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Analysis of stability and bullwhip effect in production-inventory systems

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Abstract: To discuss the relationship between stability and bullwhip effect in the supply chain system, a basic model in a production-inventory control system is developed using difference equations. Z-transform techniques are applied to investigate the production ordering and inventory dynamics. For the two operational regimes of sufficient inventory coverage and insufficient inventory coverage, the scope of decision parameters which make the system stable or instable is investigated. Under two operational regimes and the actual system, production release arates, stability/instability and bullwhip effect in the stable region and instable region are examined based on different demand functions, and then the numerical simulation results are given. The results show that reasonable choices of fractional adjustment of inventory and supply line can make the system stable and decrease bullwhip effect. It is summarized that the piecewise linearization based on the stability analysis approach is a valid approximation to the analysis of production-inventory ordering systems with nonlinearities. Some interesting results are obtained and they have important implications for improving inventory and order decisions in supply chain systems.

Key words: production-inventory systems; stability analysis; bullwhip effect; Z-transform

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The supply chain is a complex system, and there exist various types of uncertainties, e.g., demand uncertainty, production uncertainty, delivery uncertainty and lead-time delays. Analysis of production and inventory dynamics helps reveal periods of inventory build-ups, stocks-out, overtime production and production shutdown, which costs the companies in terms of profits, market position and customer satisfaction. This phenomenon, in which the variance of order quantities increases as we move away from the end customers in a supply chain, is termed as the bullwhip effect[1]. The Beer Game, developed at MIT and reported in Ref.[2], was widely used in teaching inventory management. Springer and Kim[3] determined supply chain designs and policies that minimize volatility using a system dynamics approach. Wang et al.[4] examined the influence of forecast-updating methods between order quantity and actual demand in the amplifications of the bullwhip effect. A recent study by Strozzi et al.[5] applied a control technique (state-space method), based on the system's divergence, to control the local stability of a simple supply chain in order to reduce the costs as well as the bullwhip effect. Sipahi et al.[6] investigated stability of inventory behavior controlled by an automated pipeline inventory and an order-based production control system (APIOBPCS) with respect to delays originating from different physical reasons, lead time, transportation and decision-making.

A natural choice to examine the production and inventory dynamics is the application of system dynamics and control theoretical techniques. A comprehensive literature review on the use of control theoretic concepts for the dynamic analysis of production-inventory systems can be found in Refs.[7 – 8]. The supply chain systems are analyzed through the application of control theoretic tools such as block diagrams, and functional transformations[9]. Using linear Z-transform analysis, Disney et al.[10] identified and proposed a method to eliminate the possibility of an inventory drift due to uncertain pipeline lead times. The parameter scope for fractional adjustment of inventory and fractional adjustment of the supply line is $[0, 1]$[11]. However, in this paper, we extend the parameters scope to be greater than zero.

In this research, the difference equation models of the production-inventory system are constructed. The production release ordering rules of the system are adapted from the APIOBPCS family of the models[12]. The shipment to the customers is defined as a function of demand and on-hand inventory to ensure that the shipment does not exceed the on-hand inventory. It is assumed that shortages are allowed, thus the shipment rate is a typical nonlinear function. Such nonlinear functions are linearized using a local linearization technique, in which the nonlinear function is separated into piecewise linear functions. The boundary conditions for the system stability are computed based on Jury's Test[13]. The function transformation technique is used as a basis for our research and allows us to gain important insights about the dynamic behavior of supply chain systems. Numerical results are provided to indicate the impacts of decision parameters on stability/instability in the two operational regimes. Furthermore, the relationship between bullwhip effect and stability/instability in the supply chain is created through numerical analysis.

1 Basic Model

Assume that the system is periodically reviewed. Given lead time τ, the sequence of events during one period is given as follows: satisfying consumer demand based on on-hand inventory, forecasting the demand, making the production release quantity, reviewing the remaining inventory and supply line, and receiving the production release quantities which were placed in period $t-\tau$. Other variables and notations needed for the model at time period t are given as: L_t is the external customer demand; $\hat{L}_t$ is the forecasted demand; S_t is the on-hand inventory (the actual inventory minus the backlogged); D_t is the shipments; P_t is the production release quantity; R_t is the production rate; and SL_t is the supply line.

