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Lazy loading algorithm for traffic assignment of road networks under fixed charge condition

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Abstract: The measures of path charge are important considerations in traffic assignment of road networks. Factors, such as travel time, fixed charge and traffic congestion which affect road users' choices of trip paths, are analyzed. Travelers usually decide their trip paths based on their personal habits, preferences and the information at hand. By considering both deterministic and stochastic factors which affect the value of time (VOT) during the process of path choosing, a variational inequality model is proposed to describe the problem of traffic assignment. A lazy loading algorithm for traffic assignment is designed to solve the proposed model, and the calculation steps are given. Numerical experiment results show that compared with the all-or-nothing assignment, the proposed model and the algorithm can provide more optimal traffic assignments for road networks. The results of this study can be used to optimize traffic planning and management.

Key words: traffic assignment; road networks; fixed charge; lazy loading algorithm

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As an important part of traffic planning, traffic assignment generally assumes that the selection of trip paths of road users is based on only one principle: travel time or expenses, without considering the fact that different travelers have different criteria on the effects of VOT. Establishing the traffic assignment model considering different preferences between travel time and expenses is not only the basis of the charging system of traffic networks but also the premise of traffic control and induced traffic.

In the study of the traffic assignment problem, researchers have presented various formulations[1-3]. The theory of traffic assignment considering only deterministic factors has been well-established. Beckmann et al.[4] established the optimal model corresponding to the Wardrop equilibrium principle under the assumed case that the travel cost is separable from travel time. In connection with the case of asymmetric travel cost, Smith[5] built up the equivalent formula corresponding to the Wardrop equilibrium and proved its uniqueness. The key assumption of the deterministic traffic assignment theory is that travelers are completely rational and clearly grasp the traffic conditions and information, which is usually impracticable. Generally, road users choose trip paths based on the estimation of travel cost through incomplete information, so the traffic assignment model based on the stochastic theory can reflect actual traffic conditions more rationally[6-7].

The existing traffic stochastic assignment theories are still imperfect. On the study of route cost (the key issue of traffic assignment), researchers treat travel cost and travel time as two weakly correlated parts. Gabriel et al.[8] proposed a route cost function based on travel cost, and Larsson et al.[9] established a route cost formula based on travel time. All these models have an imcomplete consideration on travel cost and travel time and ignore the fact that different individuals have different values of time. This paper builds up a traffic assignment model based on perceived time which is influenced by deterministic and stochastic factors. An actual and practical algorithm is designed to solve the stochastic traffic assignment problem. The proposed model and algorithm can distribute traffic flow more optimally.

1 Stochastic Assignment Model

1.1 Definition of network and symbol

Consider a road network $G=(N, L)$ which has node set N and arc set L, and $l=(i_l, j_l)$ is an arc directed from node i_l to j_l. Let p be a path starting at origin o, $o \in N$ and ending at destination d, $d \in N$; and let x_l and u_l be traffic flow influenced by deterministic effects and stochastic effects, respectively. b_j is node j's predecessor arc in tree T; $v_{o\text{-}d}$ is total trips from origin o to destination d.

1.2 Perceived time

The road users' perceived time is mainly influenced by two components: deterministic and stochastic. Either or both may be flow-dependent. Let d_l state the effect of the deterministic component and s_l the effect of the stochastic component, and then the perceived time of trip path l can be defined as

$$g_p(\alpha) = \sum_{l \in p} d_l(x_l) + \alpha \sum_{l \in p} s_l(x_l)$$

where α is the weight of the stochastic component. With given α, the probability density function (PDF) of the perceived travel time from node o to node d is $f(\alpha \mid o, d)$.

Assumption 1 (Wardrop's principle) Each trip from o to d has its own α and uses a path that minimizes $g_p(\alpha)$ for its particular value of α. The probability of an o-d trip using path p is

$$\text{prob}[p \mid o, d] = \int_{\alpha^{p-1}}^{\alpha^p} f(\alpha \mid o, d)\, \mathrm{d}\alpha$$

where α^{p-1} and α^p are the negative of the slopes of the efficient frontier's two edges adjacent to p. Then for all $o \in N$,

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$\alpha \in A$, a feasible traffic assignment $x = (x_{ol}(\alpha): o \in N, l \in L, \alpha \in A)$ satisfies

$$\sum_{l:j_l=d} x_{ol}(\alpha) - \sum_{l:i_l=d} x_{ol}(\alpha) = v_{o\text{-}d} f(\alpha \mid o, d)$$

It is also easy to obtain the following formulae:

$$x_l = \sum_A x_l(\alpha) = \sum_A \sum_N x_{ol}(\alpha) \qquad \forall o \in N, \alpha \in A, l \in L$$

$$u_l = \sum_A \alpha x_l(\alpha) = \sum_A \sum_N \alpha x_{ol}(\alpha) \qquad \forall o \in N, \alpha \in A, l \in L$$

where x_l is the total traffic flow through arc l, and u_l is the distributed traffic flow through arc l under the effects of stochastic factors.

