

# Effect of quality uncertainty of parts on performance of reprocessing system in remanufacturing environment

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**Abstract:** Aimed at the problem of stochastic routings for reprocessing operations and highly variable processing times, an open queuing network is utilized to model a typical reprocessing system. In the model, each server is subject to breakdown and has a finite buffer capacity, while repair times, breakdown times and service time follow an exponential distribution. Based on the decomposition principle and the expansion methodology, an approximation analytical algorithm is proposed to calculate the mean reprocessing time, the throughput of each server and other parameters of the processing system. Then an approach to determining the quality of disassembled parts is suggested, on the basis of which the effect of parts quality on the performance of the reprocessing system is investigated. Numerical examples show that there is a negative correlation between quality of parts and their mean reprocessing time. Furthermore, marginal reprocessing time of the parts decrease with the drop in their quality.

**Key words:** remanufacturing; uncertainty; reprocessing system; open queuing network

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The problem of resource and environmental conservation has received considerable attention in recent decades. In this situation, cyclic economy is advocated around the world. As a substantial option of cyclic economy, remanufacturing has been considered as the best way because it can maximize the utilization of potentials of value recovery from the used products. Remanufacturing is an industrial process whereby worn-out products, referred to as cores, are restored to like new condition<sup>[1-2]</sup>. During this process, the core passes through a number of remanufacturing operations, e. g. inspection, disassembly, reprocessing, reassembly, and testing to ensure it meets the desired product standards. Compared with the traditional manufacturing environment of the same products, there are several distinct characteristics between remanufacturing and manufacturing, which add different kinds of complexity and uncertainty in the remanufacturing processes<sup>[3]</sup>. Although the problem of stochastic routing for reprocessing operations and highly variable reprocessing time is a complicated problem among the several characteristics, there has been no paper about it until now<sup>[4-6]</sup>. To overcome this inefficiency, this paper, based on the analysis of the quality uncertainty of disassembled parts and the OQN model of a typical reprocessing system, attempts to investigate the impact of quality uncertainty of

disassembled parts on the performance of the reprocessing system.

## 1 Analysis of Quality Uncertainty of Parts

Stochastic routings and highly uncertain reprocessing times are a primary concern at the operational level in remanufacturing environments. Stochastic routings are a reflection of the uncertain condition of cores. Due to different reliability lives, the usage model and the return model of products, and the quality of returned cores is uncertain, which generally results in the quality uncertainty of parts disassembled from the cores. A part will have a maximum set of reprocesses that should be performed to restore the part to the desired specifications. However, these routings represent a worst-case scenario, and the majority of parts will only require a sub-set of these reprocessing routings. Highly variable reprocessing times are also a function of the quality uncertainty of parts for reprocessing. Parts with uncertain qualities require uncertain reprocessing operations, that is, uncertain routings, which deservedly imply uncertain reprocessing times. Shifting bottlenecks are common in this environment because parts disassembled from cores will vary from unit to unit, i. e., highly variable reprocessing time and stochastic routings. This makes resource planning, estimating material flow times, and material planning significantly difficult<sup>[7-8]</sup>.

In order to investigate the effects of quality uncertainty of disassembled parts on the performance of the reprocessing system, we define the quality of disassembled parts  $q$  as

$$q = \frac{1}{\sum_S \left[ \left( \prod_{i < j, i, j \in S} p_{ij} \right) v_{ij} \right]} \quad (1)$$

where  $p_{ij}$  is the probability that parts leaving node  $i$  join node  $j$ ;  $v_{ij}$  denotes the weight of routings; and  $S$  represents the number of paths in parallel after a split node.

## 2 OQN Model of Reprocessing Systems

Open queuing networks (OQN) are a popular type of queuing networks used for modeling production systems. Here, an OQN with finite buffers and unreliable servers is utilized to model a typical reprocessing system which can be modeled as a collection of various service areas where parts arrive at different rates and demand services with unequal times.

