Journal of Southeast University (English Edition) Vol. 27, No. 1, pp. 119 – 122 Mar. 2011 ISSN 1003—7985

Existence for positive steady states of an eco-epidemiological model

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Abstract: An eco-epidemiological model with an epidemic in the predator and with a Holling type Ⅱ function is considered. A system with diffusion under the homogeneous Neumann boundary condition is studied. The existence for a positive solution of the corresponding steady state problem is mainly discussed. First, a prior estimates (positive upper and lower bounds) of the positive steady states of the reaction-diffusion system is given by the maximum principle and the Harnack inequation. Then, the non-existence of non-constant positive steady states by using the energy method is given. Finally, the existence of non-constant positive steady states is obtained by using the topological degree.

Key words: eco-epidemiological model; existence; positive steady states

doi: 10.3969/j.issn.1003 – 7985.2011.01.025

In this paper, we study a predator-prey system with an epidemic in the predator [1]. By the following scaling form,

$$X \mapsto u_1, S \mapsto u_2, I \mapsto u_3$$

the system takes the form

$$\left.\begin{aligned} \frac{\mathrm{d}u_1}{\mathrm{d}t} &= u_1\left(r - \frac{r}{K}u_1 - \frac{au_2}{1+bu_1}\right) \\ \frac{\mathrm{d}u_2}{\mathrm{d}t} &= u_2\left(\frac{eau_1}{1+bu_1} - D_1 - \beta u_3\right) \\ \frac{\mathrm{d}u_3}{\mathrm{d}t} &= u_3(\beta u_2 - D_2) \end{aligned}\right\} \quad (1)$$

where r, K, a, b, e, β, D_1 and D_2 are positive constants.

Now, if the predator and prey are confined to a fixed bounded domain Ω in $\mathbf{R}^n$ with a smooth boundary, then their densities are spatially inhomogeneous. From (1), we consider the following reaction-diffusion system,

$$\left.\begin{array}{ll} u_{1t} - d_1\Delta u_1 = u_1\left(r - \frac{r}{K}u_1 - \frac{au_2}{1+bu_1}\right) & x\in\Omega,\ t>0 \\ u_{2t} - d_2\Delta u_2 = u_2\left(\frac{eau_1}{1+bu_1} - D_1 - \beta u_3\right) & x\in\Omega,\ t>0 \\ u_{3t} - d_3\Delta u_3 = u_3(\beta u_2 - D_2) & x\in\Omega,\ t>0 \\ \frac{\partial u_1}{\partial v} = \frac{\partial u_2}{\partial v} = \frac{\partial u_3}{\partial v} = 0 & x\in\partial\Omega, t>0 \\ u_i(x,0)\geqslant 0 & i=1,2,3; x\in\Omega \end{array}\right\} \quad (2)$$

where v is the outward unit normal vector of the boundary $\partial\Omega$. Denote $\boldsymbol{u} = (u_1, u_2, u_3)^{\mathrm{T}}$.

The existence of positive steady-state solutions has been studied by the degree theorem and the bifurcation technique in many works[2–8] for reactive diffusion predator-prey systems.

1 A Priori Estimates of Positive Steady States

The main purpose of this section is to give a priori positive lower and upper bounds for the positive steady states of (2). The corresponding steady-state problem of (2) is the elliptic system,

$$\left.\begin{array}{ll} -d_1\Delta u_1 = G_1(\boldsymbol{u}) & x\in\Omega \\ -d_2\Delta u_2 = G_2(\boldsymbol{u}) & x\in\Omega \\ -d_3\Delta u_3 = G_3(\boldsymbol{u}) & x\in\Omega \\ \frac{\partial u_1}{\partial v} = \frac{\partial u_2}{\partial v} = \frac{\partial u_3}{\partial v} = 0 & x\in\partial\Omega \end{array}\right\} \quad (3)$$

For convenience, we shall write Λ instead of the constants (r, K, a, b, e, β, D_1, D_2). We use two propositions from Refs. [9 – 10]. In this paper, by the classical solutions, we consider solutions in $C^2(\Omega)\cap C^1(\overline{\Omega})$. The results of upper bounds can be stated as follows.

