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On Gorenstein FP-injective modules

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Abstract: An R-module M is called Gorenstein FP-injective if there is an exact sequence $\cdots\to E_1\to E_0\to E^0\to E^1\to\cdots$ of FP-injective R-modules with $M=\ker(E^0\to E^1)$ and such that $\mathrm{Hom}(E, -)$ leaves the sequence exact whenever E is an FP-injective R-module. Some properties of Gorenstein FP-injective are obtained. Moreover, it is proved that a ring is left Noetherian if and only if every Gorenstein FP-injective left R-module is Gorenstein injective. Furthermore, it is shown that over an n-FC and perfect ring R, a left R-module M is Gorenstein FP-injective if and only if $M\cong F\oplus H$ for some FP-injective left R-module F and some strongly Gorenstein FP-injective R-module H. In view of this, Gorenstein FP-injective precovers and Gorenstein FP-injective preenvelopes are considered.

Key words: coherent ring; Gorenstein FP-injective dimension; Gorenstein FP-injective precover; Gorenstein FP-injective preenvelope

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Let R be a ring. A left R-module M is called FP-injective (or absolutely pure) [1-2], if $\mathrm{Ext}^1(N, M)=0$ for all finitely presented left R-modules N. The FP-injective dimension of M, denoted by FP-id(M), is defined to be the smallest nonnegative integer n such that $\mathrm{Ext}^{n+1}(F, M)=0$ for every finitely presented left R-module F. If no such n exists, set FP-id$(M)=\infty$. In what follows, we write FI for the class of all the FP-injective left R-modules. Let C be a class of left R-modules and M a left R-module. According to Ref. [3], we say that a homomorphism φ: $M\to C$ is a C-preenvelope of M if $C\in C$ and the abelian group homomorphism $\mathrm{Hom}(\varphi, C')$: $\mathrm{Hom}(C, C')\to\mathrm{Hom}(M, C')$ is subjective for each $C'\in C$. A C-preenvelope φ: $M\to C$ is called a C-envelope, if every endomorphism f: $C\to C$ such that $f\varphi=\varphi$ is an isomorphism. Dually, we have the definitions of C-precovers and C-covers. C-envelopes (C-covers) may not exist in general, but if they exist, they are unique up to isomorphisms. It has been recently proved that every left R-module has an FP-injective (pre)cover over a left coherent ring R[4].

Recall that a left R-module N is called Gorenstein injective[5], if there is a $\mathrm{Hom}(\mathrm{Inj}, -)$ exact exact sequence $\cdots\to E_1\to E_0\to E^0\to E^1\to\cdots$ of injective left R-modules such that $N=\ker(E^0\to E^1)$, where Inj stands for the class of all the injective left R-modules. A right R-module M is called Gorenstein flat[5], if there is a $-\otimes\mathrm{Inj}$ exact exact sequence $\cdots\to F_1\to F_0\to F^0\to F^1\to\cdots$ of flat right R-modules such that $M=\ker(F^0\to F^1)$. Gorenstein injective and Gorenstein flat modules have been studied by many authors[5-8]. These modules have nice properties when the ring in question is n-Gorenstein (a ring R is called n-Gorenstein if R is a left and right Noetherian ring with self-injective dimension at most n for an integer $n\geqslant 0$ on either side). In Ref. [9], an R-module M is called Gorenstein FP-injective if there is an exact sequence $\cdots\to E_1\to E_0\to E^0\to E^1\to\cdots$ of injective R-modules with $M=\ker(E^0\to E^1)$ and such that $\mathrm{Hom}(E, -)$ leaves the sequence exact whenever E is an FP-injective R-module. In this paper, we call it a strongly Gorenstein FP-injective module.

In this paper, a left R-module M is called Gorenstein FP-injective if there is an exact sequence $\cdots\to E_1\to E_0\to E^0\to E^1\to\cdots$ of FP-injective left R-modules with $M=\ker(E^0\to E^1)$ and such that $\mathrm{Hom}(E, -)$ leaves the sequence exact whenever E is an FP-injective left R-module. It is well known that a ring R is left Noetherian if and only if every FP-injective left R-module is injective[10]. There is a generalization of this classical result. It is proved that a ring is left Noetherian if and only if every FP-injective left R-module is strongly Gorenstein FP-injective if and only if every Gorenstein FP-injective left R-module is Gorenstein injective (see Theorem 1). This also shows that "a left coherent ring R" in Ref. [9] is superfluous.

