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# Gauge dependence of chiral condensate and fermion mass by an unquenched model

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**Abstract:** Based on three-dimensional quantum electrodynamics theory, a set of truncated Dyson-Schwinger (D-S) equations are solved to study photon and fermion propagators with the effect of vacuum polarization. Numerical studies show that condensation and the value of fermion mass depends heavily on how the D-S equations are truncated. By solving a set of coupled D-S equations, it is also found that the fermion propagator shows a clear dependence on the order parameter. The truncated D-S equations under unquenched approximation are used to study the mass-function and chiral condensation of the fermions. The results under the unquenched approximation are clearly different from the ones under quenched approximation. With the increase in the order parameter, the fermion condensation in the unquenched approximation decreases when $0 \leqslant \xi < 5$, while it increases when $\xi > 5$. However, nothing like this is observed in the quenched approximation, which indicates that there may be flaws in the quenched approximations.

**Key words:** quantum electrodynamics ($QED_3$); unquenched; fermion-boson vertex; fermion chiral condensate; quenched; drawback

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The gauge field plays an important role in theoretical physics. In theoretical physics, gauge symmetry reflects the fact that all the physical values should be independent of the gauge parameters. In the perturbation gauge theory, the results respect these requirements at every level of approximation, but this has not been achieved in general in the nonperturbative theory. To indicate the physical values in the strong coupled system, one should resolve Dyson-Schwinger (D-S) equations. Since the involved fermion-boson vertex is unknown, these coupled equations have to be truncated and some symmetry of the system is destroyed. A previous work showed the gauge dependence[1] by truncated D-S equations where the quenched approximation was adopted. Therefore, it is interesting to investigate the gauge dependence of physical values with different truncated schemes for D-S equations which can be solved self-consistently.

Quantum electrodynamics ($QED_3$) in $(2+1)$ dimensions with massless fermion as a field-theoretical model has been extensively studied in recent years. It has many nonperturbative features, such as confinement[2] and dynamical chiral symmetry breaking (DCSB) in the chiral limit[3-4]. Moreover, it is superrenormalizable, so that it is not plagued with the ultraviolet divergences which are present in $QCD_4$. Consequently, $QED_3$ can be used as a toy model to study some nonperturbative phenomena. $QED_3$ is also relevant to theories for some realistic microscopic models in condensed matter physics. Especially, since the discovery of the high-$T_c$ superconductivity, $QED_3$ has received more attention. It is widely accepted that $QED_3$ with $N$ flavors can be regarded as a possible effective theory for high-$T_c$ superconductivity (HTSC) in doped cuprates[5]. Therefore, the "simple" gauge theory might be an admirable model to investigate the problems proposed in this paper.

The fermion condensate is a low energy nonperturbative phenomenon, because $QED_3$ is asymptotically free and only in the infrared region is the gauge interaction strong enough to cause fermion condensation. This condensate in quantum chromodynamics in four dimensions is widely believed to account for the pion. Since the fermion mass is relational to the condensate, the gauge dependence of fermion condensate and the mass in this model is studied.

## 1 Dyson-Schwinger Equation for Propagators

The Lagrangian for the massless fermion in general covariant gauge in the Euclidean space can be written as

$$L = \bar{\psi}(\partial - \mathrm{i}eA)\psi + \frac{1}{4}F_{\rho\nu}^2 + \frac{1}{2\xi}(\partial_\rho A_\rho)^2 \tag{1}$$

where four-component spinors for the fermion are used. This chiral symmetry can be dynamically broken by generation of a nonzero mass. In this formulation, there can also be a parity-breaking mass term, which conserves the chiral symmetry, but such a mass is not dynamically generated.

From Lorentz structure analysis, the inverse fermion propagator can be written as

$$S^{-1}(p) = \mathrm{i}\gamma p A(p^2) + B(p^2) \tag{2}$$

where $A(p^2)$ is the wave-function renormalization and is dimensionless; $B(p^2)$ is the fermion self-energy which has the dimension of mass.