Theorem 1 (upper bound) For any positive classical solution (u_1, u_2, u_3) of (3), if $M_3 \geqslant 0$, then

$$\max_{\Omega} u_1 \leqslant K,\ \max_{\Omega} u_2 \leqslant M_2,\ \max_{\Omega} u_3 \leqslant M_3 \quad (4)$$

where

$$M_2 = \frac{eu_1K}{d_2} + \frac{erK}{4D_1}$$

$$M_3 = \frac{eu_1K}{d_3} + \frac{erd_2K}{4d_3D_1} + \frac{e^2ad_2K^2}{d_2D_2} + \frac{e^2arK^2}{4D_1D_2} - \frac{D_1}{D_2}$$

Proof Since $u_1\left(r - \frac{r}{K}u_1 - \frac{au_2}{1+bu_1}\right) \leqslant u_1\left(r - \frac{r}{K}u_1\right)$, the first result easily follows the maximum principle. Then let $\omega = ed_1u_1 + d_2u_2$, we can conclude that

$$\left.\begin{array}{ll} -\Delta\omega = eu_1\left(r - \frac{r}{K}u_1\right) - D_1u_2 - \beta u_2u_3 & x\in\Omega \\ \partial_v\omega = 0 & x\in\partial\Omega \end{array}\right\}$$

Let $\omega(x_0) = \max_{\Omega} \omega(x)$. By the application of the maximum principle, it yields

$$D_1u_2(x_0) \leqslant \frac{erK}{4}$$

Consequently,

Received 2010-05-12.

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Foundation item: The National Natural Science Foundation of China (No. 10601011).

Citation: Wang Yaping, Chen Wenyan. Existence for positive steady states of an eco-epidemiological model[J]. Journal of Southeast University (English Edition), 2011, 27(1): 119 – 122. [doi: 10.3969/j.issn.1003 – 7985.2011.01.025]

$$d_2\max_{\Omega} u_2 \leqslant \max_{\Omega} \omega(x) = \omega(x_0)$$

and hence

$$\max_{\Omega} u_2(x) \leqslant M_2$$

Then, let $\mu = d_2u_2 + d_3u_3$, by the same way, we obtain

$$\max_{\Omega} u_3(x) \leqslant M_3$$

Theorem 2 (lower bound) Let $\underline{\Lambda}$, $\underline{d}_1$, $\underline{d}_2$, $\underline{d}_3$ be fixed positive constants. Assume that $(d_1, d_2, d_3) \in [\underline{d}_1, \infty) \times [\underline{d}_2, \infty) \times [\underline{d}_3, \infty)$, and

$$K - \frac{Ka}{r}M_2 > \frac{D_1}{ea - bD_1} \tag{5}$$

Then there exists a positive constant $\underline{C} = \underline{C}(\underline{\Lambda}, \underline{d}_1, \underline{d}_2, \underline{d}_3)$, such that every positive classical solution (u_1, u_2, u_3) of (3) satisfies

$$\min_{\Omega} u_i(x) > \underline{C} \qquad \mathrm{i} = 1, 2, 3 \tag{6}$$

Proof Let

$$c_1(x) = d_1^{-1}\left(r - \frac{r}{K}u_1 - \frac{au_2}{1 + bu_1}\right)$$
$$c_2(x) = d_2^{-1}\left(\frac{eau_1}{1 + bu_1} - D_1 - \beta u_3\right)$$
$$c_3(x) = d_3^{-1}(\beta u_2 - D_2)$$

Then, in view of (4), there exists a positive constant $\overline{C}(d, \Lambda)$ such that $\|c_1\|_\infty, \|c_2\|_\infty, \|c_3\|_\infty \leqslant \overline{C}$, if $d_1, d_2, d_3 \geqslant d$.