Recall that R is called an n-FC ring[6], if R is a left and right coherent ring with FP-id$({}_RR)\leqslant n$ and FP-id$(R_R)\leqslant n$ for an integer $n\geqslant 0$. If R is an n-FC ring, we prove that the weak global dimension $\mathrm{wD}(R)\leqslant n$ if and only if every Gorenstein FP-injective left R-module is FP-injective (see Proposition 1). It is shown that over an n-FC and perfect ring R, a left R-module M is Gorenstein FP-injective if and only if $M\cong F\oplus H$ for some FP-injective left R-module F and some strongly Gorenstein FP-injective R-module H (see Theorem 2); and then for an n-FC and perfect ring R, every R-module has a Gorenstein FP-injective preenvelope, and every pure-injective R-module has a Gorenstein FP-injective precover. If R is a two-sided coherent and perfect ring, then R is a QF ring if and only if every R-module is Gorenstein FP-injective if and only if R is a 0-FC ring. If n is a nonnegative integer, R is a two-sided coherent and perfect ring, then R is an n-FC ring if and only if every n-th Inj-cosyzygy of an R-module is Gorenstein FP-injective if and only if every n-th minimal FI-syzygy of an R-module is Gorenstein FP-injective.

We begin with the following definition.

Definition 1 Let R be a ring. A left R-module M is called Gorenstein FP-injective if there exists an exact sequence $E=\cdots\to E_1\to E_0\to E^0\to E^1\to\cdots$ of FP-injective left R-modules such that $M=\ker(E^0\to E^1)$ and such that $\mathrm{Hom}(E, -)$ leaves the sequence exact whenever E is an FP-

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injective R-module.

Remark 1 1) Obviously, we have the following implications:

injective modules $\Rightarrow$ FP-injective modules $\Rightarrow$ Gorenstein FP-injective modules;

injective modules $\Rightarrow$ strongly Gorenstein FP-injective modules $\Rightarrow$ Gorenstein injective modules;

strongly Gorenstein FP-injective modules $\Rightarrow$ Gorenstein FP-injective modules.

2) If R is left Noetherian, then the class of Gorenstein FP-injective left R-modules coincides with that of Gorenstein injective left R-modules. What about the converse? We will give a positive answer in Theorem 1.

3) By symmetry, all the images, the kernels and the cokernels of E are Gorenstein FP-injective.

The class of Gorenstein FP-injective left R-modules is closed under direct products by definition.

It is well known that a ring R is left Noetherian if and only if every FP-injective left R-module is injective (see Theorem 3 in Ref. [10]). Now, we give a generalization.

Theorem 1 The following are equivalent for any ring R.

1) R is left Noetherian;

2) Every FP-injective left R-module is Gorenstein injective;

3) Every FP-injective left R-module is strongly Gorenstein FP-injective;

4) Every Gorenstein FP-injective left R-module is Gorenstein injective;

5) Every Gorenstein FP-injective left R-module is strongly Gorenstein FP-injective.

Proof 1)$\Rightarrow$3)$\Rightarrow$2), 1)$\Rightarrow$4)$\Rightarrow$2) and 1)$\Rightarrow$5)$\Rightarrow$3) are trivial.

2)$\Rightarrow$1). Let I be any set and $E_i(i\in I)$ be injective left R-modules. By 2), we obtain that $\oplus_{i\in I}E_i$ is Gorenstein injective. Hence, there is a Hom(Inj, $-$) exact sequence $E\to\oplus_{i\in I}E_i$ with E injective. It follows that $\mathrm{Hom}(E_i, E)\to\mathrm{Hom}(E_i, \oplus_{i\in I}E_i)\to 0$ is exact, for any E_i, $i\in I$. This induces that $\prod_{i\in I}\mathrm{Hom}(E_i, E)\to\prod_{i\in I}\mathrm{Hom}(E_i, \oplus_{i\in I}E_i)\to 0$ is exact, that is, $\mathrm{Hom}(\oplus_{i\in I}E_i, E)\to\mathrm{Hom}(\oplus_{i\in I}E_i, \oplus_{i\in I}E_i)\to 0$ is exact. Therefore, $\oplus_{i\in I}E_i$ is isomorphic to a direct summand of E, and so it is injective. Thus 1) holds by Proposition 18.13 in Ref. [11].