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1.1 Shipment

It is noted that all the unfulfilled demand is assumed to be lost. In the model presented above, the shipment rate is given as a function of the demand and the inventory available. Assuming a sharply discontinuous function, it can be seen that when the demand is less than the inventory available, the shipment rate equals the demand; and when the demand rate is more than the inventory available, the shipment rate equals the inventory available, as shown below:

$$D_t = \min(L_t, S_{t-1}) \to D_t = \begin{cases} L_t & L_t \leqslant S_{t-1} \quad (1) \\ S_{t-1} & L_t > S_{t-1} \quad (2) \end{cases}$$

In the first operational regime, it is assumed that there is always sufficient inventory coverage to meet the desired shipments (Eq. (1)). In the second operational regime, it is assumed that there is not sufficient inventory coverage to meet the desired shipments (Eq. (2)).

1.2 Demand forecast

The forecasted demand of the products is modeled as the first-order exponential smoothing of the customer demand, with a smoothing constant θ. That is

$$\hat{L}_t = \theta L_{t-1} + (1-\theta)\hat{L}_{t-1} \qquad 0 \leqslant \theta \leqslant 1$$

1.3 Production ordering

The production ordering aspect determines the production release quantities by the ordering rule which is based on the forecasted demand, the difference between the desired inventory level and the current inventory level, and the difference between the desired supply line and the current supply line.

$$P_t = \hat{L}_{t-1} + \alpha_S(S^*_{t-1} - S_{t-1}) + \alpha_{SL}(\mathrm{SL}^*_{t-1} - \mathrm{SL}_{t-1})$$

Based on Little's Law, the desired supply line in the system is set to yield the desired throughput ($\mathrm{SL}^*_{t-1} = \tau\hat{L}_{t-1}$). The desired inventory is set equal to the forecasted demand ($S^*_{t-1} = \hat{L}_{t-1}$). The fractional adjustment rate α_S for inventory describes how much of the discrepancy between the desired and current levels of inventory is to be added to the production release order. The fractional adjustment rate α_{SL} for the supply line describes how much of the discrepancy between the desired and current levels of the supply line is to be added to the production release order.

1.4 Inventory and supply line review

The inventory level accumulates the difference in the production rate and the shipment rate:

$$S_t = S_{t-1} + R_t - D_t$$

The supply line accumulates the difference in the production release rate and the production rate:

$$\mathrm{SL}_t = \mathrm{SL}_{t-1} + P_t - R_t$$

The production process is modeled as a typically fixed pipeline delay:

$$R_t = P_{t-\tau}$$

2 System Analysis

Production and inventory control systems can be readily viewed as a system sampled at regular discrete intervals, since the ordering rules are evaluated only at discrete points in time, such as every day or every week. In this paper, the Z-transform technique is used to obtain generalized transfer functions of the production release order. Now, a necessary step before the Z-transform analysis is the linearization of the non-linear functions present in the system model. Such nonlinear functions are linearized using local linearization techniques, in which the nonlinear function is separated into piecewise linear functions.

The Z-transform of the production ordering and inventory control system (refer to section 1), with the excess equation reduced, are given as follows:

$$\hat{L}[z] = \frac{\theta L[z]}{z-1+\theta}$$

$$D_t = \begin{cases} L[z] & L[z] \leqslant S[z] \\ z^{-1}S[z] & L[z] > S[z] \end{cases}$$

$$R[z] = z^{-\tau}P[z]$$

$$P[z] = \frac{(1+\tau\alpha_{SL}+\alpha_S)\hat{L}[z] - \alpha_{SL}\mathrm{SL}[z] - \alpha_S S[z]}{z}$$

$$S[z] = \frac{z(R[z]-D[z])}{z-1}, \quad \mathrm{SL}[z] = \frac{z(R[z]-P[z])}{z-1}$$