### 2.1 Model design

In this paper, we analyze a sort of part and assume that interval times for arrivals are exponentially distributed with rates  $\lambda_{ar}$ . There is one server at each workstation and the finite buffer capacity of each server is represented by  $B_i$ . The

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service rate  $\mu_i$  for each operation is exponentially distributed and the service obeys a first-come-first-serve discipline. The breakdown rate  $\alpha_i$  and the repair time rate  $\beta_i$  for broken machines are also exponentially distributed. The blocking mechanism in the reprocessing system is “Block after service”. When a part is ready to join a workstation, either the buffer is not full, in which case the part joins the queue for the reprocessing operations, or the buffer is full and the part cannot join the queue, in which case it stays where it has come from and blocks that server. A blocked part is released to the downstream workstation as a space becomes available there. The transfer time of parts between buffers and workstation are assumed to be negligible. When a failure of a workstation occurs during the reprocessing of a part, the part stays there while the workstation is being repaired. After the repair of the workstation, the part is reprocessed from the beginning. Based on the above assumption, a queuing network representation of a typical reprocessing system is depicted in Fig. 1.

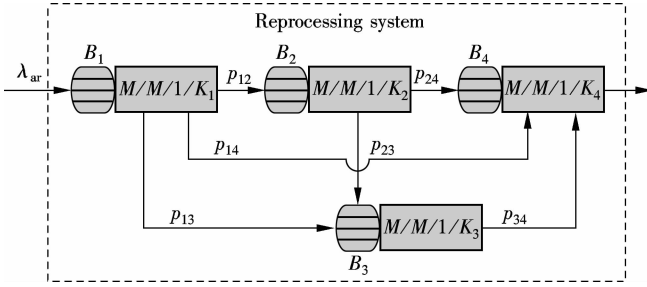


Fig. 1 OQN model of the reprocessing system

## 2.2 Solution algorithm

In order to analyze the queuing network, the decomposition principle and the expansion methodology are used. The decomposition principle is widely used in the analysis of a queuing network when a closed form solution for the network does not exist. The idea is to partition the network into individual nodes so that one can analyze and estimate the necessary parameters of each node independent of the rest of the network. When the analysis of each node is complete, the interaction of each node with the rest of the network can be reviewed. After decomposing the network, the expansion methodology is utilized to analyze each node individually<sup>[9]</sup>.

We analyze the reprocessing system by monitoring the mean reprocessing time of the system. The analysis is based on the steady-state behavior of the given reprocessing system with the following three stages:

### Stage 1 Decomposition and expansion of the network

In order to analyze the system, we first decompose the network and study each node in isolation as shown in Fig. 2, where  $\lambda_{ti}$  is the arrival rate to node  $i$ ,  $M/M/1/K_i$  represents an  $M/M/1/K_i$  queuing system with service interruption. Then we expand the network by adding a holding node in front of the finite buffer as shown in Fig. 3, where  $K_i$  is the part holding capacity of node  $i$  ( $K_i = B_i + 1$ ),  $p_{K_i}$  denotes the probability of having  $K_i$  parts at the destination node  $i$ , and  $p'_{K_i}$  is the feedback blocking probability in the expansion method.

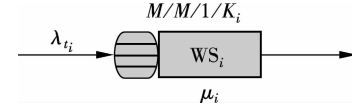


Fig. 2 Isolated node  $i$

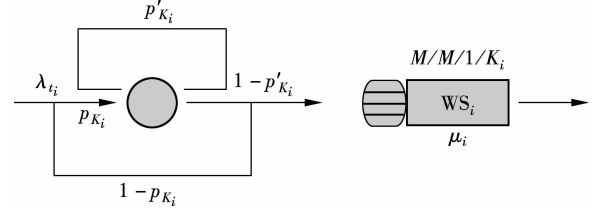


Fig. 3 Expanded node  $i$

At the time of arrival of a part, if the buffer is not full, the part is routed directly to the buffer at node  $i$  with probability  $1 - p_{K_i}$ . On the other hand, at the time of arrival of a part, if the buffer at node  $i$  is full and the part is denied into the node, it is routed to the holding node. The probability that this event occurs is denoted by  $p_{K_i}$ . Once the part enters the holding node, it stays there until a space becomes available at the buffer of node  $i$ . While residing at the holding node, the part checks the status of the buffer of node  $i$  in the Bernoulli manner at every  $\Delta t$  ( $\Delta t \rightarrow 0$ ). After every check, if the buffer is still full, the part returns the holding node with probability  $p'_{K_i}$ . One can view the holding node as an  $M/M/\infty$  node with zero processing time. In this way the holding node can accommodate any number of parts rejected by the full buffer of node  $i$ . When a space is available at the buffer of node  $i$ , the part joins the buffer. The probability of this event occurring is  $1 - p'_{K_i}$ .