Remark 2 By the previous theorem, we obtain that a left coherent ring R in Ref. [9] is superfluous. For any ring R, if the class of strongly Gorenstein FP-injective left R-modules is closed under direct sums, then R is left Noetherian. A strongly Gorenstein FP-injective module need not be FP-injective. Let $R=Z/4Z$, where Z is the ring of integers. Then R is a quasi-Frobenius ring, every projective (resp. injective) R-module is injective (resp. projective). Thus, $2R$ is strongly Gorenstein FP-injective, since there is an exact sequence $\cdots\xrightarrow{f}R\xrightarrow{f}R\xrightarrow{f}\cdots$, where $f(x)=2x$ for $x\in 2R$. It is clear that $\mathrm{im}(f)=\ker(f)=2R$. But $2R$ is not injective.

Lemma 1 Let R be a left coherent ring and M be a Gorenstein FP-injective left R-module. Then the FP-injective dimension of M is zero or infinite.

Proof Assume that FP-id$(M)<\infty$. Then there is a Hom(FI, $-$) exact exact sequence $0\to M\to E^0\to E^1\to\cdots\to E^n\to 0$ with each E^i FP-injective. It is easy to see that M is FP-injective.

Remark 3 If R is a left coherent ring with $\mathrm{wD}(R)<\infty$, then the class of Gorenstein FP-injective left R-modules coincides with that of FP-injective left R-modules. What about the converse? This is a partial answer in the following proposition.

Proposition 1 The following are equivalent for an n-FC ring R.

1) $\mathrm{wD}(R)\leqslant n$;

2) Every Gorenstein flat right R-module is flat;

3) Every Gorenstein FP-injective left R-module is FP-injective;

4) Every strongly Gorenstein FP-injective left R-module is FP-injective;

5) Every strongly Gorenstein FP-injective left R-module is injective.

Proof 1)$\Leftrightarrow$2)$\Leftrightarrow$4)$\Leftrightarrow$5) follows from Refs. [6, 9]. 1)$\Rightarrow$3) and 3)$\Rightarrow$4) follow from Lemma 1 and 1) of Remark 1.

Recall that a left R-module M is called FI-injective if $\mathrm{Ext}^1(G, M)=0$ for any FP-injective left R-module G[12]. For convenience, we shall write "R-module" to mean "left and right R-module" in the rest of this paper unless otherwise specified.

Lemma 2 Let R be a left coherent ring and M be a Gorenstein FP-injective left R-module. Then $M=F\oplus H$ for some FP-injective left R-module F and some FI-injective left R-module H.

Proof Since M is Gorenstein FP-injective, there is a Hom(FI, $-$) exact exact sequence $\cdots\to E_1\xrightarrow{d_1}E_0\xrightarrow{d_0}E^0\xrightarrow{d^0}E^1\to\cdots$ of FP-injective modules with $M=\ker(E^0\to E^1)$. Set $K=\mathrm{coker}(M\to E^0)$. By Ref. [4], K has an FP-injective cover: $G^0\to K$ with kernel M^0. So we have the commutative diagram with exact rows:

$$\begin{tikzcd}
0 \arrow[r] & M^0 \arrow[r, "f"] \arrow[d, "\phi"] & G^0 \arrow[r] \arrow[d, "\gamma"] & K \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M \arrow[r, "\alpha"] \arrow[d, "\sigma"] & E^0 \arrow[r] \arrow[d, "\beta"] & K \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M^0 \arrow[r, "f"] & G^0 \arrow[r] & K \arrow[r] & 0
\end{tikzcd}$$

Note that $\beta\gamma$ is an isomorphism, and so $E^0=\ker(\beta)\oplus\mathrm{im}(\gamma)$. Thus $\mathrm{im}(\gamma)\cong G^0$ and $\ker(\beta)$ are FP-injective. Moreover, M^0 is an FI-injective module by Ref. [13]. Since $\sigma\phi$ is an isomorphism by the Five Lemma, we have $M=\ker(\sigma)\oplus\mathrm{im}(\phi)$, where $\mathrm{im}(\phi)\cong M^0$. In addition, we obtain the commutative diagram:

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & \ker(\sigma) \arrow[r] \arrow[d] & \ker(\beta) \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & M \arrow[r, "\alpha"] \arrow[d, "\sigma"] & E^0 \arrow[r] \arrow[d, "\beta"] & K \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & M^0 \arrow[r, "f"] & G^0 \arrow[r] & K \arrow[r] & 0
\end{tikzcd}$$

Hence $\ker(\sigma)\cong\ker(\beta)$ by the 3×3 Lemma in Ref. [14].

This completes the proof.

Theorem 2 Let R be an n-FC and perfect ring. Then M is a Gorenstein FP-injective left R-module if and only if $M = F \oplus H$ for some FP-injective left R-module F and some strongly Gorenstein FP-injective left R-module H.

Proof "$\Leftarrow$" is clear.

