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On the critical conditions of traffic jams

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Abstract: Traffic wave theory is used to study the critical conditions for traffic jams according to their features. First, the characteristics of traffic wave propagation is analyzed for the simple signal-controlled lane and the critical conditions for oversaturation is established. Then, the basic road is decomposed into a series of one-way links according to its topological characteristics. Based on the decomposition, traffic wave propagation under complex conditions is studied. Three complicated factors are considered to establish the corresponding critical conditions of jam formation, namely, dynamic and insufficient split, channelized section spillover and endogenous traffic flow. The results show that road geometric features, traffic demand structures and signal settings influence the formation and propagation of traffic congestion. These findings can serve as a theoretical basis for future network jam control.

Key words: traffic engineering; traffic jam; traffic signal; road network; traffic wave

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The urban road network has been considered as the lifeline of urban daily operations. It serves as the basis of the urban economy. So, assurance of its normal operation is important. However, with the development of urbanization, the number of vehicles exceeds the capacity of the road network, which often results in large scale traffic jams during peak hours and leads to many problems such as pollution, noise and so on. A large scale traffic jam often forms from a local jam which takes place in a road or a single node. According to Ref. [1], there are three sources of a traffic jam: temporary obstruction, stochastic fluctuation in demand, and permanent capacity bottleneck. These types of congestion cannot be efficiently prevented because they are tightly related with road topology, land usage and other factors. However, if we can control the formation and propagation of traffic jams, then, theoretically, network traffic jams will be prevented. Literature focusing on this subject currently can be classified into the following three types.

The first type of research is conducted with a lack of theoretical analysis tools; many researchers use simulation methods such as the cell transmission model[2] and the cellular automata model[3] to study traffic jam dynamics[1] and the physical features[4–5]. This kind of research does not require much theoretical deduction and it can set simulation parameters easily. However, the used road networks in previous studies are usually regular. So we cannot obtain universal conclusions.

The second type of research is called "gridlock" proposed by Daganzo[6]. By dividing the network into neighborhood-sized "reservoirs", the analysis is simplified and the monitor or control measures can be taken on the neighborhood level. This method is too simple to take network control parameters such as cycle, split and offset into account.

The third type of research mainly focuses on the individual responses to a traffic jam[7] and usually adopts networks which are more simple than those referred to above.

All these researches are based on road networks with simple topology. However, this simplification is too coarse. For example, traffic conflicts in the intersection cannot be analyzed by the traditional framework. Roads always consist of several lanes including channelized sections, and traffic flow may be left-turn, run-through or right-turn. These features cannot be revealed with a simple state equation.

In this paper, we deal with this problem using traffic wave theory together with the decomposition method for a basic road. Hence, the critical conditions for various scenarios can be established, including the simplest one-way road and the complicated condition such as endogenous flow and channelized section spillover and so on.

1 Simple Condition

Traffic congestion always stems from the local congestion of a single road. Consider a one-lane road that is controlled by a signal without endogenous flow as shown in Fig. 1, which is the simplest condition in a road network. The relationship between flow and density is assumed to be a parabolic function as shown in Fig. 2. Where q_{m} denotes the maximum flow and k_{j} denotes the congestion density. A traffic queue forms under signal control. Assume that g is the effective green time, C is the cycle, and λ is the split, $\lambda = g/C$. The upstream flow is q (point A in Fig. 2), and k can be obtained from a parabolic function. The density of the upstream flow is always smaller than optimum density under a maximum flow, that is, point A should be located on the left side of the curve. Otherwise, the stopping wave speed will be greater than the starting wave speed, which denotes that the queue will never disperse (see Fig. 2).

$$k = \frac{k_{\mathrm{j}} - \sqrt{k_{\mathrm{j}}^2 - 4\dfrac{k_{\mathrm{m}}^2}{q_{\mathrm{m}}}q}}{2} \tag{1}$$