The Harnack inequality in Ref. [9] shows that there exists a positive constant $C_* = C_*(d, \Lambda)$ such that

$$\max_{\Omega} u_i \leqslant C_* \min_{\Omega} u_i \qquad i = 1, 2, 3 \tag{7}$$

Now, on the contrary, suppose that (6) does not hold. Then there exists a sequence $\{d_{1i}, d_{2i}, d_{3i}\}_{i=1}^{\infty}$ with $d_{1i}, d_{2i}, d_{3i} \in [\underline{d}_1, \infty) \times [\underline{d}_2, \infty) \times [\underline{d}_2, \infty)$ such that the corresponding positive solutions (u_{1i}, u_{2i}, u_{3i}) of (3) satisfy

$$\max_{\Omega} u_{1i} \to 0 \text{ or } \max_{\Omega} u_{2i} \to 0 \text{ or } \max_{\Omega} u_{3i} \to 0 \qquad i \to \infty \tag{8}$$

Integrating by parts, we obtain that

$$\left.\begin{aligned} &\int_{\Omega} u_{1i}\left[\left(r - \frac{r}{K}u_{1i} - \frac{au_{2i}}{1 + bu_{1i}}\right)\right]\mathrm{d}x = 0 \\ &\int_{\Omega} u_{2i}\left[\left(\frac{eau_{1i}}{1 + bu_{1i}} - D_1 - \beta u_{3i}\right)\right]\mathrm{d}x = 0 \\ &\int_{\Omega} u_{3i}[(\beta u_{2i} - D_2)]\,\mathrm{d}x = 0 \end{aligned}\right\} \tag{9}$$

for $i = 1, 2, \dots$. The standard regularity theorem for the elliptic equations yields that there exists a subsequence of (u_{1i}, u_{2i}, u_{3i}), which we shall still denote by (u_{1i}, u_{2i}, u_{3i}), and three non-negative functions u_1, u_2, $u_3 \in C^2(\overline{\Omega})$, such that $(u_{1i}, u_{2i}, u_{3i}) \to (u_1, u_2, u_3)$ in $\lceil C^2(\overline{\Omega}) \rceil^3$ as $i \to \infty$. By (8), we note that $u_1 \equiv 0$ or $u_2 \equiv 0$ or $u_3 \equiv 0$. Moreover, we assume that $(d_{1i}, d_{2i}, d_{3i}) \to (\overline{d}_1, \overline{d}_2)$, $\overline{d}_3 \in [\underline{d}_1, \infty) \times [\underline{d}_2, \infty) \times [\underline{d}_3, \infty)$. Let $i \to \infty$ in (9), and we obtain that

$$\left.\begin{aligned} &\int_{\Omega} u_1\left[\left(r - \frac{r}{K}u_1 - \frac{au_2}{1 + bu_1}\right)\right]\mathrm{d}x = 0 \\ &\int_{\Omega} u_2\left[\left(\frac{eau_1}{1 + bu_1} - D_1 - \beta u_3\right)\right]\mathrm{d}x = 0 \\ &\int_{\Omega} u_3[(\beta u_2 - D_2)]\,\mathrm{d}x = 0 \end{aligned}\right\} \tag{10}$$

We now consider the following three cases.

Case 1 $u_1 \equiv 0$. Since $u_{1i} \to u_1$ as $i \to \infty$, then $eau_{1i}/(1 + bu_{1i}) - D_1 - \beta u_{3i} < 0$ on $\overline{\Omega}$, for all $i \gg 1$.

Integrating the differential equation for u_{2i} over Ω by parts, we have

$$0 = d_{2i}\int_{\partial\Omega} \partial_\nu u_{2i}\mathrm{d}s = \int_{\Omega}\left(\frac{eau_{1i}}{1 + bu_{1i}} - D_1 - \beta u_{3i}\right)\mathrm{d}x < 0$$

for all $i \gg 1$, which is a contradiction.

