

Time varying congestion pricing for multi-class and multi-mode transportation system with asymmetric cost functions

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Abstract: This paper considers the problem of time varying congestion pricing to determine optimal time-varying tolls at peak periods for a queuing network with the interactions between buses and private cars. Through the combined applications of the space-time expanded network (STEN) and the conventional network equilibrium modeling techniques, a multi-class, multi-mode and multi-criteria traffic network equilibrium model is developed. Travelers of different classes have distinctive value of times (VOTs), and travelers from the same class perceive their travel disutility or generalized costs on a route according to different weights of travel time and travel costs. Moreover, the symmetric cost function model is extended to deal with the interactions between buses and private cars. It is found that there exists a uniform (anonymous) link toll pattern which can drive a multi-class, multi-mode and multi-criteria user equilibrium flow pattern to a system optimum when the system's objective function is measured in terms of money. It is also found that the marginal cost pricing models with a symmetric travel cost function do not reflect the interactions between traffic flows of different road sections, and the obtained congestion pricing toll is smaller than the real value.

Key words: time varying congestion pricing; asymmetric; multi-class; multi-mode; multi-criteria

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In most of the previous congestion pricing models, attention has been paid to the static pricing schemes, where travel demand and costs do not change with time. The study of time varying traffic congestion pricing has significance in theory and practice. Recently, there has been renewed interest in the formulation, analysis, and computation of multi-criteria traffic network equilibrium problems^[1]. In the tolled network, traffic flow distribution is forecasted using bi-criterion traffic assignment models in which users select their routes according to two criteria: travel time and travel cost (for example, toll charge). It is well known that value of time (VOT) plays an important role in the bi-criterion network equilibrium model and network performance evaluation. Conventionally, VOTs are assumed to be identi-

cal for all the users (i. e., homogeneous users). Therefore, these models cannot describe how users make trade-offs between cost and time in response to toll charges, and the welfare implications on different classes of users become difficult to assess^[2]. It is necessary to extend the traditional homogenous user assumption to transportation networks with multi-class users. Note that the term of multi-class users refers to two distinct situations in Ref. [2]. The first situation is that the flows in a transportation network are divided into different classes of vehicles or modes, each of which has an individual cost-flow function, and at the same time contributes to its own and other class cost functions in an individual way^[3]. The second situation is that all the users or drivers are assumed to use the same types of vehicles and to behave identically when using a road facility, but users differ from one another in unobservable ways such as the value they place on time. Note that multi-class hereinafter refers to the second case of users with different VOTs, while multi-mode hereinafter refers to the first case of different types of vehicles (private transport and public transit). In transportation science, the literature on congestion pricing focuses on systems with a single mode of transportation-automobiles. However, estimating the effects of congestion pricing on other forms of transportation requires models that address systems with multiple modes^[4]. In this paper, we develop such congestion pricing models and create a congestion pricing strategy under the mixed traffic that contains buses and private cars. In addition to attending to multi-class users and multi-modes, we also address the congestion pricing problem with asymmetric cost functions in this paper.

1 Space-Time Extended Network (STEN)

The STEN approach was originally proposed for a study of the dynamic maximal flow problem^[5]. Drissi-Kaitouni et al.^[6-7] brought improvements to the STEN approach by setting queues on links so that vehicles in the queue of a link share the physical space with the other vehicles on the link. We apply the STEN approach to model our joint departure time, travel mode and route choice problem in a multi-class, multi-mode and multi-criteria queuing network with schedule costs. The introduction of the STEN is omitted due to the length constraint of this paper.

2 Equilibrium with Independent Modes

In this section, we study the situation when these two transportation modes (private car or bus) are independent. To model this multi-mode problem, we assume that each flow direction on a street is represented by two links: a private car link and a bus link. Assume that the set of bus links A and the set of private car links \hat{A} have been ordered so that

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a and \hat{a} represent bus and private car links corresponding to the same physical street and the same direction of flow. This can be easily achieved by adopting two different links in the STEN for private cars and buses, respectively.