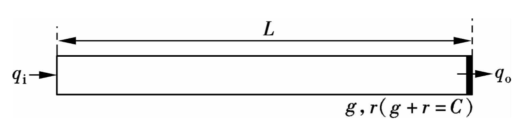

Fig. 1 Single one-way link

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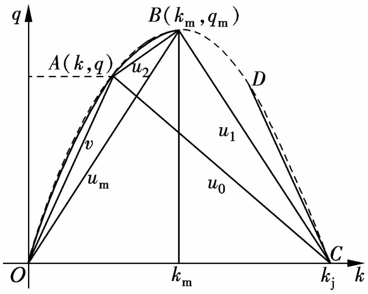

Fig. 2 Fundamental diagram

Formation and dispersion of a queue behind the stop line at a signal controlled road is shown in Fig. 3(a). At the beginning of the red time, the stopping wave (line *OB* in Fig. 3(a) which represents the queue back) propagates upstream with velocity u_0 and a queue forms. When the effective green time begins, a starting wave (line *AB* in Fig. 3(a)) emerges and also propagates upstream with a greater speed u_1. After t', the starting wave catches up with the stopping wave, the queue has dispersed and a new wave u_2 forms. It takes t'' for wave u_2 to run through the stop line. If the effective green time g is greater than $t' + t''$, then the traffic of each cycle will be the same. However, if g is smaller than $t' + t''$, the wave propagation profile of each cycle is different (see Fig. 3(b)). Suppose that there is no vehicle at the beginning. During the second cycle, the stopping wave first propagates with speed u_1, which makes the queue back of this cycle further away from the stop line than the former cycle. So the queue length will be longer and longer. It is an unstable condition which represents oversaturation.

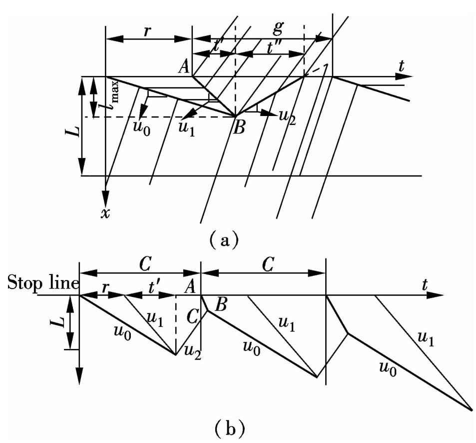

Fig. 3 Traffic wave propagation. (a) Unsaturated condition; (b) Saturated condition

Based on the above analysis, some formulae can be obtained as follows:

$$u_1 = \frac{q_m}{k_j - k_m},\ u_0 = \frac{q}{k_j - k},\ u_2 = \frac{q_m - q}{k_m - k} \tag{2}$$

$$u_0(r + t') = u_1 t' \Rightarrow t' = \frac{u_0 r}{u_1 - u_0} \tag{3}$$

$$l_{max} = u_1 t' = \frac{u_1 u_0 r}{u_1 - u_0} \tag{4}$$

$$t'' = \frac{L_{max}}{u_2} \tag{5}$$

When the effective green time $g = t' + t''$, a stable state forms. Given a flow rate q, λ can be determined. If λ decreases, oversaturation will occur. Critical condition is defined such that the maximal queue length equals the road length. Larger q or smaller split both will result in queue spillover. It can be easily expressed by $g = t' + t''$ and $l_{max} = L$, i. e.,

$$l_{max} = u_1 t' = \frac{u_1 u_0 r}{u_1 - u_0},\ g = \frac{l_{max}}{u_1} + \frac{l_{max}}{u_2},\ l_{max} = L \tag{6}$$

2 Spillover Condition for Basic Road

The problems discussed above are mainly focused on the ideal scenario of one-way links while the case in reality shows more topological complexity. We define the road that always appears in an urban road network as a basic road (see Fig. 4). It is controlled by a signal with a channelized section of length l_2. Overall upstream traffic demand, q_i is assumed to be uniformly distributed between two lanes, i. e., $q_{i1} = q_{i2}$. The proportion of left-turn flow, right-turn flow and through-flow is p_{ol}, p_{os} and p_{or} (and $p_{or} = 1 - p_{ol} - p_{os}$), respectively. So we can obtain

$$q_i = q_{ol} + q_{os} + q_{or},\ q_{ol} = q_i p_{ol},\ q_{os} = q_i p_{os},\ q_{or} = q_i p_{or} \tag{7}$$