The full photon propagator with covariant gauge is given by

$$D_{\rho\nu}(q) = \frac{\delta_{\rho\nu} - q_\rho q_\nu / q^2}{q^2[1 + \Pi(q^2)]} + \xi \frac{q_\rho q_\nu}{q^4} \tag{3}$$

where $\Pi(q^2)$ is the vacuum polarization for the photon and $\xi$ is the gauge parameter.

The full inverse fermion propagator and the full photon

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propagator satisfy the following Dyson-Schwinger equation (DSE):

$$S^{-1}(p) = S_0^{-1}(p) + e^2\int \frac{d^3k}{(2\pi)^3}\gamma_\rho S(k)\Gamma_\nu(p,k)D_{\rho\nu}(p-k) \tag{4}$$

where $S_0^{-1}(p)$ is the bare inverse propagator for the massless fermion; $\Gamma_\nu(p,k)$ is the full fermion-photon vertex and is reduced to $\gamma_\nu$ in rainbow approximation. However, using this tensor, Ward-Takahashi identity (WTI) is destroyed. Ansatze, beyond rainbow approximation, should be introduced.

DSEs for the photon and fermion propagator form a set of coupled equations for three scalar functions if we know the full fermion-photon-vertex $\Gamma_\nu(p,k)$. Unfortunately, although several works have attempted to resolve the problem, none of them is completely satisfactory.

In this paper, adopting reasonable approximation for the full vertex function, we employ several ansatzes to obtain the availably truncated DSEs. For the reason of simplicity, we choose the following ansatze[6]:

$$\Gamma_\nu(p,k) = f(A(p^2),A(k^2))\gamma_\nu \tag{5}$$

where the form of function $\Gamma_\nu(p,k)$ is: $\gamma_\nu$, $[A(p^2)+A(k^2)]\gamma_\nu/2$, or $A(p^2)A(k^2)\gamma_\nu$. The first one is the bare vertex. This structure plays the most dominant role in the full vertex in the high energy region and the full fermion-boson vertex reduces to it in large momentum limit. The second form is chosen from the BC-vertex. Previous works show that the numerical results of DSEs employing the choice are as satisfactory as those of the DSEs employing BC and CP vertices which are obtained from WTI and multiplicative renormalizability[7]. Since the numerical results obtained using this ansatze coincide very well with earlier investigations, we choose the third one. So it is reasonable to adopt those ansatzes to investigate our problems, though those ansatzes destroy WTI.

Form Eq. (2) and Eq. (4), we obtain the equations satisfied by $A(p^2)$ and $B(p^2)$ in those ansatzes,

$$A(p^2) = 1 - \frac{e^2}{4p^2}\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}[i(\gamma p)\gamma_\rho S(k)\gamma_\nu f[A(p^2),A(k^2)]D_{\rho\nu}(p-k)] \tag{6}$$

$$B(p^2) = \frac{e^2}{4}\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\gamma_\rho S(k)\gamma_\nu f[A(p^2),A(k^2)]D_{\rho\nu}(p-k)] \tag{7}$$

where the notation Tr denotes trace over the Dirac indices. In principle, there are two solutions for the above equations, the Nambu-Goldstone solution ($B(p^2)\neq 0$) and the Wigner solution ($B(p^2)\equiv 0$). If Eqs. (5) and (6) have only the trivial solution (the Wigner solution), the fermions will remain massless and DCSB will not occur. If besides the trivial solution, it has a nontrivial solution (the Nambu-Goldstone solution), then the original massless fermion will acquire a nonzero mass and chiral symmetry will be broken spontaneously.