2.1 Problem description

Consider a network $G = (N, A \cup \hat{A})$, where N is the set of nodes in the network and A, \hat{A} are the set of links in the bus network and the private car network, respectively. Let the set of origin nodes be $R, R \subset N$, and destination nodes $F, F \subset N$. A typical O-D pair is represented by $rs \in R \times F$. The set of routes between the O-D pair $r-s$ is denoted by P_{rs} , $rs \in R \times F$. Note that the origin and destination nodes are assumed to be common to both the bus and the private car networks. Let the overall time horizon $[0, T]$ be divided into equally spaced discrete time periods sequentially numbered $t = 0, 1, \dots, T$. Suppose that the time horizon $[0, T]$ is chosen to be large enough so that it does not affect the results of the analysis. We divide the total number of travelers into M classes according to their respective VOTs that are well related to their income levels, and let a typical class of travelers be denoted by m . Let $\beta_m (\beta_m > 0)$ be the average VOT for travelers of class m , and the classes be ordered according to an increasing VOT, namely, $\beta_1 < \beta_2 < \dots < \beta_M$. Let $f_{p,m}^{rs}$ ($\hat{f}_{p,m}^{rs}$) denote the nonnegative flow of bus (private car) travelers of class m on path p that enters the network from origin r to destination s , and let x_a^m (\hat{x}_a^m) denote the flow of bus (private car) travelers of class m on link a (\hat{a}). Note that the flow on the bus and private car links is measured in terms of vehicular trips. The relationship between the link loads by class and the path flows is

$$x_a^m = \sum_r \sum_s \sum_p f_{p,m}^{rs} \delta_{a,p}^{rs} \quad \forall a \in A; m = 1, 2, \dots, M \quad (1)$$

$$\hat{x}_a^m = \sum_r \sum_s \sum_p \hat{f}_{p,m}^{rs} \delta_{a,p}^{rs} \quad \forall \hat{a} \in \hat{A}; m = 1, 2, \dots, M \quad (2)$$

where $\delta_{a,p}^{rs} = 1$, if route p between O-D pair $r-s$ uses link $a \in A$, and 0 otherwise. The total flow \bar{x}_a^m of both bus and private car travelers of class m on link a and \hat{a} is

$$\bar{x}_a^m = x_a^m + \hat{x}_a^m \quad \forall a \in A \cup \hat{A}; m = 1, 2, \dots, M \quad (3)$$

In addition, let \bar{x}_a denote the total flow on link a and \hat{a} ,

$$\bar{x}_a = \sum_{m=1}^M \bar{x}_a^m \quad a \in A \cup \hat{A} \quad (4)$$

Let q_m^{rs} (\hat{q}_m^{rs}) denote the demand of bus (private car) travelers of class m for O-D pair $r-s$, thus the amount of the private car flow is calculated by \hat{q}_m^{rs}/γ_p , $m = 1, 2, \dots, M$. The factor γ_p in the function represents the average occupancy rate for private cars. Likewise, the amount of the bus flow is given by q_m^{rs}/γ_b , $m = 1, 2, \dots, M$. The factor γ_b is the average occupancy rate for buses. Therefore, one must obtain

$$\frac{q_m^{rs}}{\gamma_b} = \sum_{p \in P_{rs}} f_{p,m}^{rs} \quad \forall r, s; m = 1, 2, \dots, M \quad (5)$$

$$\frac{\hat{q}_m^{rs}}{\gamma_p} = \sum_{p \in P_{rs}} \hat{f}_{p,m}^{rs} \quad \forall r, s; m = 1, 2, \dots, M \quad (6)$$

Let \bar{q}_m^{rs} denote the total demand of both bus and private car

travelers of class m for O-D pair $r-s$ and \bar{q}_m^{rs} be assumed to be fixed and given as

$$\bar{q}_m^{rs} = q_m^{rs} + \hat{q}_m^{rs} \quad \forall r, s; m = 1, 2, \dots, M \quad (7)$$

2.2 Link disutility functions and route travel cost

We are now ready to describe the disutility functions associated with the links in the bus (private car) network. We know that in the STEN, the total time spent by a vehicle on link a is equal to the sum of a fixed travel time τ_a and a waiting time spent in the exit queue of the link. Thus, we assume a travel time function $t_a(\hat{t}_a)$ associated with each link $a(\hat{a})$ in the bus (private car) network is

$$t_a = \tau_a + \tau_a^{\text{wait}} \quad \forall a \in A \quad (8)$$

$$\hat{t}_a = \tau_a + \tau_a^{\text{wait}} \quad \forall \hat{a} \in \hat{A} \quad (9)$$

where τ_a is the free (uncongested) travel time on link $a(\hat{a})$; τ_a^{wait} is the waiting time on link $a(\hat{a})$. And a travel cost function $v_a(\hat{v}_a)$ associated with each link $a(\hat{a})$ in the bus network (private car network) is

$$v_a = v_a(x_a) \quad \forall a \in A \quad (10)$$

$$\hat{v}_a = \hat{v}_a(\hat{x}_a) \quad \forall \hat{a} \in \hat{A} \quad (11)$$