1.3 Variational inequality

Because the effects of deterministic factors and stochastic factors are asymmetric, we can build a variational inequality model to represent stochastic traffic assignments.

$$\sum_{l \in L} (d_l(x_l^*)(x_l - x_l^*) + s_l(x_l^*)(u_l - u_l^*)) \geqslant 0 \quad (1)$$

s. t.

$$x_l = \sum_A x_l(\alpha) = \sum_A \sum_N x_{ol}(\alpha) \qquad \forall o \in N, \alpha \in A, l \in L$$

$$u_l = \sum_A \alpha x_l(\alpha) = \sum_A \sum_N \alpha x_{ol}(\alpha) \qquad \forall o \in N, \alpha \in A, l \in L$$

$$d_l \geqslant 0, s_l \geqslant 0 \qquad \forall l \in L$$

2 Lazy Loading Algorithm for Traffic Assignment

In order to solve the variational inequality problem of expression (1), the lazy loading algorithm for traffic assignment can be used. The basic steps are as follows.

Step 1 Set upper bound $\alpha^{\max} = 1$ on VOT weight α;

Step 2 Calculate the lower bound $a^{\min}$ of VOT, $\alpha^{\min} = (d_{jl} - d_{il} - d_l^o)/(s_{il} + s_l^o - s_{jl})$, and execute the program of scanning nodes. If $\alpha_{n'}^{lb} = \alpha^{\max}$, assign trips destined for these nodes n'; if $\alpha_n^{lb} = \alpha^{\min}$, correspond nodes n to set D, $D = \{n \mid n \in N \cap \alpha_n^{lb} = \alpha^{\min}\}$.

Step 3 If $D = \varnothing$, quit and return (x, u), update the origin node o and weight α_n^{lb}; otherwise, turn to step 4.

Step 4 Let $j' = D$, compute trips destined for j' from origin o:

$$(\mathrm{d}x_{j'}, \mathrm{d}u_{j'}) = v_{oj}\Big(\int_{\alpha^{lb}}^{\alpha_{j'}} f(\alpha \mid o, j)\,\mathrm{d}\alpha, \int_{\alpha^{lb}}^{\alpha_{j'}} \alpha f(\alpha \mid o, j)\,\mathrm{d}\alpha, \Big)$$

Step 5 Define $l = b_j$; cascade node $j's$ trips: $(x_l, u_l) = (x_l + x_{j'} + \mathrm{d}x_{j'}, u_l + x_{j'} + \mathrm{d}u_{j'})$, $(x_{il}, u_{il}) = (x_{il} + u_{j'} + \mathrm{d}x_{j'}, u_{il} + u_{j'} + \mathrm{d}u_{j'})$; and initialize $(x_{j'}, u_{j'}) = (0, 0)$.

Step 6 Decrease α-bound of node j', set $\alpha_{j'} = \alpha^{lb}$, and go to step 2.

3 Numerical Experiment

A numerical example is carried out to clarify the mechanics of the lazy loading algorithm for traffic assignment based on a 9-node and 12-arc road network. This network has four arcs which charge the fixed fee as shown in brackets in Fig. 1 and the arc time is in minutes. This simple example has exactly 500 trips going from node 1 to each of the remaining 8 nodes. α is the VOT weight, and we assume that trips cannot distinguish VOT below 0.001 yuan and the α-weight PDF for all the OD pairs is uniform from 0 to 1.

$$v_{1j}\int_{\alpha^{lb}}^{\alpha_j} f(\alpha \mid o, j)\,\mathrm{d}\alpha = 500(\alpha_j - \alpha^{lb})$$

$$v_{1j}\int_{\alpha^{lb}}^{\alpha_j} \alpha f(\alpha \mid o, j)\,\mathrm{d}\alpha = 250(\alpha_j^2 - (\alpha^{lb})^2)$$

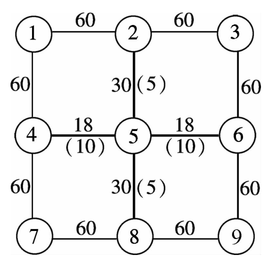

Fig. 1 Diagram of a road network

Fig. 2 explains the annotation of several variables in the algorithm. The tree's node labels α_j show the exclusive lower bound of VOT, dashed arcs show links that will swap into the next tree, and double circles are nodes whose minimal path will change. The network's arc labels (x_{ij}, u_{ij}) represent assigned traffic flows, and node labels $(\mathrm{d}x_j, \mathrm{d}u_j)$ are the destination flows accounted for in this step.