### Stage 2 Calculation of network parameters

Consider the isolated  $(M/M/1/K_i)$  node  $i$ ,  $p_{s_i}$ ,  $p_{0s_i}$ ,  $p_{1s_i}$ , and  $p_{K_i}$  can be calculated using the following equations recursively<sup>[10-11]</sup>, where  $p_{s_i}$  represents the probability that there are  $s_i$  ( $0 < s_i < K_i$ ) parts at node  $i$ , and  $p_{rs_i}$  denotes the probability that there are  $s_i$  parts at node  $i$  and the machine is either in failure ( $r = 0$ ) or severing ( $r = 1$ ):

$$p_{01_i} = \frac{\lambda_{ti}}{\mu_i} p_{00_i} \quad (2)$$

$$p_{0s_i} = \left( \frac{\lambda_{ti}}{\mu_i} \right) \left\{ p_{0, s_i-1} + \frac{\alpha_i}{\lambda_{ti} + \beta_i} \left[ \sum_{m=1}^{s_i-1} p_{0m} \left( \frac{\lambda_{ti}}{\lambda_{ti} + \beta_i} \right)^{s_i-m-1} \right] \right\} \quad \forall 2 \leq s_i \leq K_i \quad (3)$$

$$p_{10_i} = 0 \quad (4)$$

$$p_{11_i} = \frac{\alpha_i}{\lambda_{ti} + \beta_i} p_{01_i} \quad (5)$$

$$p_{1s_i} = \frac{\alpha_i}{\lambda_{ti} + \beta_i} \left[ \sum_{m=1}^{s_i} p_{0m} \left( \frac{\lambda_{ti}}{\lambda_{ti} + \beta_i} \right)^{s_i-m} \right] \quad \forall 2 \leq s_i \leq K_i - 1 \quad (6)$$

$$p_{1K_i} = \frac{\lambda_{ti}}{\beta_i} \frac{\alpha_i}{\lambda_{ti} + \beta_i} \left[ \sum_{m=1}^{K_i-1} p_{0m} \left( \frac{\lambda_{ti}}{\lambda_{ti} + \beta_i} \right)^{K_i-m-1} \right] + \frac{\alpha_i}{\beta_i} p_{0K_i} \quad (7)$$

$$p_{00_i} = 1 - \sum_{r=0}^1 \sum_{s_i=1}^{K_i} p_{rs_i} \quad (8)$$

$$p_{s_i} = p_{0s_i} + p_{1s_i} \quad 1 \leq s_i \leq K_i \quad (9)$$

By setting  $s_i = K_i$ , we can obtain  $p_{K_i}$ ,  $p'_{K_i}$  and  $\lambda_i$  can be simultaneously derived by solving the following equations:

$$\lambda_i = \lambda_{j_i} - \lambda_{h_i}(1 - p'_{K_i}) + \alpha_i + \beta_i \quad (10)$$

$$p'_{K_i} = \left[ 2 - \frac{\lambda_i(r_{2_i}^{K_i} - r_{1_i}^{K_i}) - (r_{2_i}^{K_i-1} - r_{1_i}^{K_i-1})}{\mu_i(r_{2_i}^{K_i+1} - r_{1_i}^{K_i+1}) - (r_{2_i}^{K_i} - r_{1_i}^{K_i})} \right]^{-1} \quad (11)$$

where  $\lambda_i$  is the accumulated rate of node  $i$ ;  $\lambda_{h_i}$  represents the arrival rate of parts to the holding node after being rejected by the full buffer at the destination node; and  $\lambda_{j_i}$  is the arrival rate of those parts that are not rejected by the system.

$$\lambda_{j_i} = \lambda_{i_i}(1 - p_{K_i}) \quad (12)$$

$$\lambda_{h_i} = \lambda_{i_i} p_{K_i} \quad (13)$$

$$r_{1_i} = \frac{(\lambda_i + 2\mu_i) - \sqrt{z_i}}{2\mu_i} \quad (14)$$

$$r_{2_i} = \frac{(\lambda_i + 2\mu_i) + \sqrt{z_i}}{2\mu_i} \quad (15)$$

$$z_i = (\lambda_i + 2\mu_i)^2 - 4\lambda_{i_i}\mu_i \quad (16)$$

### Stage 3 Calculation of the throughput

Since each node is analyzed independently in isolation, the throughput of each node is also calculated independently. The throughput of each node is calculated as