"$\Rightarrow$". Since M is Gorenstein FP-injective, there is a Hom(FI, $-$) exact exact equence $\cdots \to E_1 \xrightarrow{d_1} E_0 \xrightarrow{d_0} E^0 \xrightarrow{d^0} E^1 \to \cdots$ of FP-injective modules with $M = \ker(E^0 \to E^1)$. Set $K = \operatorname{coker}(M \to E^0)$. By Theorem 2. 6 in Ref. [4], K has an FP-injective cover $G^0 \to K$ with kernel M^0. By Lemma 2, we obtain that $M = M^0 \oplus H^0$, where H^0 is FP-injective and M^0 is FI-injective. Let $\cdots \to F_1 \to F_0 \to M^0 \to 0$ be a minimal left FI-resolution of M^0. From Lemma 3. 13 in Ref. [12], we obtain that each F_i is injective. This means that $\cdots \to F_1 \to F_0 \oplus H^0 \to M^0 \oplus H^0 \to 0$ is an FI-resolution of M. Since M has an exact left FI-resolution by hypothesis, in terms of Theorem 8. 2. 14 in Ref. [5], we obtain that $\cdots \to F_1 \to F_0 \to M^0 \to 0$ is exact. Therefore, M^0 is strongly Gorenstein FP-injective by Lemma 3. 1 in Ref. [9].

Corollary 1 Let R be an n-FC and perfect ring, and M be an R-module.

1) If M has a strongly Gorenstein FP-injective preenvelope, then M has a Gorenstein FP-injective preenvelope;

2) If M has a strongly Gorenstein FP-injective precover, then M has a Gorenstein FP-injective precover.

Proof 1) Let M be an R-module. Then M has an FP-injective preenvelope f: $M \to F$ by Ref. [5]. On the other hand, by hypothesis, there is a strongly Gorenstein FP-injective preenvelope g: $M \to G$. We claim that h: $M \to F \oplus G$ $(h(m) = f(m) + g(m),\ \forall m \in M)$ is a Gorenstein FP-injective preenvelope. For any Gorenstein FP-injective N, $N \cong F_1 \oplus G_1$ for some FP-injective left R-module F_1 and some strongly Gorenstein FP-injective left R-module G_1 by Theorem 2. Let t_1: $F_1 \oplus G_1 \to F_1$ and t_2: $F_1 \oplus G_1 \to G_1$ be projections. Hence, for any homomorphism α: $M \to N$, there are β: $F \to F_1$ and γ: $G \to G_1$ such that $t_1\alpha = \beta f$ and $t_2\alpha = \gamma g$. This shows that $\alpha = (\beta \oplus \gamma)h$, and so 1) holds.

2) Let M be an R-module. Then M has an FP-injective cover f: $F \to M$ by Theorem 2. 6 in Ref. [4]. On the other hand, by hypothesis, there is a strongly Gorenstein FP-injective precover g: $G \to M$. We claim that h: $F \oplus G \to M$ $(h(a+b) = f(a) + g(b),\ \forall a \in F,\ b \in G)$ is a Gorenstein FP-injective precover. For any Gorenstein FP-injective N, $N \cong F_1 \oplus G_1$ for some FP-injective left R-module F_1 and some strongly Gorenstein FP-injective left R-module G_1 by Theorem 2. Let η_1: $F_1 \to F_1 \oplus G_1$ and η_2: $G_1 \to F_1 \oplus G_1$ be injections. Hence, for any homomorphism α: $N \to M$, there are β: $F_1 \to F$ and γ: $G_1 \to G$ such that $\alpha\eta_1 = f\beta$ and $\alpha\eta_2 = g\gamma$. This shows that $\alpha = h(\beta \oplus \gamma)$, and so 2) holds.

Corollary 2 Let R be an n-FC and perfect ring. Then

1) Every R-module M has a Gorenstein FP-injective preenvelope f: $M \to F \oplus G$ such that $f\pi_1$ and $f\pi_2$ are monic, $\mathrm{id}(L) \leqslant n-1$ whenever $n \geqslant 1$, where π_1: $F \oplus G \to F$ and π_2: $F \oplus G \to G$ are projections, $L = \operatorname{coker}(f\pi_2)$. Moreover, if $\mathrm{id}(M) < \infty$, then G is injective.

2) Every pure-injective R-module has a Gorenstein FP-injective precover.

Proof This result follows from Corollary 1, Theorem 3. 3 and Theorem 4. 1 in Ref. [9].

Proposition 2 The following are equivalent for a two-sided coherent and perfect ring R.