where x_a, \hat{x}_a denote the bus and private car flows on link $a(a \in A)$ and $\hat{a}(\hat{a} \in \hat{A})$, respectively. It should be pointed out that in this paper we take multi-criteria traffic network equilibrium problems into consideration, and thus we allow for the general situation in which the travel cost depends on the link load, so it can effectively overcome the situation that in the STEN the link travel time is constant and link disutilities are not dependent on the link load pattern. We assume that each of a bus (private car) traveler of class m has his/her own perception of the trade-off between travel time and travel cost, which are represented by the nonnegative weights φ_a^m ($\hat{\varphi}_a^m$) and $1 - \varphi_a^m$ ($1 - \hat{\varphi}_a^m$). The weights φ_a^m ($\hat{\varphi}_a^m$) and $1 - \varphi_a^m$ ($1 - \hat{\varphi}_a^m$) are link-dependent and, thus, can incorporate such link-dependent factors as safety, view and comfort. These link-dependent weights make travel decision-making more flexible than weights that are identical for the travel time and for the travel cost on all the links for a given class. We can now construct the travel disutility of bus (private car) travelers of class m associated with link $a(\hat{a})$, and denoted by c_a^m (\hat{c}_a^m),

$$c_a^m(x_a) = \beta_m \varphi_a^m t_a + (1 - \varphi_a^m) v_a(x_a) \quad \forall m, \forall a \quad (12)$$

$$\hat{c}_a^m(\hat{x}_a) = \beta_m \hat{\varphi}_a^m \hat{t}_a + (1 - \hat{\varphi}_a^m) \hat{v}_a(\hat{x}_a) \quad \forall m, \forall \hat{a} \quad (13)$$

Now, we establish the route travel cost by O-D pair. In this paper, the travel cost of bus (private car) travelers of class m is given by

$$c_{p,m}^{rs}(f) = \sum_{a \in A} \delta_{a,p}^{rs} c_a^m \quad p \in P_{rs}; rs \in R \times F; m = 1, 2, \dots, M \quad (14)$$

$$\hat{c}_{p,m}^{rs}(\hat{f}) = \sum_{\hat{a} \in \hat{A}} \delta_{\hat{a},p}^{rs} \hat{c}_a^m \quad p \in P_{rs}; rs \in R \times F; m = 1, 2, \dots, M \quad (15)$$

We assume that travelers' choices of departure-time and

route are inter-related decisions. The route travel cost associated with the schedule delay costs of arriving times at destinations is formulated as below, following the approach in the previous related works^[8-11].

$$\varphi_{p,m}^{rs}(f,t) = c_{p,m}^{rs}(f) + \pi_{p,m}^s(t) = \sum_{a \in A} \delta_{a,p}^{rs} c_a^m + \pi_{p,m}^s(t) \quad (16)$$

$$p \in P_{rs}; rs \in R \times F; m = 1, 2, \dots, M; t = 0, 1, \dots, T$$

$$\hat{\varphi}_{p,m}^{rs}(\hat{f},t) = \hat{c}_{p,m}^{rs}(\hat{f}) + \pi_{p,m}^s(t) = \sum_{a \in \hat{A}} \delta_{a,p}^{rs} \hat{c}_a^m + \pi_{p,m}^s(t) \quad (17)$$

$$p \in P_{rs}; rs \in R \times F; m = 1, 2, \dots, M; t = 0, 1, \dots, T$$

where $\varphi_{p,m}^{rs}(f,t)$ ($\hat{\varphi}_{p,m}^{rs}(\hat{f},t)$) denotes the route travel cost of bus (private car) travelers of class m departing at interval t from origin r via route p toward destination s . $\pi_{p,m}^s(t)$ describes the penalty which will be incurred if one arrives early or late and,

$$\pi_{p,m}^s(t) = \begin{cases} \alpha_{s,m} \{ (t_{s,m}^* - \Delta_{s,m}) - [t + T_{p,m}^{rs}(t)] \} & t + T_{p,m}^{rs}(t) < t_{s,m}^* - \Delta_{s,m} \\ \vartheta_{s,m} \{ [t + T_{p,m}^{rs}(t)] - (t_{s,m}^* + \Delta_{s,m}) \} & t + T_{p,m}^{rs}(t) > t_{s,m}^* + \Delta_{s,m} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