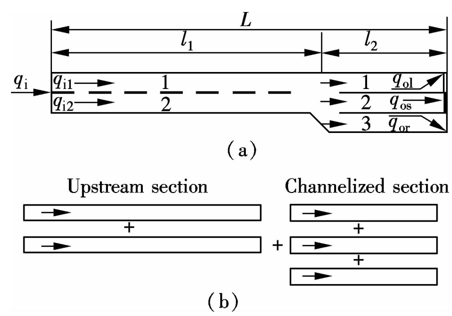

Fig. 4 Sketch map of basic road and its decomposition. (a) Basic road topology; (b) Decomposition

Decompose the basic road as shown in Fig. 4(b). The basic road is divided into two sections: the upstream section and the channelized section. Both include some single one-way links. The channelized sections are controlled by the signal (except the right-turn lane). So we only need to analyze spillover of the channelized section using the method in section 1.

3 Complex Circumstances

During daily operation, circumstances often are more complex than those described above. In this section, we will deal with this problem by analyzing the following three factors: dynamic and insufficient split, endogenous flow of the basic road, and the spillover of the channelized section.

3.1 Dynamic and insufficient split

Under oversaturated conditions, queue length becomes longer and longer. Each green signal shortens queue length while each red signal prolongs queue length. Furthermore, the length shortened or prolonged is proportional to the time duration of traffic signal because the wave speed including the starting-wave and the stopping-wave is fixed.

Given the effective green time g_i and the red time r_i of cycle i, based on the geometric relationship, the queue length shortened h_{g_i} and the queue length prolonged h_{r_i} in Fig. 5 can be computed by

$$h_{r_i}=\frac{r_i u_0 u_1}{u_1-u_0},\quad h_{g_i}=\frac{g_i u_2 u_1}{u_1+u_2} \tag{8}$$

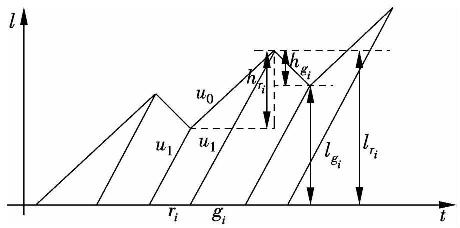

Fig. 5 Stopping wave and starting wave under oversaturation

So, the locations of queue back after red time r_i and after effective green time g_i respectively are

$$l_{r_i} = l_0 + \sum_{i-1}(h_{r_i}-h_{g_i}) + h_{r_i} = l_{g_{i-1}} + h_{r_i} \tag{9}$$

$$l_{g_i} = l_0 + \sum_{i}(h_{r_i}-h_{g_i}) \tag{10}$$

where l_0 denotes the length of the initial oversaturated queue. It can also be calculated easily by assuming that traffic is under saturation at first. From the geometric relationship in Fig. 6, we can obtain $t' = u_0 r/(u_1-u_0)$ and $l_t = u_1 u_0 r_i/(u_1-u_0)$. Derivations are as follows:

$$l_{u_2}+l_{u_1}=l \Rightarrow u_2 t_x + (t_x + t' - g_i)u_1 = 1 \tag{11}$$

$$t_x=\frac{l-(t'-g_i)u_1}{u_1+u_2} \tag{12}$$

$$l_0=(t_x+t'-g_i)u_1=\left(\frac{l-(t'-g_i)u_1}{u_1+u_2}+t'-g_i\right)u_1 \tag{13}$$

Substituting l_0 in Eq. (13) into Eqs. (9) and (10), queue back at any cycle can be calculated. When queue back exceeds road length, i. e., $l_{r_i} > L$, a spillover emerges.