Case 2 $u_2 \equiv 0$, $u_1 \neq 0$, on Ω, then the Hopf boundary lemma gives $u_1 > 0$ on $\overline{\Omega}$. Since $u_{2i} \to u_2$ as $i \to \infty$, then $\beta u_{2i} - D_2 < 0$ on $\overline{\Omega}$, for all $i \gg 1$.

Integrating the differential equation for u_{3i} over $\overline{\Omega}$ by parts, we have

$$0 = d_{3i}\int_{\partial\Omega} \partial_\nu u_{3i}\mathrm{d}s = \int_{\Omega} u_{3i}(\beta u_{2i} - D_2)\,\mathrm{d}x < 0$$

for all $i \gg 1$, which is a contradiction.

Case 3 $u_3 \equiv 0$, $u_2 \neq 0$, $u_1 \neq 0$, on $\overline{\Omega}$, then the Hopf boundary lemma gives $u_1 > 0$, $u_2 > 0$ on $\overline{\Omega}$, and u_1, u_2 satisfy

$$-\overline{d}_1\Delta u_1 = u_1\left(r - \frac{r}{K}u_1 - \frac{au_2}{1 + bu_1}\right)$$

Let $u_1(x_1) = \min_{\Omega} u_1(x)$, then, we have

$$\min_{\Omega} u_1(x) \geqslant K - \frac{aKM_2}{r}$$

Integrating by parts, we have

$$0 = d_{2i}\int_{\partial\Omega} \partial_\nu u_{2i}\mathrm{d}s = \int_{\Omega} u_{2i}\left(\frac{eau_{1i}}{1 + bu_{1i}} - D_1 - \beta u_{3i}\right)\mathrm{d}x > 0$$

for all $i \gg 1$, which is a contradiction.

2 Non-Existence of Non-Constant Positive Solutions

In this section we shall discuss the non-constant positive solutions to problem (3) when the diffusion coefficient d_1 varies and the other parameters d_1, d_3, Λ are fixed.

Theorem 3 Let d_1^* and d_3^* be fixed positive constants and satisfy $d_1^*\mu_1 > r$, $d_3^*\mu_1 > \beta M_2 - D_2$. Then there exists a positive constant $F_2 = F_2(\Lambda, d_1^*, d_3^*)$, such that when $d_1 > d_1^*$, $d_2 > F_2$, $d_3 > d_3^*$, problem (3) has no non-constant positive solution.

Proof For any $\varphi \in L^1(\Omega)$, let

$$\bar{\varphi} = \frac{1}{|\Omega|}\int_\Omega \varphi \mathrm{d}x \tag{11}$$

Multiplying the differential equation (3) by $\boldsymbol{u}-\bar{\boldsymbol{u}}$, and then integrating over Ω by parts, we have

$$\begin{aligned}\sum_{i=1}^{3}\int_\Omega d_i |\nabla u_i|^2 \mathrm{d}x = \sum_{i=1}^{3}\int_\Omega (\boldsymbol{G}_i(\boldsymbol{u}) - \boldsymbol{G}_i(\bar{\boldsymbol{u}}))(u_i - \bar{u}_i) \leqslant \\ (r+\varepsilon_1')(u-\bar{u}_1)^2 + \\ \left(\frac{ea}{1+b} - D_1 + C_1' + C_2'\right)(u_2-\bar{u}_2)^2 + \\ (\beta u_2^* - D_2 + \varepsilon_2')(u_3 - \bar{u}_3)^2\end{aligned} \tag{12}$$

for some positive constants $C_1' = C_1'(\Lambda, d_1^*, d_3^*, \varepsilon_1')$, $C_2' = C_2'(\Lambda, d_1^*, d_3^*, \varepsilon_2')$, where ε_1' and ε_2' are the arbitrary small positive constants arising from Young's inequality.