In addition, the DSE satisfied by the photon vacuum polarization tensor is written as

$$\Pi_{\rho\nu}(q^2) = -e^2\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}[S(k)\gamma_\rho S(q+k)\gamma_\nu f[A(p^2),A(k^2)]] \tag{8}$$

Using the relationship between the vacuum polarization $\Pi_{\rho\nu}(q^2)$ and $\Pi(q^2)$,

$$\Pi_{\rho\nu}(q^2) = (q^2\delta_{\rho\nu} - q_\rho q_\nu)\Pi(q^2) \tag{9}$$

we can obtain an equation for $\Pi(q^2)$ which has ultraviolet divergence. Fortunately, it is present only in the longitudinal part. One can remove this divergence by applying the projection operator

$$P_{\rho\nu} = \delta_{\rho\nu} - 3\frac{q_\rho q_\nu}{q^2} \tag{10}$$

and obtain a finite vacuum polarization.

Finally, the important parameter $\xi$ is involved. From the above ansatze for the full vertex and setting $e^2=1$, the coupled DSEs for the fermion propagator and photon vacuum polarization in covariant gauge reduce to the following:

$$A(p^2) = 1 + \frac{1}{p^2}\int \frac{d^3k}{(2\pi)^3}\frac{A(k^2)[2(1-\xi')(pq)(kq)/q^2 + \xi' pk/q^2]}{q^2[A^2(k^2)k^2 + B^2(k^2)][1+\Pi(q^2)]}f[A(p^2),A(k^2)] \tag{11}$$

$$B(p^2) = \int \frac{d^3k}{(2\pi)^3}\frac{f[A(p^2),A(k^2)]B(k^2)(2+\xi')}{q^2[A^2(k^2)k^2 + B^2(k^2)][1+\Pi(q^2)]} \tag{12}$$

$$\Pi(q^2) = \frac{2}{q^2}\int \frac{d^3k}{(2\pi)^3}\frac{A(k^2)A(p^2)[2k^2 - 4(kq) - 6(kq)^2/q^2]}{[A^2(k^2)k^2 + B^2(k^2)][A^2(p^2)p^2 + B^2(p^2)]}f[A(p^2),A(k^2)] \tag{13}$$

where $p=q+k$ and $\xi' = \xi[1+\Pi(q^2)]$. $\Pi(q^2)$ from Eq. (12) is apparently affected by the parameter $(2+\xi')$ and is proportional to the fermion self-energy, in the sense that covariant gauge alters the coupling constant in this framework. One expects that the self-energy can increase with the accretion of $\xi$. In the next section, the numerical results for the influence of $\xi$ on the fermion mass and condensate are given, where the fermion mass is defined as

$$m(p^2) = \frac{B(p^2)}{A(p^2)} \tag{14}$$

and the condensate is trivially obtained by[8]

$$\langle\bar\psi\psi\rangle = \mathrm{Tr}[S(x\equiv 0)] = 4\int \frac{d^3p}{(2\pi)^3}\frac{B(p^2)}{A^2(p^2)p^2 + B^2(p^2)} \tag{15}$$

## 2 Results

First, we consider the DCSB phase, i. e. Eq. (12) has a solution $B(p^2) \neq 0$. From Eq. (15), it can be found that chiral symmetry of the system is broken where the originally massless bare fermion will acquire a dynamical mass through nonperturbative effects.

The task now is to obtain $A(p^2)$ and $B(p^2)$. We numerically solve the three coupled equations (11) to (13). Starting from $A(p^2)=1$, $B(p^2)=1$ and $\Pi(q^2)=1$, we iterate the three coupled equations for any ansatze until all the three functions converge to a stable solution, and the typical behaviors of the mass functions $m(p^2)$ and $\Pi(q^2)$ in the DCSB phase for several $\xi$ are plotted in Fig. 1. In the case of $\xi=0$, the choice of a covariant gauge is called as a Landau gauge which has frequently been adopted in previous works. Another choice is the Feymann gauge where $\xi=1$. The others are chosen to see how much the results depend on our particular choice.