where $T_{p,m}^{rs}(t)$ is the travel time incurred by a traveler of class m who departs from origin $r \in R$, in time period $t, t = 0, 1, \dots, T$, and travels on route $p \in P_{rs}$ to destination $s \in F$ by bus or private car. $\alpha_{s,m}$ ($\vartheta_{s,m}$) is the traveler of class m 's unit cost of schedule delay early (late) at the destination $s \in F$. $[t_{s,m}^* - \Delta_{s,m}, t_{s,m}^* + \Delta_{s,m}]$ is the window of arrival times at destination s without the schedule delay penalty for the traveler of class m , and $t_{s,m}^*$ is the middle point of the time window and can represent the official work starting time in destination s for the traveler of class m . In accordance with the previous empirical results^[12], we assume that $\vartheta_{s,m} > \beta_m > \alpha_{s,m}, m = 1, 2, \dots, M$. Up to now, we have presented the route travel cost of a traveler, in determining when to depart from home and by which transportation mode to make a trip. Each traveler trades off travel time, travel cost, schedule delay and congestion toll (if any) by minimizing his/her generalized travel costs.

2.3 System optimum model

We consider a system problem that maximizes a partial net benefit of travelers who make trips using buses and derive the optimal tolls over the STEN $G = (N, A \cup \hat{A})$. We can obtain the minimization problem:

$$\min \sum_{a \in A} \sum_{m=1}^M c_a^m x_a^m \gamma_b + \sum_{\hat{a} \in \hat{A}} \sum_{m=1}^M \hat{c}_a^m \hat{x}_a^m \gamma_p - \sum_{rs \in R \times F} \sum_{m=1}^M \int_0^{q_m^{rs}} \frac{1}{\eta} \left[\ln \left(\frac{q_m^{rs} - z}{z} \right) - (\varepsilon_m - \hat{\varepsilon}_m) \right] dz \quad (19)$$

s. t.

$$\sum_{t=0}^T q_m^{rs}(t) = q_m^{rs} \quad rs \in R \times F; m = 1, 2, \dots, M \quad (20)$$

$$\sum_{t=0}^T \hat{q}_m^{rs}(t) = \bar{q}_m^{rs} - q_m^{rs} \quad rs \in R \times F; m = 1, 2, \dots, M \quad (21)$$

$$\sum_{b \in P_{rs}^t} f_{p,m}^{rs}(t) = \frac{q_m^{rs}(t)}{\gamma_b} \quad rs \in R \times F; t = 0, 1, \dots, T; m = 1, 2, \dots, M \quad (22)$$

$$\sum_{b \in P_{rs}^t} \hat{f}_{p,m}^{rs}(t) = \frac{\hat{q}_m^{rs}(t)}{\gamma_p} \quad rs \in R \times F; t = 0, 1, \dots, T; m = 1, 2, \dots, M \quad (23)$$

$$x_b \leq S_a \quad b \in A_a^e; a \in A \quad (24)$$

$$\hat{x}_b \leq \hat{S}_a \quad \hat{b} \in \hat{A}_a^e; \hat{a} \in \hat{A} \quad (25)$$

$$f_{p,m}^{rs} \geq 0 \quad p \in P_{rs}; rs \in R \times F; m = 1, 2, \dots, M \quad (26)$$

$$\hat{f}_{p,m}^{rs} \geq 0 \quad p \in P_{rs}; rs \in R \times F; m = 1, 2, \dots, M \quad (27)$$

where P_{rs}^t is the set of paths available between $rs \in R \times F$ during time period $t, t = 0, 1, \dots, T$, in STEN. S_a (\hat{S}_a) is the link capacity. A_a^e (\hat{A}_a^e) is the set of exit links in STEN corresponding to the link $a \in A$ ($\hat{a} \in \hat{A}$) of the original bus (private car) network. $\varepsilon_m, \hat{\varepsilon}_m, \eta$ are the parameters.

2.4 Externality and congestion pricing

From the Kuhn-Tucker condition, we can obtain that

$$\bar{c}_{p,m}^{rs}(t) = \sum_{b \in A} \delta_{b,p}^{rs} \bar{c}_b^m \quad p \in P_{rs}^t; rs \in R \times F; t = 0, 1, \dots, T; m = 1, 2, \dots, M$$

$$\bar{c}_b^m = \begin{cases} c_b^m + \sum_{m=1}^M \left\{ (1 - \varphi_a^m) x_b^m \frac{dv_b(x_b)}{dx_b} \right\} + \lambda_b & b \in A_a^e; a \in A \\ c_b^m + \sum_{m=1}^M \left\{ (1 - \varphi_a^m) x_b^m \frac{dv_b(x_b)}{dx_b} \right\} & \text{otherwise} \end{cases} \quad (28)$$

$$\hat{\bar{c}}_{p,m}^{rs}(t) = \sum_{b \in \hat{A}} \delta_{b,p}^{rs} \hat{\bar{c}}_b^m \quad p \in P_{rs}^t; rs \in R \times F; t = 0, 1, \dots, T; m = 1, 2, \dots, M$$

$$\hat{\bar{c}}_b^m = \begin{cases} \hat{c}_b^m + \sum_{m=1}^M \left\{ (1 - \hat{\varphi}_a^m) \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b)}{d\hat{x}_b} \right\} + \hat{\lambda}_b & \hat{b} \in \hat{A}_a^e; \hat{a} \in \hat{A} \\ \hat{c}_b^m + \sum_{m=1}^M \left\{ (1 - \hat{\varphi}_a^m) \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b)}{d\hat{x}_b} \right\} & \text{otherwise} \end{cases} \quad (29)$$