1) R is a QF ring (i. e., 0-Gorenstein ring);

2) Every R-module is Gorenstein FP-injective;

3) R is an FC ring.

Proof 1) $\Rightarrow$ 2) follows by Proposition 2. 6 in Ref. [15].

2) $\Rightarrow$ 3). By 2), R is Gorenstein FP-injective, and so there is an epimorphism $E \to R$ with E FP-injective. Therefore, FP-id$(R) = 0$, that is, R is an FC ring.

3) $\Rightarrow$ 1). By Theorem 8. 4. 31 in Ref. [5], the flat dimension of any FP-injective R-module $E \leqslant 0$. Note that R is perfect. Hence each FP-injective R-module is projective, that is, R is a QF ring.

Proposition 3 Let n be a nonnegative integer. Then the following are equivalent for a two-sided coherent and perfect ring R.

1) R is an n-FC ring;

2) Every n-th Inj-cosyzygy of an R-module is Gorenstein FP-injective;

3) Every n-th minimal FI-syzygy of an R-module is Gorenstein FP-injective.

Proof 2) $\Leftrightarrow$ 1). By Theorem 8. 4. 31 in Ref. [5], R is an n-FC ring if and only if the flat dimension of any FP-injective R-module $E \leqslant n$. Note that R is perfect. Then R is an n-FC ring if and only if the projective dimension of any FP-injective R-module $E \leqslant n$ if and only if every n-th Inj-cosyzygy of an R-module is Gorenstein FP-injective since $\mathrm{Ext}^{n+1}(E, M) = 0$ for any R-module M.

3) $\Rightarrow$ 1) follows from Theorem 8. 4. 31 in Ref. [5].

2) $\Rightarrow$ 3). If $n = 0$, 1) $\Rightarrow$ 3) by Proposition 2.

Suppose that $n \geqslant 1$. Let M be any left R-module. By Theorem 2. 6 in Ref. [4], M has an FP-injective cover: $F_0 \to M$ with kernel K_0. Thus K_0 is FI-injective by Lemma 2. 1. 1 in Ref. [13]. From Lemma 3. 13 in Ref. [12], we obtain that K_0 has an FP-injective cover $F_1 \to K_0$ with F_1 injective. Hence, if $\cdots \to F_1 \to F_0 \to M \to 0$ is the minimal left FI-resolution of M, then each $F_i\,(i \geqslant 1)$ is injective. Note that R is an n-FC ring by the equivalence of 1) and 2). By Theorem 8. 4. 31 in Ref. [5], every left FI-resolution is exact at F_i, for every $i \geqslant n-1$. This implies that the n-th syzygy K_n is Gorenstein FP-injective by 2).

Remark 4 Enochs and Jenda have proved that a two-sided Noetherian ring R is n-Gorenstein if and only if every n-th Inj-cosyzygy of an R-module is Gorenstein injective (see Theorem 12. 3. 1 in Ref. [5]), while Proposition 3 shows that a two-sided coherent and perfect ring R is n-FC if and only if every n-th Inj-cosyzygy of an R-module is Gorenstein FP-injective. The following is an example of a two-sided coherent and perfect ring which is not two-sided Noetherian. Let F be a field, V an infinite dimensional vector space over F, $R = \begin{pmatrix} F & 0 \\ V & F \end{pmatrix}$. Then R is a two-sided coherent and perfect ring but which is neither left Noetherian nor right Noetherian by Exercise 5B. 6 in Ref. [16].

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关于 Gorenstein FP-内射模

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摘要: R-模 M 称为是 Gorenstein FP-内射的, 如果存在一个 FP-内射 R-模正合列 $\cdots\to E_1\to E_0\to E^0\to E^1\to\cdots$, 其中 $M=\ker(E^0\to E^1)$, 使得对任意 FP-内射模 E, $\mathrm{Hom}(E,-)$ 保持正合列正合. 根据定义讨论了 Gorenstein FP-内射模的性质, 并且证明了若环 R 是左 Noetherian 环当且仅当每个 Gorenstein FP-内射左 R-模是 Gorenstein 内射左 R-模. 此外, 证明了在 n-FC 完全环 R 上, 若左 R-模是 Gorenstein FP-内射的当且仅当 $M\cong F\oplus H$, 其中 F 是 FP-内射左 R-模, H 是 Gorenstein FP-内射左 R-模. 由此考察了 Gorenstein FP-内射预覆盖与 Gorenstein FP-内射预包络的存在性.

关键词: 凝聚环; Gorenstein FP-内射维数; Gorenstein FP-内射预覆盖; Gorenstein FP-内射预包络
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