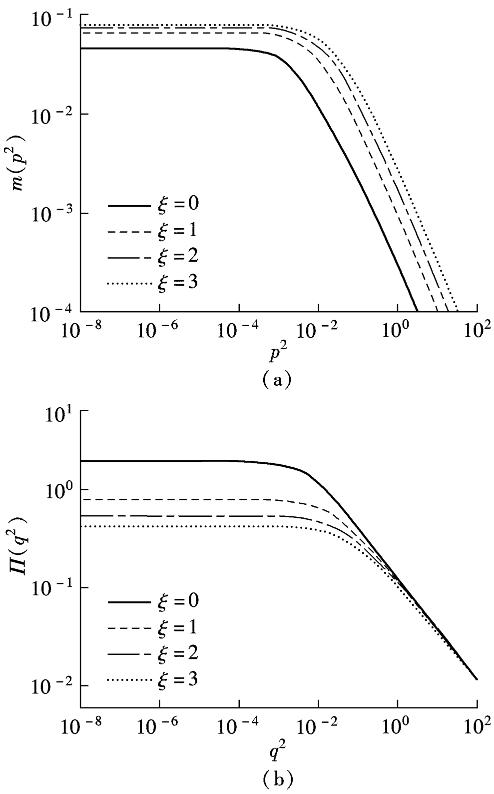


**Fig. 1** Typical behavior of $m(p^2)$ and $\Pi(q^2)$ for several $\xi$ for ansatze 2. (a) $m(p^2)$; (b) $\Pi(q^2)$

Generally, using the above explicit solution, we find from Fig. 1 that the two functions decrease with $p^2$ and approach zero as $p^2 \to \infty$ for all $\xi$ and we also observe that, for any choices of covariant gauge, the two functions are almost constant in the infrared region while the functions behave like $m(p^2) \to 1/p^2$, and $\Pi(q^2) \propto 1/q$ in the ultraviolet region. From Fig. 1(b), it can be seen that the infrared values of the two functions apparently decrease with $\xi$ increasing, but $B(p^2)$ is affected apparently by the choice of covariant gauge in all the energy region, while $\Pi(q^2)$ is affected little only in the high energy region.

In the following, we shall employ our numerical results to study the influence on fermion condensate in $QED_3$. First, we recall some known results in the literature. The Lagrangian of massless $QED_3$ is chirally symmetric due to the absence of the bare fermion mass term $m_0 \bar{\psi}\psi$, but chiral symmetry may be broken spontaneously when a fermion mass is generated dynamically. Since the condensate is defined via a fermion propagator, it should change with $\xi$. Based on the bare vertex, the trend of function $\langle \bar{\psi}\psi \rangle_\xi$ is shown in Fig. 2.

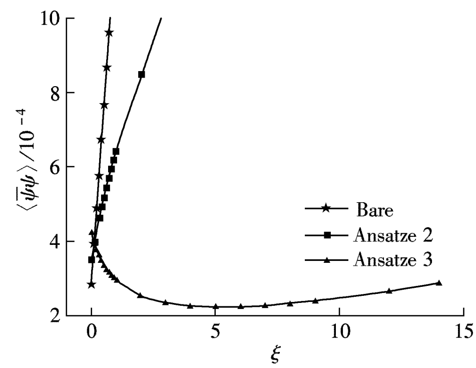


**Fig. 2** Gauge dependence of the fermion chiral condensate

The condensate decreases with $\xi$ increasing when $0 \leqslant \xi < 5$ and increases with covariant gauge for $\xi > 5$. Nevertheless, the condensate alters quickly from the Landau gauge to the Feymann gauge, but slowly in the right of Fig. 2. Because of the complexity of QCD, rainbow approximation is wildly used in hadron theory where Landau and Feymann gauge are adopted. Comparing two condensates for the two gauges, we find that the value of the condensate at Landau gauge $\langle \bar{\psi}\psi \rangle_L$ is greater than the corresponding value at Feymann gauge $\langle \bar{\psi}\psi \rangle_F$. $\langle \bar{\psi}\psi \rangle_L$ enlarges about 1.5 times more than $\langle \bar{\psi}\psi \rangle_F$.