$\lambda_b, \hat{\lambda}_b$ are the Lagrange multipliers associated with the link exit capacity constraints (24) and (25), respectively. In

Eqs. (28) and (29), $\sum_{m=1}^M \left\{ (1 - \varphi_a^m) x_b^m \frac{dv_b(x_b)}{dx_b} \right\}$ and $\sum_{m=1}^M \left\{ (1 - \hat{\varphi}_a^m) \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b)}{d\hat{x}_b} \right\}$ represent the marginal-cost toll evaluated at

traffic flow $x_b, b \in A_a^e, a \in A$ and $\hat{x}_b, \hat{b} \in \hat{A}_a^e, \hat{a} \in \hat{A}$ for bus and private car travelers, respectively. $\lambda_a(t), a \in A, t = 0, 1, \dots, T$ and $\hat{\lambda}_a(t), \hat{a} \in \hat{A}, t = 0, 1, \dots, T$ denote queue delay occurred in the bus network and the private car network. The optimal toll to be charged at each exit link for a system optimization should be equal to the user externality. In view of the above, we know, $\sum_{m=1}^M \left\{ (1 - \varphi_a^m) x_b^m \frac{dv_b(x_b)}{dx_b} \right\} + \lambda_b$ or

$\sum_{m=1}^M \left\{ (1 - \hat{\varphi}_a^m) \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b)}{d\hat{x}_b} \right\} + \hat{\lambda}_b$ is nothing but the optimal toll to be charged for each original link in the bus and private car network during each time period. Then we prove that there exist uniform link tolls across all the individuals that can support a cost-based multi-class and multi-mode user equilibrium flow pattern as a system optimum.

$$\zeta_b = \sum_{m=1}^M \left\{ (1 - \varphi_b^m) x_b^m \frac{dv_b(x_b)}{dx_b} \right\} + \lambda_b = \left(\sum_{m=1}^M (1 - \varphi_b^m) \frac{x_b^m}{x_b} \right) \cdot \left(x_b \frac{dv_b(x_b)}{dx_b} \right) + \lambda_b \quad b \in A_a^e; a \in A \quad (30)$$

Clearly, in link toll ζ_b for the bus travelers, the queue delay λ_b and $x_b \frac{dv_b(x_b)}{dx_b}$ are identical for all classes. The resultant toll charge is uniform and equal to this common travel cost externality multiplied by the average $(1 - \varphi_b^m)$ of all the travelers traversing the link:

$$(1 - \varphi_b^m)_{\text{bus, ave}} = \sum_{m=1}^M (1 - \varphi_b^m) \frac{x_b^m}{x_b} \quad b \in A_a^e; a \in A \quad (31)$$

The mean $(1 - \varphi_b^m)_{\text{bus, ave}}$ is link dependent and is the arithmetic mean of the $(1 - \varphi_b^m)$ of all the bus travelers using link $a \in A$. A similar conclusion can be obtained for private car travelers. By far we find that there exist uniform link

tolls across all the individuals that can support a cost-based multi-class and multi-mode user equilibrium flow pattern as a system optimum. Note that the link tolls are not unique due to the non-uniqueness of link flows by class^[2].

3 Equilibrium with Interdependent Modes

In section 2, we assume that no interactions exist between the bus links and the private car links. This assumption is not generally applicable, so in this section, we extend the model proposed in section 2 to cases in which the travel cost/delay of either mode depends on the flows of both modes. It is known that there is no known mathematical program, the solution of which is the equilibrium flow pattern when the link-travel-cost Jacobian is assumed to be asymmetric. In this section, we first give the generalized travel cost of bus (private car) travelers under the assumption that the link-travel-cost Jacobian is asymmetric, and then a direct externality and congestion pricing is given.

