °/h, the accelerometer constant bias is $10^{-3}g$, and the gyroscope random noise and the accelerometer random noise are colored noise, which originate from SINS actual measurement data. In the robust UKF, the given positive number γ is 2.0 which is based on experience; the given matrix L_k is a unit matrix, and the other filter parameters are the same as those of the simplified UKF.

The estimation error curves are shown in Fig. 2 and Fig. 3. From the figures, it can be seen that the robust UKF algorithm is of more rapid convergence and higher accuracy. The estimated error of misalignment angle ϕ_u of the robust UKF algorithm is about 4.3′. In contrast, the estimated error of the simplified UKF algorithm is 16.9′ (The statistical data is from 300 to 600 s). The simulation results show that when the actual system noise is not Gaussian noise, the precision of the simplified UKF algorithm declines significantly. On the other hand, the robust UKF algorithm in this paper is of high precision and strong robustness.

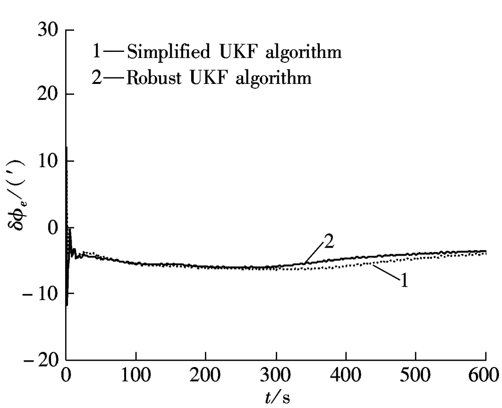

Fig. 2 Estimation error of misalignment angle ϕ_e

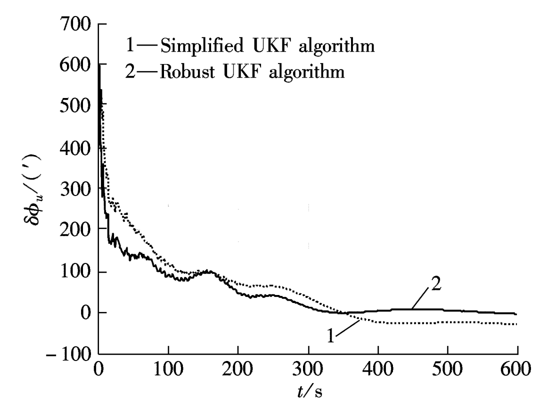

Fig. 3 Estimation error of misalignment angle ϕ_u

4 Conclusion

The accuracy and robustness of the simplified UKF decline when uncertain disturbances exist in the practical dynamic system. This paper presents a robust UKF algorithm based on the *H*-infinity norm, which adds a robust element. The robust element adjusts the filtering gain by the given positive number γ, and the system robustness is improved. The simulation results of SINS initial alignment with the large heading misalignment angle demonstrate that compared with the simplified UKF algorithm, the robust UKF algorithm in this paper can improve the accuracy, and the estimation error of the misalignment angle ϕ_u is reduced from 16.9′ to 4.3′.

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鲁棒 UKF 滤波算法在 SINS 初始对准中的应用

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摘要:针对系统存在不确定性扰动时传统 UKF 滤波算法的滤波精度和鲁棒性均下降的问题,提出了一种基于 H^{∞} 范数的鲁棒 UKF 滤波算法. 该算法在 Krein 空间内对简化 UKF 滤波算法进行改进,增加了一个鲁棒环节. 鲁棒环节通过引入给定正常数调整滤波增益从而提高滤波算法的鲁棒性能. 在 SINS 大方位失准角初始对准中对简化 UKF 滤波算法和鲁棒 UKF 滤波算法进行了对比研究. 仿真结果表明:与简化 UKF 滤波算法相比,鲁棒 UKF 滤波算法的方位失准角估计误差由 16.9′缩小到 4.3′. 鲁棒 UKF 滤波算法降低了系统对扰动的敏感度,具有更好的滤波性能.

关键词:无迹卡尔曼滤波;鲁棒性;Krein 空间;初始对准;大方位失准角

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