Simplifying and collecting in powers of z, the transfer functions result in the following expressions. The sufficient inventory coverage and insufficient inventory coverage are described as

$$\frac{P[z]}{L[z]} = \frac{(\theta(z-1)(1+\tau\alpha_{SL}) + \alpha_S(z-1+\theta z))(z\tau-\tau+3)^3}{(z-1+\theta)(27\alpha_S - 27\alpha_{SL} + (z-1)(z\tau-\tau+3)^3 + \alpha_{SL}(z\tau-\tau+3)^3)} \quad (3)$$

$$\frac{P[z]}{L[z]} = \frac{\theta z(z-1)(1+\tau\alpha_{SL}+\alpha_S)(z\tau-\tau+3)^3}{(z-1+\theta)(27\alpha_S(z-1) - 27\alpha_{SL}z + z(z-1)(z\tau-\tau+3)^3 + \alpha_{SL}z(z\tau-\tau+3)^3)} \quad (4)$$

The control parameters including the fractional adjustment rate for inventory, the fractional adjustment rate for the supply line, the exponential smoothing constant for the forecasted demand and the production lead time are identified as affecting the system stability. The stability conditions for the two different considered operational regimes are obtained in terms of the above control parameters. The difference in the transfer functions between Eq. (3) and Eq. (4), and the respective resultant system become explicit when studied in terms of system stability, as detailed in section 3.

3 Analysis of System Stability

Now, it is important to understand how the production ordering and the inventory control system responds to any change in its input (i.e., demand), especially under a fluctuating market. Does the response result in increasing amplitude oscillations in general, or does the response appear controllable and damped? Thus, it becomes essential to know under what conditions the system is stable or instable. Hence, the general conditions for the system stability from the $P(t)$ transfer functions in terms of the various design parameters are derived.

In this paper, Jury's test is employed to determine the location of the roots. A system given by its closed form transfer function is said to be stable if all the roots (poles) of the transfer function's denominator polynomial lie within the unit circle in the complex plane. Systems with poles that are outside the unit circle or with repeated poles in the unit circle are said to be instable as they expand. Also it can be observed that, the further inside the unit circle the poles are, the faster the damping and, hence, the higher the stability. Though Jury's test enables a solution, it still involves tedious calculations, which are hence performed using Matlab (R2009a). The Jury test for stability of a discrete-time system is: Given a transfer function $H(z) = b(z)/a(z)$, the system is stable if and only if all the roots of $a(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_n$ are inside the unit circle.

These two distinct regimes of inventory operations are separately analyzed using Z-transform techniques. It is noted that the dynamic behavior of the system often results in a transition between the operational regimes, which are not captured in such a separate analysis. However, useful insights can be drawn from such a segregated analysis.

3.1 Sufficient inventory coverage (Case 1)

For the case of sufficient inventory coverage ($L_t \leqslant S_{t-1}$), it is assumed that there is always sufficient inventory coverage to meet the desired shipments. The coefficients a_0 to a_5 refer to the coefficients of Eq. (3) and yield the following equation:

$$\begin{aligned} a(z) = {} & \tau^3 z^5 + [9\tau^2 + (-5 + \alpha_{SL} + \theta)\tau^3] z^4 + [27\tau + (-36 + \\ & 9\alpha_{SL} + 9\theta)\tau^2 + (10 - 4\theta - 4\alpha_{SL} + \theta\alpha_{SL})\tau^3] z^3 + \\ & [27 + (-81 + 27\alpha_{SL} + 27\theta)\tau + (54 - 27\alpha_{SL} - 27\theta + \\ & 9\theta\alpha_{SL})\tau^2 + (-10 + 6\theta + 6\alpha_{SL} - 3\theta\alpha_{SL})\tau^3] z^2 + \\ & [-54 + 27\alpha_S + 27\theta + (81 - 54\alpha_{SL} - 54\theta + 27\theta\alpha_{SL})\tau + \\ & (-36 + 27\alpha_{SL} + 27\theta - 18\theta\alpha_{SL})\tau^2 + \\ & (5 - 4\theta - 4\alpha_{SL} + 3\theta\alpha_{SL})\tau^3] z + 27 - 27\alpha_S - 27\theta + \\ & 27\theta\alpha_{SL} + (-27 + 27\alpha_{SL} + 27\theta - 27\theta\alpha_{SL})\tau + \\ & (9 - 9\alpha_{SL} - 9\theta + 9\theta\alpha_{SL})\tau^2 + (-1 + \theta + 4\alpha_{SL} - \theta\alpha_{SL})\tau^3 \end{aligned}$$