3.1 Generalized travel cost

We assume that the travel disutility function satisfies the following two conditions: 1) The travel disutility on a link is a strictly increasing function of the flow on that link; 2) The main dependence of a link's travel disutility is on its own flow. In this section, the link-travel-cost Jacobian is assumed to be asymmetric, so the generalized travel cost $\bar{c}_{p,m}^{rs}(t)$ and $\bar{\hat{c}}_{p,m}^{rs}(t)$ need to be modified and given below:

$$\begin{aligned} \bar{c}_{p,m}^{rs}(t) &= \sum_{b \in A} \delta_{b,p}^{rs} \bar{c}_b^m(x_b, \hat{x}_b) \quad p \in P_{rs}^t; rs \in R \times F; t = 0, 1, \dots, T; m = 1, 2, \dots, M \\ \bar{c}_b^m &= \begin{cases} c_b^m(x_b, \hat{x}_b) + \sum_{m=1}^M \left\{ (1 - \varphi_a^m) \left[x_b^m \frac{dv_b(x_b, \hat{x}_b)}{dx_b} + \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b, x_b)}{d\hat{x}_b} \right] \right\} + \lambda_b & b \in A_a^e \cup \hat{A}_a^e; a \in A; \hat{a} \in \hat{A} \\ c_b^m(x_b, \hat{x}_b) + \sum_{m=1}^M \left\{ (1 - \varphi_a^m) \left[x_b^m \frac{dv_b(x_b, \hat{x}_b)}{dx_b} + \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b, x_b)}{d\hat{x}_b} \right] \right\} & \text{otherwise} \end{cases} \quad (32) \end{aligned}$$

$$\begin{aligned} \bar{\hat{c}}_{p,m}^{rs}(t) &= \sum_{b \in A} \delta_{b,p}^{rs} \bar{\hat{c}}_b^m(\hat{x}_b, x_b) \quad p \in P_{rs}^t; rs \in R \times F; t = 0, 1, \dots, T; m = 1, 2, \dots, M \\ \bar{\hat{c}}_b^m &= \begin{cases} \hat{c}_b^m(\hat{x}_b, x_b) + \sum_{m=1}^M \left\{ (1 - \hat{\varphi}_a^m) \left[\hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b, x_b)}{d\hat{x}_b} + x_b^m \frac{dv_b(x_b, \hat{x}_b)}{dx_b} \right] \right\} + \hat{\lambda}_b & b \in A_a^e \cup \hat{A}_a^e; a \in A; \hat{a} \in \hat{A} \\ \hat{c}_b^m(\hat{x}_b, x_b) + \sum_{m=1}^M \left\{ (1 - \hat{\varphi}_a^m) \left[\hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b, x_b)}{d\hat{x}_b} + x_b^m \frac{dv_b(x_b, \hat{x}_b)}{dx_b} \right] \right\} & \text{otherwise} \end{cases} \quad (33) \end{aligned}$$

3.2 Externality and congestion pricing

It should be noted that when considering the interactions between the bus and the private car modes, the optimal link toll will be different from ζ_b in Eq. (30). When the Jacobian is assumed to be asymmetric, the optimal link toll for the bus (private car) travelers not only contains the marginal-

cost toll that a marginal bus (private car) traveler imposes on other bus (private car) travelers already traveling on the link $a \in A (\hat{a} \in \hat{A})$, but also the marginal-cost toll that a marginal bus (private car) traveler imposes on other private car (bus) travelers already traveling on the link $\hat{a} \in \hat{A} (a \in A)$. Thus, we can obtain

$$\zeta_b = \sum_{m=1}^M \left\{ (1 - \varphi_b^m) x_b^m \frac{dv_b(x_b, \hat{x}_b)}{dx_b} + (1 - \hat{\varphi}_b^m) \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b, x_b)}{d\hat{x}_b} \right\} + \lambda_b \quad b \in A_a^e \cup \hat{A}_a^e; a \in A; \hat{a} \in \hat{A} \quad (34)$$

$$\hat{\zeta}_b = \sum_{m=1}^M \left\{ (1 - \hat{\varphi}_b^m) \hat{x}_b^m \frac{d\hat{v}_b(\hat{x}_b, x_b)}{d\hat{x}_b} + (1 - \varphi_b^m) x_b^m \frac{dv_b(x_b, \hat{x}_b)}{dx_b} \right\} + \hat{\lambda}_b \quad b \in A_a^e \cup \hat{A}_a^e; a \in A; \hat{a} \in \hat{A} \quad (35)$$

where $\zeta_b(\hat{\zeta}_b)$ is the link toll for the bus (private car) travelers. Note that the same conclusions can be obtained as in section 2.4. For the bus (private car) travelers with differ-

ent VOTs, the link tolls $\zeta_b(\hat{\zeta}_b)$ are the same. Moreover, $\zeta_b(\hat{\zeta}_b)$ are also not unique due to the non-uniqueness of link flows by class^[2].