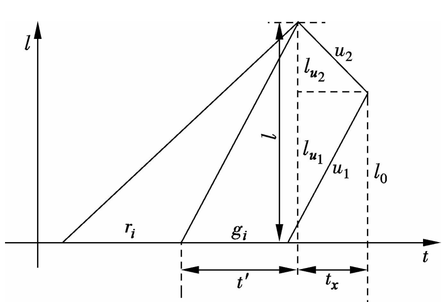

Fig. 6 Initial oversaturated queue

3.2 Influence of endogenous flow

It is generally assumed that traffic flow is generated at nodes, i. e., intersections within the network or originations of a road network[8]. However, intersections inside the road network actually do not generate flow. Of course, from the viewpoint of a downstream intersection, flow is "produced" at upstream intersection. In fact, most traffic jam forms from the entrance of a road. So it is necessary to take endogenous traffic flow into account. Due to the complexity in analyzing a multi-lane road, we only deal with a one-way road with an entrance inside (see Fig. 7). In Fig. 7, q_e is the entrance flow rate. We assume that vehicles on major roads have a priority to cross the intersection.

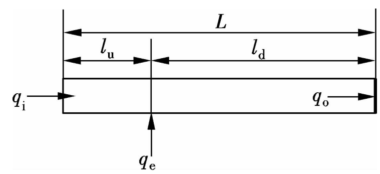

Fig. 7 Single road with endogenous traffic flow

When the queue back does not reach the entrance location, the overall arriving flow is $q_i + q_e$; when the queue length exceeds the distance between the stop line and the entrance location of endogenous flow l_d, upstream flow becomes q_i. So the stopping wave and starting wave with endogenous flow propagate differently from that without endogenous flow (see Fig. 3(a)). Given q_i and q_e, the stopping wave profile and the starting wave profile can be derived. u_{01} is the stopping wave when the queue length does not exceed l_d and u_{02} is the stopping wave when queue length exceeds l_d. The definition of u_{21} and u_{22} are similar to u_{01} and u_{02} as shown in Fig. 8.

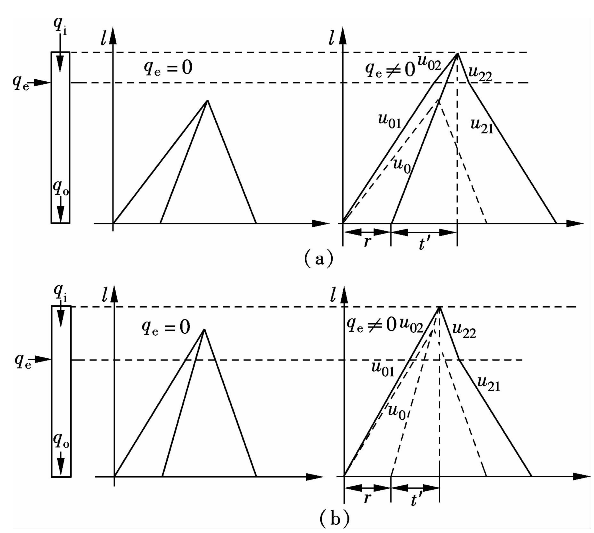

Fig. 8 Stopping wave and starting wave with endogenous flow

By the traffic wave theory, we can obtain

$$u_{01}=\frac{(q_i+q_e)}{k_j-k},\quad u_{02}=\frac{q_i}{k_j-k} \tag{14}$$

$$u_{21}=\frac{q_m-q_i}{k_m-k},\quad u_{22}=\frac{q_m-(q_i+q_e)}{k_m-k} \tag{15}$$

The starting wave speed is the same as that in the one-way signal controlled road. The critical spillover condition is defined as follows: 1) Stopping wave and starting wave meet exactly at the road tail; 2) Maximum queue length equals road length L; 3) After waves disperse, effective green time ends. The conditions above can be expressed mathematically as

$$\frac{l_d}{u_{01}} + \frac{l_u}{u_{02}} = r + t',\ u_1 t' = L,\ \frac{l_d}{u_{21}} + \frac{l_u}{u_{22}} + t' = g \qquad (16)$$