Plotting this function on the parameter plane yields Fig. 1, which shows the boundary of stability for different α_S and α_{SL} in the production-inventory system when $\tau = 3$. In Fig. 1, the gray area refers to the common stable region of two cases. It illustrates the stable region (black area plus gray area) on the parameter plane for case 1. The system is guaranteed to be stable when the values of α_S and α_{SL} are restricted to the stable region.

3.2 Insufficient inventory coverage (Case 2)

For the case of insufficient inventory coverage ($L_t > S_{t-1}$), it is assumed that there is not sufficient inventory coverage to meet the desired shipments. The coefficients a_0 to a_6 refer to the coefficients of Eq. (4) and yield the following equation:

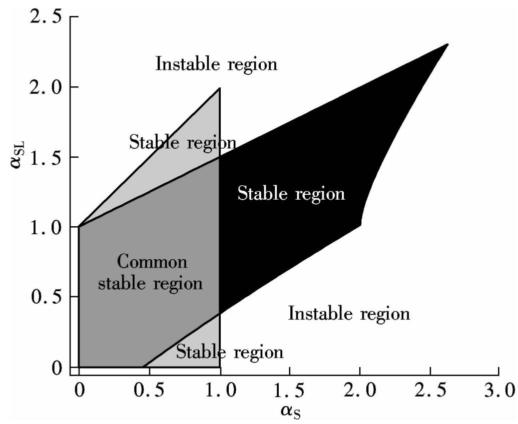

Fig. 1 Stable and instable regions for two cases

$$\begin{aligned} a(z) = {} & \tau^3 z^6 + [9\tau^2 + (-5 + \alpha_{SL} + \theta)\tau^3] z^5 + [27\tau + (-36 + \\ & 9\alpha_{SL} + 9\theta)\tau^2 + (10 - 4\theta - 4\alpha_{SL} + \theta\alpha_{SL})\tau^3] z^4 + \\ & [27 + (-81 + 27\alpha_{SL} + 27\theta)\tau + (54 - 27\alpha_{SL} - 27\theta + \\ & 9\theta\alpha_{SL})\tau^2 + (-10 + 6\theta + 6\alpha_{SL} - 3\theta\alpha_{SL})\tau^3] z^3 - \\ & 54 + 27\alpha_S + 27\theta + (81 - 54\alpha_{SL} - 54\theta + 27\theta\alpha_{SL})\tau + \\ & (-36 + 27\alpha_{SL} + 27\theta - 18\theta\alpha_{SL})\tau^2 + \\ & (5 - 4\theta - 4\alpha_{SL} + 3\theta\alpha_{SL})\tau^3] z^2 + [27 - 54\alpha_S - 27\theta + \\ & 27\theta\alpha_{SL} + (-27 + 27\alpha_{SL} + 27\theta - 27\theta\alpha_{SL})\tau + (9 - 9\alpha_{SL} - \\ & 9\theta + 9\theta\alpha_{SL})\tau^2 + (-1 + \theta + \alpha_{SL} - \theta\alpha_{SL})\tau^3] z + 27\alpha_S - \\ & 27\theta\alpha_S \end{aligned}$$

Fig. 1 illustrates the stable region (light gray area plus gray area) on the parameter plane for case 2. The system is guaranteed to be stable when the values of α_S and α_{SL} are restricted to the stable region. According to the numerical analysis, the stability of two operation regimes depends on two parameters α_S and α_{SL}. Meanwhile, the smoothing constant θ for demand forecasting has no effect on the stability of the control system.