$$TH_i = (L_i - L_{q,i})\mu_i + \lambda_{j_i}(1 - p'_{K_i})^{\rho_i + \rho_{i-1}}(1 - p_{K_i}) \quad (17)$$

where  $L_i$  is the expected number of parts at node  $i$ ;  $L_{q,i}$  is the expected number of parts in queue at node  $i$ ; and  $\rho_i$  denotes the utilization rate at node  $i$ . They can be derived as

$$L_i = \sum_{s_i=0}^{K_i} s_i p_{s_i} \quad (18)$$

$$L_{q,i} = \sum_{s_i=1}^{K_i} (s_i - 1) p_{0s_i} + \sum_{s_i=1}^{K_i} s_i p_{1s_i} \quad (19)$$

$$\rho_i = \frac{\lambda_{i_i}}{\mu_i} \quad (20)$$

Once the throughput of node  $i$  is calculated, this value becomes the arrival rate for the downstream node. However, if the network is splitted into  $S$  parallel paths after node  $i$ , then the arrival rate to each path is the parts of the branching probability  $p_{ij}$  and the throughput for node  $i$ . Also, if several parallel paths are merged into one node, the arrival rate to the downstream node is the sum of the throughput rates from each parallel path. The throughput of the last node is just the throughput of the entire system.

The algorithm to determine the throughput of a finite buffer reprocessing system with arbitrary topology where service is subject to interruptions due to machine breakdown

can be summarized as follows:

- 1) Read in the value of  $\lambda_{ar}$ ,  $K_i$ ,  $\alpha_i$ ,  $\beta_i$  and  $\mu_i$ , set  $i=0$ ,  $\rho_0=0$  and  $TH_0 = \lambda_{ar}$ ;
- 2) Set  $i = i + 1$ ,  $\lambda_{i_i} = TH_{i-1}$ ;
- 3) Calculate  $p_{rs_i}$  using Eqs. (2) to (8);
- 4) Calculate  $p_{K_i} = p_{0K_i} + p_{1K_i}$ ;
- 5) Calculate  $p'_{K_i}$  and  $\lambda_i$  using Eqs. (10) to (16);

$$6) \text{ Calculate } L_i - L_{q,i}, L_i - L_{q,i} = \sum_{s_i=1}^{K_i} p_{0s_i};$$

$$7) \text{ Calculate } \rho_i;$$

- 8) Calculate the throughput by the following equations:  
If the node is branched into  $S$  parallel paths,

$$TH_i = p_{ij}[(L_i - L_{q,i})\mu_i + \lambda_{j_i}(1 - p'_{K_i})^{\rho_i + \rho_{i-1}}(1 - p_{K_i})]$$

If  $S$  parallel paths are merged into a single node,

$$TH_i = \sum_{j=1}^S [(L_j - L_{q,j})\mu_j + \lambda_{j_j}(1 - p'_{K_j})^{\rho_j + \rho_{j-1}}(1 - p_{K_j})]$$

If there is no branching and merging,

$$TH_i = (L_i - L_{q,i})\mu_i + \lambda_{j_i}(1 - p'_{K_i})^{\rho_i + \rho_{i-1}}(1 - p_{K_i})$$

- 9) If  $i < I$ , go to step 1), where  $I$  is the number of nodes in the network; otherwise, if  $i = I$ , the throughput of the system is  $TH_i$ ;

- 10) Calculate mean reprocessing time of the system PT,

$$PT = \sum_{i=1}^I L_i / \lambda_{ar}.$$

## 3 Numerical Examples

### 3.1 Description and computation

In order to analyze the reprocessing system given in Fig. 1, we assume that  $\lambda_{ar} = 4$ . Tab. 1 and Tab. 2 provide the values of the system parameters.

**Tab. 1** Routing probabilities  $p_{ij}$  and weight  $v_{ij}$

$j$	$i$			
	1	2	3	4
1		0.5/2	0.4/2	0.1/1
2			0.8/4	0.2/2
3				1/2
4				

**Tab. 2** System parameters

Node	1	2	3	4
$B_i$	7	5	5	5
$\mu_i$	4	2	6	4
$\alpha_i$	1	1	5	3
$\beta_i$	8	7	8	6

According to the above-mentioned algorithm, the results of computation using LINGO 11 are as follows:  $q = 10/53$ ;  $L_1 = 4.684$ ,  $TH_1 = 3.969$ ;  $L_2 = 3.426$ ,  $TH_2 = 1.638$ ;  $L_3 = 2.021$ ,  $TH_3 = 3.317$ ;  $L_4 = 4.219$ ,  $TH_4 = 3.018$ ;  $PT = 3.587$ .