In view of the Poincaré inequality [11],

$$\mu_1 \int_\Omega (f-\bar{f})^2 \leqslant \int_\Omega |\nabla f|^2 \mathrm{d}x$$

where $\bar{f}$ is similar to (11), it follows from (12) that

$$\begin{aligned}\sum_{i=1}^{3}\int_\Omega d_i(u_i-\bar{u}_i)^2\mathrm{d}x \leqslant (r+\varepsilon_1')(u-\bar{u}_i)^2 + \\ \left(\frac{ea}{1+b} - D_1 + C_1' + C_2'\right)(u_2-\bar{u}_2)^2 + \\ (\beta u_2^* - D_2 + \varepsilon_2')(u_3-\bar{u}_3)^2\end{aligned} \tag{13}$$

Choose $\varepsilon_1', \varepsilon_2' > 0$ to be very small such that

$$d_1^*\mu_1 > r + \varepsilon_1',\ d_3^*\mu_1 > \beta M_2 - D_2 + \varepsilon_2'$$

Then (13) implies that $u_1 = \bar{u}_1 =$ constant, $u_3 = \bar{u}_3 =$ constant, and $u_2 = \bar{u}_2 =$ constant if

$$d_2 > F_2 \triangleq \mu^{-1}\left(\frac{ea}{1+b} - D_1 + C_1' + C_2'\right)$$

The proof is complete.

3 Existence of Non-Constant Positive Steady States

Let $0 = \mu_0 < \mu_1 \leqslant \mu_2 \leqslant \ldots$ be the eigenvalues of the operator $-\Delta$ on Ω with the homogeneous Neumann boundary condition, and $E(\mu_i)$ be the eigenspace corresponding to μ_i in $C^2(\bar{\Omega})$.

In this section, we discuss the existence of non-constant positive classical solutions to (3) when the diffusion coefficient d_3 varies and the parameters Λ, d_1, d_2 are kept fixed. When $a_{11} < 0$, (1) has no non-constant positive classical solutions[1]. In view of this reason, we shall restrict this discussion to the case where $a_{11} > 0$.

First, we shall study the linearization of (3) at $\tilde{\boldsymbol{u}}$. Define

$$X^+ = \{\boldsymbol{u} \in X \mid u_i > 0 \text{ on } \bar{\Omega},\ i = 1, 2, 3\}$$
$$B(C) = \{\boldsymbol{u} \in \boldsymbol{X} \mid C^{-1} < u_i < C \text{ on } \bar{\Omega},\ i = 1, 2, 3\}$$

where C is a positive constant that is guaranteed to exist by Theorems 1 and 2. Denote the matrix $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, d_3)$, then (3) can be written as

$$-\boldsymbol{D}\Delta\boldsymbol{u} = \boldsymbol{G}(\boldsymbol{u}) \text{ in } \Omega,\ \partial_\nu \boldsymbol{u} = 0 \text{ on } \Omega \tag{14}$$

and $\boldsymbol{u}$ is a positive solution to (14) if and only if

$$\boldsymbol{F}(\boldsymbol{u}) \triangleq \boldsymbol{u} - (\boldsymbol{I} - \boldsymbol{\Delta})^{-1}\{\boldsymbol{D}^{-1}\boldsymbol{G}(\boldsymbol{u}) + \boldsymbol{u}\} = 0 \text{ in } \boldsymbol{X}^+$$

We denote

$$H(d_1, d_2, d_3; \mu) \triangleq \det\{\mu \boldsymbol{I} - \boldsymbol{D}^{-1}\boldsymbol{G}_u(\tilde{\boldsymbol{u}})\} = \frac{1}{d_1 d_2 d_3}\det\{\mu\boldsymbol{D} - \boldsymbol{G}_u(\tilde{\boldsymbol{u}})\} \tag{15}$$

By the similar arguments as in Ref. [12], we have the following proposition.