4 Supply Chain Stability and Bullwhip Effect

Now, it is important to present the dynamic behavior of the original nonlinear system in response to time-varying system inputs. It is of interest to validate the applicability of such a separate analysis when the system switches from one operational regime to another. To see if mode instability or stability impacts the bullwhip metric, we calculate the bullwhip effect. The delays and the nonlinearity in the supply chain model result in the bullwhip effect. The bullwhip effect (BWE) is quantified as follows:

$$\mathrm{BWE} = \frac{\mathrm{var}(P)}{\mathrm{var}(L)} \tag{5}$$

To find the numerical results of the bullwhip effect calculated in Eq. (5), we rely on a simulation approach. As before, the parameters for forecasting demand do not affect the stability of the system, but they influence the bullwhip effect in the control system. Without loss of generality, we set $\theta = 0.5$, $\tau = 3$. Two types of demand patterns are considered, i.e., a step input function and a random function.

4.1 Step input demand

Our simulations are designed to mimic the behavior of the beer game. In the first four weeks, the demand of the customers is four units per week. In week five, the demand increases to eight units per week and then stays constant at eight units per week for the rest of the simulation.

Fig. 2 and Tab. 1 highlight six possible designs that are to be used as test cases of the stability criteria via the system response to step demand input for various settings of α_S and α_{SL}. It can be seen that the actual system clearly demarcates itself into stable and instable regions. Some of the most interesting results are obtained:

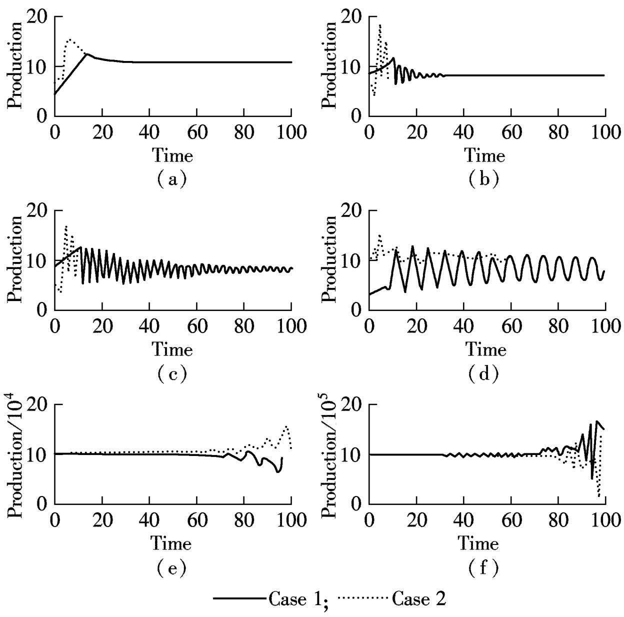

Fig. 2 Production release rates for different settings of α_S and α_{SL}. (a) $\alpha_S=0.2, \alpha_{SL}=0.3$; (b) $\alpha_S=0.7, \alpha_{SL}=1.3$; (c) $\alpha_S=0.5, \alpha_{SL}=1.3$; (d) $\alpha_S=1.4, \alpha_{SL}=0.6$; (e) $\alpha_S=1.2, \alpha_{SL}=0.3$; (f) $\alpha_S=0.4, \alpha_{SL}=1.5$

1) When (α_S, α_{SL}) is in a common stable region ($\alpha_S=0.2$ and $\alpha_{SL}=0.3$; $\alpha_S=0.7$ and $\alpha_{SL}=1.3$), it can be observed that, the further inside the unit circle the poles are, the faster the damping, and, hence, the higher the stability. Meanwhile, the less response time to a stable condition is, the further inside the unit circle the poles are. In the initial stage, higher values of α_S and α_{SL} can induce the amplitude of variation in production ordering. In the stable region, the decision makers choose the smaller values in the stable region to make the production ordering stable at 8 units quickly. Therefore, the actual system is stable.