3.3 Channelized section spillover

Up to now, most research does not take the spillover of a channelized section into account. But at peak hours, due to length constraints, the channelized section often cannot accommodate excessive vehicles, which makes spillover of a channelized section inevitable. According to Ref. [1], we assume that the traffic flow of the upstream section is uniformly distributed (i. e., velocity, density and flow are uniformly distributed) and lane changing behavior is instantly executed at the interface between the channelized section and the upstream section. This is acceptable from the viewpoint of the system description. From the fundamental traffic theory, we can easily obtain the flow-density relationship (see Fig. 9). The parabolic curve below is that of a left-turn channelized section. Fundamental diagrams of the right-turn and the through channelized section are the same. The upper curve is that of the upstream section.

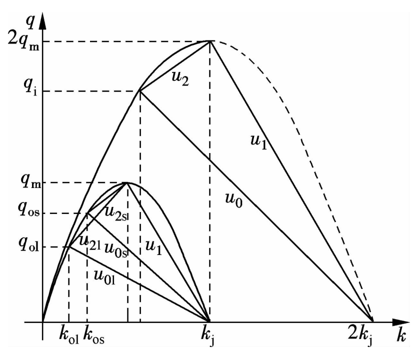

Fig. 9 *q-k* relationship of two sections

When the channelized section spillover exists, the effect can be considered as "virtual red time" (see Fig. 10). Once there is a spillover, a queue will propagate upstream and block the upstream section. The result is similar to the situation with the red signal. So a virtual signal is set at the interface between two sections as shown in Fig. 10. Now the analysis of the spillover can be carried out by the following two steps:

1) The traffic wave of the channelized section is analyzed to obtain virtual red time. Note that the queue of the through section and the left-turn section may be overflowed at the same time or may be not, so the block time of these spillovers may be separated, overlapped or partly overlapped.

2) After the virtual time (interface block time) is obtained, we analyze the traffic wave of the upstream section. If the upstream section queue back exceeds the road length, then the upstream intersection will be blocked.

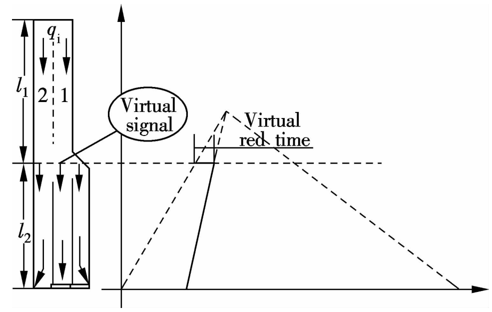

Fig. 10 Virtual signal and virtual red time

Due to the fact that both channelized sections may block the interface, we first deal with only one section spillover and then two.

3.3.1 One section spillover

Since results of spillover of the two channelized sections are the same, we simply assume that only one left-turn section spillover takes place. When $p_{ol} = p_{os}$, i. e., left-turn traffic flow q_{ol} is equal to through flow q_{os}. It can be deduced from Fig. 9 that the starting wave and the stopping wave keep their speed after spreading over the interface. This condition is the same as that in section 1.

When $p_{ol} > p_{os}$ (i. e. $q_{ol} > q_{os}$), the upstream section holds more flow averagely than the left turn lane. So the wave propagation profile should be the pattern B in Fig. 11. Its stopping wave speed u_{0a} is greater than u_0 and u_{2a} is smaller than u_2. When the effective green time g extends over point z, the queue will disperse completely. So the critical conditions for a domino formation can be deduced:

$$\left(\frac{l_2}{u_0} + \frac{l_1}{u_{0a}}\right) + \left(\frac{l_1}{u_{2a}} + \frac{l_2}{u_2}\right) = r + g,\ \frac{l_2}{u_0} + \frac{l_1}{u_{0a}} = r + \frac{l_1 + l_2}{u_1} \qquad (17)$$