Proposition 1 Suppose that, for all $i \geqslant 0$, the matrix $\mu_i \boldsymbol{I} - \boldsymbol{D}^{-1}\boldsymbol{G}_u(\tilde{\boldsymbol{u}})$ is nonsingular.

Then index $(\boldsymbol{F}(\cdot), \tilde{\boldsymbol{u}}) = (-1)^\gamma$, where

$$\gamma = \sum_{i \geqslant 0,\, H(d_1, d_2, d_3; \mu_i) < 0} \dim E(u_i)$$

To compute index $(\boldsymbol{F}(\cdot), \tilde{\boldsymbol{u}})$, we have to consider the sign of $H(d_1, d_2, d_3; \mu)$. The direct calculation gives

$$\det\{\mu\boldsymbol{D} - \boldsymbol{G}_{\mathbf{u}}(\tilde{\boldsymbol{u}})\} = A(d_3; \mu)$$

$$\lim_{d_3 \to \infty} \frac{A(d_3; \mu)}{d_3} = d_2\mu^2(d_1\mu - a_{11})$$

We can establish the following proposition.

Proposition 2 There exists a positive constant D_3^* such that when $d_3 \geqslant D_3^*$, the three roots $\tilde{\mu}_i(d_3; d_1, d_2)$, $i = 1, 2, 3$ of $A(d_3; \mu) = 0$ are all real and satisfy

$$\left.\begin{aligned}\lim_{d_3\to\infty}\tilde{\mu}_1(d_3; d_1, d_2) \mid = \mu_2(d_3; d_1, d_2) = 0 \\ \lim_{d_3\to\infty}\tilde{\mu}_3(d_3; d_1, d_2) \mid = \frac{a_{11}}{d_1} > 0\end{aligned}\right\} \tag{16}$$

Moreover, when $d_3 \geqslant D_3^*$,

$$\left.\begin{aligned}-\infty < \tilde{\mu}_1(d_3; d_1, d_2) < 0 < \tilde{\mu}_2(d_3; d_1, d_2) < \tilde{\mu}_3(d_3; d_1, d_2) \\ A(d_3; \mu) < 0 \text{ if } \mu \in (-\infty, \tilde{\mu}_1(d_3; d_1, d_2)) \cup \\ (\tilde{\mu}_2(d_3; d_1, d_2), \tilde{\mu}_3(d_3; d_1, d_2)) \\ A(d_3; \mu) > 0 \text{ if } \mu \in (\tilde{\mu}_3(d_3; d_1, d_2), \infty)\end{aligned}\right\} \tag{17}$$

Then we discuss the global existence and bifurcation of non-constant positive solutions of (3) with respect to the diffusion coefficient d_3, respectively, while the other parameters are fixed.

Theorem 4 Assume that all the parameters except d_3 are fixed, $a_{11} > 0$ holds, $\mu_3^*(d_1, d_2) \in (\mu_j, \mu_{j+1})$ and $\sum_{i=1}^{j} \dim E(\mu_i)$ is odd. Then there exists a positive constant $\tilde{D}_3$ such that, if $d_3 \geqslant \tilde{D}_3$, (3) has at least one nonconstant positive solution.

Proof By Proposition 2, there exists a positive constant $\tilde{D}_3$, such that when $d_3 \geqslant \tilde{D}_3$, (17) holds and

$$\mu_j < \tilde{\mu}_3(d_3; d_1, d_2) < \mu_{j+1} \tag{18}$$

By Theorem 3, for $\hat{d}_1$ and $\hat{d}_3$ satisfying $\mu_1\hat{d}_1 > r$, $\mu_1\hat{d}_3 > \beta M_2 - D_2$ there exists large $\hat{d}_2$ such that (3) has no constant positive solutions. In addition, since $\det\{\boldsymbol{G}_{\boldsymbol{u}}(\tilde{\boldsymbol{u}})\}<0$ and $\lim_{i\to\infty}\mu_i=\infty$, it is easy to see that we can further choose $\hat{d}_1$, $\hat{d}_2$, and $\hat{d}_3$ to be so large that

$$H(\hat{d}_1,\hat{d}_2,\ \hat{d}_3;\ \mu_i)>0 \text{ for all } i\geqslant 0 \tag{19}$$

Now, we can claim that for any $d_3\geqslant\tilde{D}_3$ (3) has at least one non-constant positive solution. The proof, which is accomplished by contradiction, is based on the homotopy invariance of the topological degree. Suppose on the contrary that the assertion is not true for some $d_3=\tilde{d}_3\geqslant\tilde{D}_3$.