4 Numerical Example

The model proposed in this paper can be actually regarded as a multi-class and multi-mode network equilibrium problem with capacity constraints. Thus, the model can be solved by the inner penalty function method and the streamlined diagonalization algorithm. Fig. 1 shows a network consisting of four nodes and six directed links, and link 6 and link 5 represent bus and private car links corresponding to the same physical street and the same direction of the flow. There are two classes of travelers with different values of time. There are two O-D pairs for each class, one is from nodes 1 to 4 and the other one from nodes 2 to 4. The demand and parameters for each class of travelers are given in Tab. 1. The bottleneck capacities, link free travel times and link travel cost function are presented in Tab. 2, where the large number “9999” means no capacity constraint and the number “0” means no travel cost on this link. Note that links 1 to 5 are used for private cars, and link 6 is used for buses. Link 5 and link 6 are interdependent, that is we take interactions between the bus links and the private car links into consideration. Link 2 and link 4 are free of charge and the other links have time-varying tolls. For simplicity, we assume φ_a^m is identical for all the links and all the travelers of class m , $m = 1, 2$, and $\gamma_p = 1$, $\gamma_b = 1$. The preferred work start time and schedule delay parameters and costs are presented in Tab. 3.

Fig. 2 to Fig. 4 depict the departure rates, the exit flow rates and the time-varying tolls, respectively, for the situation that private cars and buses are interdependent. Generally speaking, for the same O-D pair, travelers of class 2 will

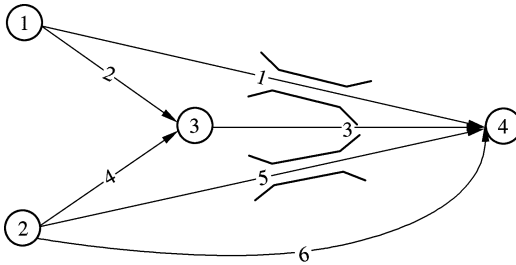


Fig. 1 General network with two O-D pairs and two bottlenecks

Tab. 1 Parameters for two classes of travelers

Class	Travel demand		$\beta_m/$ (yuan · period ⁻¹)	Parameters				
	1 to 4	2 to 4		φ_a^m	$1 - \varphi_a^m$	ε_m	$\hat{\varepsilon}_m$	$1/\eta$
1	200	400	2	0.55	0.45	-0.3	-1.25	0.1
2	100	350	1	0.5	0.5	-0.2	-1.05	0.1

Tab. 2 Link free travel time, link travel cost function and bottleneck capacity

Link	Free travel time	Link travel cost function	Capacity
1	2	$0.02x + 2$	60
2	1	0	9999
3	2	$0.01x + 1.5$	9999
4	2	0	9999
5	3	$0.0125x + 1 + 0.005\hat{x}$	130
6	4	$0.01\hat{x} + 1 + 0.003x$	9999

Tab. 3 Preferred work start time and schedule delay cost for two classes of travelers

Class	Preferred work start time	Early arrival penalty	Late arrival penalty
1	7 to 8	$c_{4,1}^*(t) = 0.45 \times (7 - t)$ for $t < 7$	$c_{4,1}^*(t) = 0.95 \times (t - 8)$ for $t > 8$
2	7 to 8	$c_{4,2}^*(t) = 0.225 \times (7 - t)$ for $t < 7$	$c_{4,2}^*(t) = 0.475 \times (t - 8)$ for $t > 8$

choose to depart earlier or put off a departure to avoid the high tolls during peak periods, while most of the travelers of class 1 will choose to travel during peak periods and care nothing about the high tolls. No travelers travel from node 2 to node 4 by link 4 and link 3, and no travelers of class 1 travel from node 1 to node 4 by link 2 and link 3 because of the high cost.

In consideration of the interaction of buses and private cars, the exit flow rates and tolls at link 5 and link 6 in Fig. 3 and Fig. 4 are obviously different from the curves in the independent situation. As a result of the influence of the buses, there is an undulation of the tolls at link 5 for private cars. In the same way, due to the influence of private cars, the tolls at link 6 for buses do not monotonously increase with the bus