Beyond rainbow approximation, using another two reasonable ansatzes for the full vertex, we obtain the values of condensate for a range of $\xi$, and also plot them in Fig. 2. By all the appearances, based on ansatzes 2 and 3, the trend of condensate about $\xi$ is generally incremental and is apparently different from that obtained above. However, the D-S equation with the two ansatzes gives the smaller value of condensate than that with a bare vertex at $\xi=0$. Near the Landau gauge, the condensate obviously increases with $\xi$ and the value is greater than that obtained when we use a bare vertex for $\xi > 0.1$.

We expect that this difference due to different choices of covariant gauge may qualitatively change the results in gauge theory. These can also be derived by Landau-Khalatnikov-Fradkin transformations. Nevertheless, no phase transition occurs, though the values of $B(p^2)$ and $\Pi(q^2)$ in low energy and condensate all change with $\xi$ altering for each ansatze.

From the above, we see that the two physical values all depend on the gauge parameter for any truncated scheme. In rainbow approximation, the bare vertex is compatibly used only in the high energy region, but the condensate and dynamical fermion mass is the low energy phenomenon. So it is obvious that the physical values cannot be obtained. However, the other two ansatzes, beyond bare vertex, are adopted. One of them is considered to play the dominant role in the BC vertex, whereas $\langle \bar{\psi}\psi \rangle$ increases with $\xi$. The results show that one also cannot obtain real physical parameters by those familiar truncated schemes since the tensors destroy the symmetry of the system. Moreover, from Fig. 2, the condensate obtained in ansatze 2 alters more rapidly than that in rainbow approximation. This result implies that the ansatze which is obtained only from WTI is incompetent for the full fermion-boson vertex.

## 3 Conclusion

We investigate the influence of the vacuum polarization on the photon propagator, in massless $QED_3$, using the truncated Dyson-Schwinger equations for the fermion and photon propagators with a range of the choices of covariant gauge. Numerical results show that the values of condensate and fermion mass depend on the truncated scheme for the D-S equation. Based on the finite coupled equation, it is also found that the functions of the fermion propagator apparently depend on $\xi$, but the boson propagator is only in the infrared region. Since $QED_3$ is nonperturbative, the difference in low energy can alter numerical results. With the increase in $\xi$, the fermion condensate decreases in bare vertex for $0 \leqslant \xi < 5$ and increases beyond rainbow approximation for all $\xi$.

Since the condensate should be gauge invariant, it is necessary to adopt a fitter fermion boson vertex which keeps the symmetry of the system to study a nonperturbative system. After investigating the previous works, these studies appear to suggest that the Landau-gauge boson propagator is finite and nonzero at $q^2 = 0$. This is consistent with an analysis of an approximate D-S equation for the boson vacuum polarization using the gauge technique before we find the satisfied ansatze for the full vertex; the Landau gauge should be appropriate. Compared with unquenched $QED_3$, quenched approximation little affects the critical behavior at familiar covariant gauge in nonperturbative systems. In addition, although the truncated schemes destroy the WTI, our results show that gauge dependence in unquenched $QED_3$ is partly different from that in quenched $QED_3$.

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# 采用非淬火模型研究手征凝聚和费米子质量的规范依存性

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**摘要:**基于三维量子电动力学理论,用截断的 Dyson-Schwinger (D-S)方程研究带有真空极化效应的光子传播子和费米传播子. 数值计算表明,凝聚和费米质量的数值取决于截断方案. 在有限耦合 D-S 方程基础上,还发现费米传播子函数明显依赖于序参量 $\xi$. 采用非淬火近似,利用截断的费米子所满足的 D-S 方程研究了费米子手征凝聚和费米子的质量函数. 在不同的截断方案下,发现采用非淬火近似得到的结果均明显不同于采用淬火近似所得的结果:前者有明显的回弯现象,即 $0 \leqslant \xi < 5$ 范围内,随 $\xi$ 的增加,费米凝聚减少,在 $\xi > 5$ 的所有范围内,费米凝聚增加;而淬火近似没有此现象. 这说明淬火近似可能有缺陷.

**关键词:**三维量子电动力学;非淬火;费米子-玻色子顶角;费米子手征凝聚;淬火;缺陷

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