2) If (α_S, α_{SL}) is in the stable region of one case, and in the instable region of other cases($\alpha_S=0.5$ and $\alpha_{SL}=1.3$; $\alpha_S=1.4$ and $\alpha_{SL}=0.6$), the system response is in fluctuation, such as periodic fluctuation. The actual system is instable. When the value of α_S is lower than that of α_{SL}, the response is less volatile. But, otherwise, the response is much more volatile. This can be a reference for making strategic decisions.

3) When (α_S, α_{SL}) is in instable regions of two cases ($\alpha_S=1.2$ and $\alpha_{SL}=0.3$; $\alpha_S=0.4$ and $\alpha_{SL}=1.5$), the system produces an instable response, such as periodic fluctuation, chaos, bifurcations, and so on.

4.2 Random demand

In particular, we assume that the customer demand follows a normal distribution with an average of 10 units and standard deviation of 3 units. We simulate each scenario 100 times and calculate the average values of the simulation results. And the initial inventory level is set to 8 units and each simulation run is 1 000-period long.

Tabs. 2, 3 and 4 highlight ten possible designs in Fig. 1 that are to be used as test cases of the stability criteria via sample time domain responses to normal demand input, where S/I denotes stable/instable, and BWE denotes the bullwhip effect. When BWE is greater than one, it means that there is a bullwhip effect.

Tab. 1 Stability response in operational regimes and actual system

Operational regimes and actual system	Decision making (α_S, α_{SL})					
	(0.2, 0.3)	(0.7, 1.3)	(0.5, 1.3)	(1.4, 0.6)	(1.2, 0.3)	(0.4, 1.5)
Sufficient inventory	Stable	Stable	Instable	Stable	Instable	Instable
Insufficient inventory	Stable	Instable	Stable	Instable	Instable	Instable
Actual system	Stable	Stable	Instable	Instable	Instable	Instable

Tab. 2 Stability response and bullwhip effect in stable regions

Operational regimes and actual system	Decision making (α_S, α_{SL})									
	(0.2, 03)		(0.9, 0.8)		(0.1, 0.9)		(0.9, 0.4)		(0.7, 1.3)	
	S/I	BWE	S/I	BWE	S/I	BWE	S/I	BWE	S/I	BWE
Sufficient inventory	S	1.67	S	9.30	S	8.89	S	7.08	S	42.89
Insufficient inventory	S	0.91	S	7.59	S	19.87	S	6.07	S	26.36
Actual system		1.70		11.18		10.06		12.95		39.97

Tab. 3 Stability response and bullwhip effect in stable and instable regions

Operational regimes and actual system	Decision making (α_S, α_{SL})									
	(1.2, 1.3)		(1.2, 0.7)		(0.3, 1.2)		(0.9, 1.5)		(0.9, 0.2)	
	S/I	BWE	S/I	BWE	S/I	BWE	S/I	BWE	S/I	BWE
Sufficient inventory	S	29.64	S	15.14	I	74.38	I	118.02	I	121.69
Insufficient inventory	I	25.94	I	11.72	S	71.56	S	53.80	S	124.91
Actual system		30.01		19.78		74.69		106.13		335.92

Tab. 4 Stability response and bullwhip effect in instable regions

Operational regimes and actual system	Decision making (α_S, α_{SL})							
	(1.1, 1.6)		(0.1, 1.2)		(1.2, 0.3)		(0.6, 1.7)	
	S/I	BWE	S/I	BWE	S/I	BWE	S/I	BWE
Sufficient inventory	I	114.56	I	6.91×10^5	I	2.28×10^6	I	1.45×10^{12}
Insufficient inventory	I	34.63	I	1.63×10^6	I	1.46×10^6	I	1.03×10^{12}
Actual system		99.13		1.31×10^6		2.30×10^6		1.26×10^{12}

If α_S and α_{SL} are chosen such that they satisfy the stability conditions of the two cases in Fig. 2, too small values of (α_S, α_{SL}) still induce the bullwhip effect. The bullwhip effect increases as the two parameters increase. It coincides with the results in section 4.1. Increasing or decreasing values which are given to the inventory and supply line has little effect on the bullwhip effect. But, when one of the two parameters is greater than one, the bullwhip effect suddenly increases. This implies that the stability of the system cannot represent low bullwhip effect.