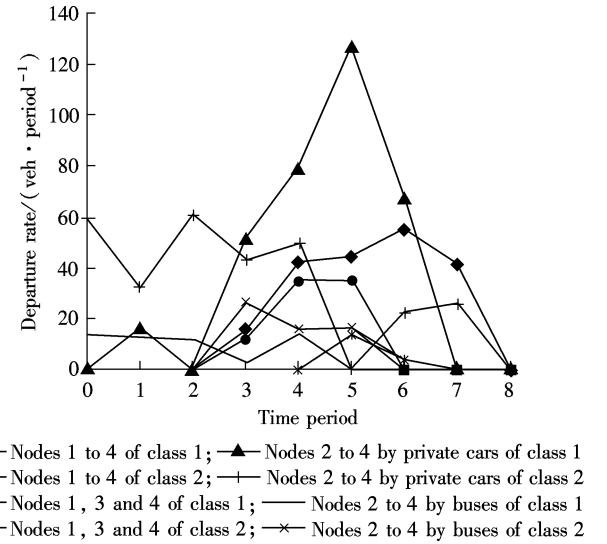


Fig. 2 Departure rate of two classes of travelers between two O-D pairs for interdependent situation

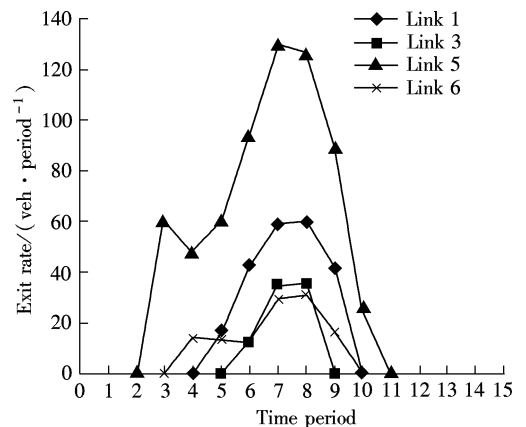


Fig. 3 Exit flow rate at the four toll links for interdependent situation

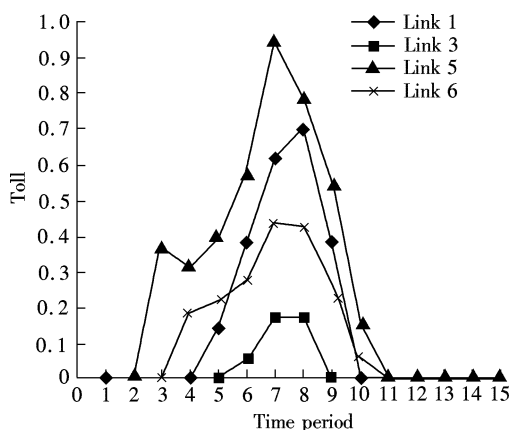


Fig. 4 Tolls for the four links over time period for interdependent situation

flow. Moreover, though there is no bus on link 6 since time 10, there still exist tolls until time 11. Note that the tolls for travelers of different classes are the same and this proves the conclusion we deduced in section 3.2. Although the transit service provider should adaptively adjust bus frequencies according to the time-varying tolls in the short-term, in the long-term the bus frequencies will be fixed values and in deciding when to depart from home and by which transportation mode to make a trip, travelers with different VOTs trade off among travel time, travel cost, schedule delay and congestion toll (if any) based on his/her self-situation by minimizing his/her generalized travel cost.

5 Conclusion

In this paper, we study the time varying congestion pricing for multi-class, multi-mode and multi-criteria transportation systems with asymmetric cost functions. In order to overcome the shortcomings that in the STEN the link travel time is constant and the link disutility is not dependent on the link load pattern, we take multi-criteria problems into consideration. Moreover, we find that marginal cost pricing models with symmetric travel cost functions only reflect the external costs that new users impose on the users who are already in the same road, but ignore the external costs that

new users impose on the users who are on a nearby road. It does not reflect the interactions between traffic flows of different road sections, and the obtained congestion pricing toll is smaller than the real value.

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阻抗函数不对称下多用户类型和多模式时变拥挤收费模型

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摘要:研究了公交车和私家车相互作用下的排队网络在高峰时段随时间变化的最优拥挤收费问题. 将时空拓展网络(STEN)与传统的网络平衡模型技术相结合,建立了多用户类型、多模式和多准则交通网络平衡模型. 不同类型的用户具有不同的时间价值(VOTs),同一类用户根据出行时间和成本的不同权重确定出行负效用或一般出行成本. 此外,将对称成本函数模型进行拓展,考虑公交车和私家车间的相互作用. 发现当系统目标函数采用费用作为计量单位时,存在统一的路段收费模式,使多用户类型、多模式和多准则用户平衡流转变为系统最优流模式. 此外,发现阻抗函数对称的边际成本收费模型不能反应不同路段上交通流的相互作用,得到的收费值偏小.

关键词:时变拥挤收费; 不对称; 多用户类型; 多模式; 多准则

中图分类号:U491.2