We fix $d_3=\tilde{d}_3$. Let $d_i(t)=td_i+(1-t)\hat{d}_i$, $i=1,2,3$ and define $\boldsymbol{D}(t)=\mathrm{diag}(d_1(t),d_2(t),d_3(t))$, $\mu_i\boldsymbol{I}-\boldsymbol{D}^{-1}\boldsymbol{G}_{\boldsymbol{u}}(\tilde{\boldsymbol{u}})$ for all $i\geqslant 0$, and

$$\sum_{i\geqslant 0,\,H(d_1,d_2,d_3;\mu_i)<0}\dim E(\mu_i)=\sum_{i=1}^{j}\dim E(\mu_i)=\text{an odd number}$$

Then by Proposition 1,

$$\mathrm{index}(\boldsymbol{F}(1;\ \cdot),\tilde{\boldsymbol{u}})=(-1)^{\gamma}=-1 \tag{20}$$

On the other hand, by Proposition 1, we obtain that

$$\mathrm{index}(\boldsymbol{F}(0;\ \cdot),\tilde{\boldsymbol{u}})=(-1)^{0}=1 \tag{21}$$

In view of $\tilde{d}_3>\tilde{D}_3$, by Theorems 1 and 2, there exists a positive constant $C=C(\tilde{D}_3,d_1,d_2,\hat{d}_1,\hat{d}_2,\hat{d}_3,\Lambda)$ such that, for all $0\leqslant t\leqslant 1$, the positive solutions satisfy $1/C<u_1,u_2,u_3<C$. Therefore, $\boldsymbol{F}(t;\ \boldsymbol{u})\neq 0$ on $\partial B(C)$ for all $0\leqslant t\leqslant 1$. By the homotopy invariance of the topological degree,

$$\deg(\boldsymbol{F}(1;\ \cdot),0,B(C))=\deg(\boldsymbol{F}(0;\ \cdot),\ 0,B(C)) \tag{22}$$

Moreover, by our supposition, both equations $\boldsymbol{F}(1;\boldsymbol{u})=0$ and $\boldsymbol{F}(0;\boldsymbol{u})=0$ have only the positive solution $\tilde{\boldsymbol{u}}$ in $B(C)$, and hence, by (20) and (21),

$$\deg(\boldsymbol{F}(0;\ \cdot),0,B(C))=\mathrm{index}(\boldsymbol{F}(0;\ \cdot),\tilde{\boldsymbol{u}})=1$$

$$\deg(\boldsymbol{F}(1;\ \cdot),0,B(C))=\mathrm{index}(\boldsymbol{F}(1;\ \cdot),\tilde{\boldsymbol{u}})=-1$$

This contradicts (22) and the proof is complete.

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一类生态-流行病模型正平衡解的存在性

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摘要：考虑一类捕食者带有传染病并具有 Holling-Ⅱ功能性反应函数的生态-流行病模型．讨论其带有扩散项的在齐次 Neumann 边界条件下问题．主要考虑其对应的平衡态问题的正解的存在性．首先应用最大值原理和 Harnack 不等式给出其反应扩散问题的正平衡解的先验估计（正的上下界估计），然后应用能量方法给出了该问题非常数正平衡解的不存在性，最后应用拓扑度理论研究了该问题非常数正平衡解的存在性．

关键词：生态-流行病模型；存在性；正平衡解

中图分类号：O175.25