Tab. 3 shows that, if two parameters are located in the stable region of sufficient inventory coverage, and in the instable region of the insufficient inventory coverage (α_S = 1.2 and α_{SL} = 1.3; α_S = 1.2 and α_{SL} = 0.7), the bullwhip effect is relatively small. But, when the parameters are located in the instable region of case 1 and the stable region of case 2 (α_S = 0.3 and α_{SL} = 1.2; α_S = 0.9 and α_{SL} = 1.5; α_S = 0.9 and α_{SL} = 0.2), the bullwhip effect is high (100 or so). This implies that when the system operates under sufficient inventory coverage, the actual system is relatively stable. Meanwhile, the higher value of α_S can induce the bullwhip effect of two operational regimes and the actual system. To make the actual system stay in sufficient inventory coverage, the decision maker can make the initial inventory level higher. Decreased values should be given to supply line levels, as long as the chosen α_S and α_{SL} satisfy the stability conditions in the sufficient inventory case.

If α_S and α_{SL} are chosen such that they satisfy instable regions of sufficient and insufficient inventory coverages in Fig. 4, the bullwhip effect is high. We can also see that when the two parameters are nearly the same (α_S = 1.1 and α_{SL} = 1.6), the bullwhip effect is relatively low (above 99). But, when the two parameters have great disparity, the bullwhip effect is very high. When α_S = 0.6 and α_{SL} = 1.7, the bullwhip effect is 1.26×10^{12}. When the system operates under insufficient inventory coverage, the bullwhip effect in the actual system is relatively low.

Comparing Tabs. 2 and 3 with Tab. 4, we can observe that the bullwhip effect is still present when the system is stable, but the value is much smaller than that when the system is instable. To avoid system instability and a high bullwhip effect, decision makers should carefully select the adjustment parameters for inventory discrepancy and supply line discrepancy to make α_S and α_{SL} stay in common stable regions. It is hence concluded that the proposed piecewise linearization based stability analysis approach is a valid approximation to the analysis of production and inventory ordering systems with nonlinearities.

5 Conclusion

This paper focuses on analyzing the stability of a production-inventory system using difference equations and Z-transforms. The results of theoretical analysis and numerical study indicate that the bullwhip effect is relatively low when the system is stable and a careful choice of decision parameters can improve the financial performance of the system. In order to effectively implement a production-inventory system, the supply chain partners have to align their production order policies and forecast methods. Finally, guidance for the selection of parameters to guarantee system stability is presented. The research can be extended to three or more echelon-supply chains, where more complex behaviors such as periodicity, quasi periodicity, chaos, bifurcations and others may exist, creating a large space for researchers to adopt various methods and generate more insights.

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生产库存系统的稳定性和牛鞭效应分析

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摘要:为了讨论供应链系统稳定性与牛鞭效应之间的关系,建立了生产库存控制系统的离散微分方程模型. 基于Z-变换理论研究了生产库存系统的动态性. 在库存充足和库存不充足2种运作机制下,研究了使生产库存系统稳定和不稳定的参数范围; 在2种运作机制以及实际系统下,分别在稳定区域和不稳定区域中,基于不同的需求模式研究了生产交付速率波动性、系统稳定与不稳定性以及牛鞭效应,并给出了相应的数值仿真. 结果显示,合理地选择库存和订单决策参数,不仅能够保证系统稳定,而且可以减小系统的牛鞭效应. 基于稳定性分析的分段线性化是研究具有非线性特性生产库存订购系统的一种有效方法. 该研究为改善供应链系统的库存和订单决策提供了有益的建议.

关键词: 生产库存系统;稳定性分析;牛鞭效应;Z-变换

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