### 3.2 Sensitivity analysis

In order to analyze the impact of parts quality on the performance of the reprocessing system, routing probabilities are now changed, which deservedly imply the change in

parts quality. If  $p_{12} = 0.1$ ,  $p_{13} = 0.4$ ,  $p_{14} = 0.5$ ,  $p_{23} = 0.8$ ,  $p_{24} = 0.2$ ,  $p_{34} = 1$ , while other parameters are unchanged. The results are  $q = 50/141$ ,  $PT = 2.829$ .

If  $p_{12} = 0.5$ ,  $p_{13} = 0.4$ ,  $p_{14} = 0.1$ ,  $p_{23} = 0.2$ ,  $p_{24} = 0.8$ ,  $p_{34} = 1$ , the results obtained are  $q = 5/18$  and  $PT = 3.386$ .

If  $p_{12} = 0.5$ ,  $p_{13} = 0.5$ ,  $p_{14} = 0$ ,  $p_{23} = 0.8$ ,  $p_{24} = 0.2$ ,  $p_{34} = 1$ , the results obtained are  $q = 5/28$  and  $PT = 3.656$ .

If  $p_{12} = 0.5$ ,  $p_{13} = 0.5$ ,  $p_{14} = 0$ ,  $p_{23} = 1$ ,  $p_{24} = 0$ ,  $p_{34} = 1$ , the results obtained are  $q = 1/6$  and  $PT = 3.670$ .

Through the above-mentioned analysis, the impact of quality uncertainty on performance of the reprocessing system is illustrated in Fig. 4.

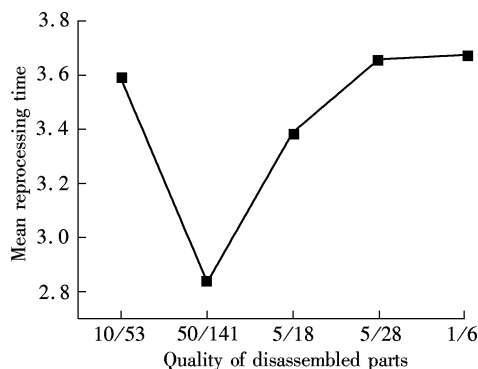


Fig. 4 Effect of quality uncertainty on performance of system

Fig. 4 shows that there is a negative correlation between quality of parts and its mean reprocessing time, that is, if the parts quality is high, their mean reprocessing time is short; otherwise, if parts quality becomes lower, the mean reprocessing time will become accordingly longer. Furthermore, its marginal reprocessing time decreases with the drop of parts quality.

#### 4 Conclusion

In remanufacturing operations, the problem of stochastic routing for reprocessing operations and highly variable reprocessing time is a complicated problem that remanufacturing firms have to deal with because they have a significant influence on resource planning, estimating material flow times, material planning, and so forth. In this paper,

through the analysis of calculation methods about quality and mean reprocessing time of disassembled parts, we obtain the quantitative regularity about the impact of quality uncertainty on the performance of the reprocessing system, which intuitively shows a negative correlation between quality of parts and its mean reprocessing time.

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## 零部件质量不确定对其再制造加工的影响

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**摘要:**针对零部件再加工路线与再加工时间的不确定性问题,建立了零部件再加工系统的开环排队网络模型.该模型考虑了再加工机器故障和有限缓冲能力,并假定机器故障率、维修率和再加工时间服从指数分布.基于分解原理和扩展方法,给出了求解模型的近似分析算法以求解零部件的平均再加工时间、再加工效率等系统参数.提出了零部件质量的标定方法,并以此为基础探讨了零部件质量不确定性对其再加工时间的影响.算例分析表明,零部件的再加工时间与其质量呈负相关关系,且随着零部件质量的不断降低其边际再加工时间逐渐减少.

**关键词:**再制造;不确定性;再加工系统;开环排队网络

**中图分类号:**F252