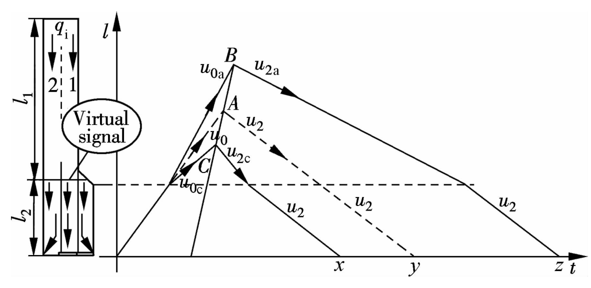

Fig. 11 Wave trajectory under one spillover condition

When $p_{ol} < p_{os}$ (i. e., $q_{ol} < q_{os}$), the upstream section holds less flow than the left turn lane. So the wave propagation profile should be the pattern C in Fig. 11. When the effective green time g ends after time x, the queue disperses completely. Critical conditions for the domino formation are deduced as

$$\left(\frac{l_2}{u_0} + \frac{l_1}{u_{0c}}\right) + \left(\frac{l_1}{u_{2c}} + \frac{l_2}{u_2}\right) = r + g,\ \frac{l_2}{u_0} + \frac{l_1}{u_{0c}} = r + \frac{l_1 + l_2}{u_1} \qquad (18)$$

After the analyses above, the problem of the domino formation with a channelized section spillover degrades to a single-way link spillover but with variational traffic waves. It can be solved using the above method.

3.3.2 Two section spillovers

Sometimes, during peak hours, the left-turn demand and the through demand may both exceed their respective capacity, which results in spillover of both sections. In such cases, the problem may be complicated but can be explained conveniently. The spillovers of two sections are just the same as on the situation when we set two virtual signals in the interface at the same time. When both virtual signals are green, the overall virtual signals are green; otherwise, the overall signals are red (see Fig. 12).

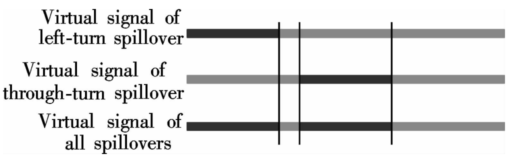

Fig. 12 Virtual signal at interface when two spillovers exist

4 Conclusion

Due to the complexity of urban traffic jams, much research has been done under simple assumptions. In this paper, a critical condition for urban traffic jam formation is studied. From the research, we can see that many factors such as road geometry, flow features and signal settings contribute to the traffic jam formation and propagation, which should be taken into account in the control of jams during peak hours.

However, limited by the relatively macroscopic method adopted, we cannot deeply investigate the mutual influence of different flows such as the left-turn flow, the right-turn flow and the through flow, which may contribute to the formation and propagation of traffic jams. This may be solved by microscopic analysis methods. Furthermore, the formation of congestion is only the beginning of large scale urban traffic jams. Propagation of an urban traffic jam itself is influenced by many factors, such as road network topology, network traffic demand, control measures and so on. The propagation is more difficult to analyze but more important for traffic jam control.

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交通拥堵形成的临界条件

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摘要:根据网络交通拥挤的特征,利用交通波理论研究了交通拥堵形成的临界条件.首先分析了单车道信号控制路段的交通波传播特性,建立了过饱和形成的临界条件.然后针对基本路段拓扑结构特征,将基本路段分解为一系列单车道路段,在此基础上研究了复杂条件下交通波传播规律,并考虑了3种复杂因素,建立了相应的交通拥挤形成临界条件,包括不足的绿信比、渠化区上溯以及路段内部出入口交通流的影响.研究结果显示,路段的物理结构、交通流需求结构以及信号设置均会影响到交通拥堵的形成和扩散.研究结果为路网拥堵的控制提供了一定的理论基础.

关键词:交通工程;交通拥堵;交通信号;路网;交通波
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