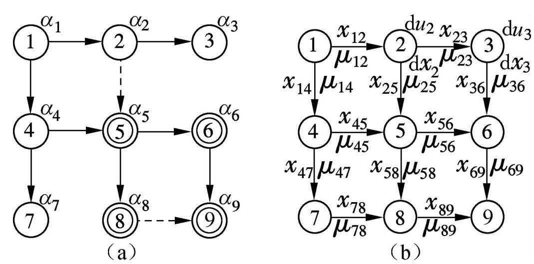

Fig. 2 Annotation of several variables in the algorithm. (a) Tree; (b) Network

Step 1 Set upper bound $\alpha^{\max} = 1$. The lower bound of calculated VOT weight falls on node 5; $\alpha_5^{\min} = (2 - 1 - 0)/(6 + 0 - 3.6) = 0.416$; tree 3 has new paths to nodes 5, 6, 8 and 9.

Step 2 Execute program of scanning nodes, $\alpha_i^{lb} = a^{\max} = 1$, $i = 2, 3, 4, 7$, so assign trips(500, 250) to the corresponding nodes; $\alpha_i^{lb} = a^{\min} = 0.416$, $i = 5, 6, 8, 9$, correspond nodes to set D, and $D = \{5, 6, 8, 9\}$.

Step 3 Compute the loaded traffic flow

$$(\mathrm{d}x, \mathrm{d}u) = v_{oj}\Big(\int_{\alpha^{lb}}^{\alpha_{j'}} f(\alpha \mid o, j)\, d\alpha, \int_{\alpha^{lb}}^{\alpha_{j'}} \alpha f(\alpha \mid o, j)\,\mathrm{d}\alpha, \Big) = (500(\alpha_{j'} - \alpha^{lb}), 250(\alpha_{j'}^2 - (\alpha^{lb})^2) = (292, 207)$$

Step 4 Update traffic flow on arcs, see Fig. 3.

Step 5 Decrease α-bound of node j', set $\alpha_{j'} = 0.416$, and go to step 2.

After the first cycle, the feasible tree and assigned traffic flow is shown in Fig. 3.

After the fourth cycle, the final feasible tree and traffic flow assignment can be obtained (see Fig. 4).

Tab. 1 shows the comparison between all-or-nothing algorithm and the lazy loading algorithm for traffic assignment under fixed charge conditions.

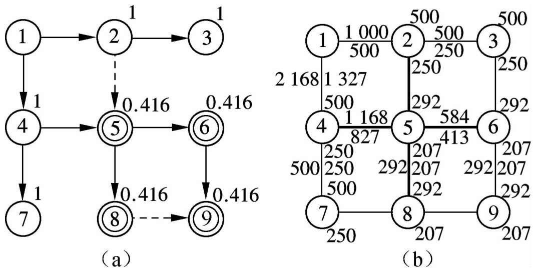

Fig. 3 Feasible tree and traffic flow assignment after first cycle. (a) Tree; (b) Network

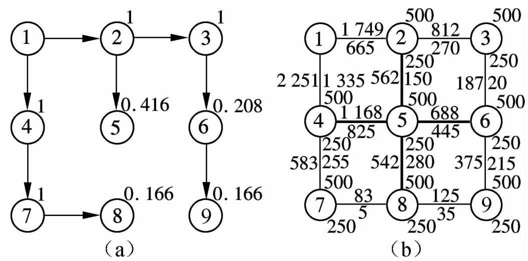

Fig. 4 Final feasible tree and traffic flow assignment. (a) Tree; (b) Network

Tab. 1 Comparison of all-or-nothing algorithm and lazy loading algorithm for traffic assignment

OD pair	All-or-nothing assignment		Lazy loading traffic assignment	
	Paths	Traffic flow	Paths	Traffic flow
1-2	1-2	500	1-2	500
1-3	1-2-3	500	1-2-3	500
1-4	1-4	500	1-4	500
1-5	1-4-5	500	1-4-5	292
			1-2-5	208
1-6	1-4-5-6	500	1-4-5-6	292
			1-2-5-6	104
			1-2-3-6	104
1-7	1-4-7	500	1-4-7	500
1-8	1-4-5-8	500	1-4-5-8	292
			1-2-5-8	125
			1-4-7-8	83
1-9	1-4-5-6-9	500	1-4-5-6-9	292
			1-2-5-8-9	125
			1-2-3-6-9	83

4 Conclusion

Under the condition of road networks exacting fixed charged paths, traffic assignment is more complex compared with uncharged networks. The traffic assignment model and its algorithm are studied in this paper to solve the traffic assignment problem of charged networks. As a variational inequality, the proposed model can reflect different habits, preferences, and the VOT of different travelers. The lazy loading algorithm for traffic assignment uses an effective node-scanning method and traffic flow loaded program to assign traffic flow. The operational speed of the algorithm is ideal and the result of traffic assignment is more actual compared with all-or-nothing assignment.

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固定收费路网条件下交通分配的延迟加载算法

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摘要:路径收费措施是影响道路网络交通分配的重要因素. 受出行时间、道路收费及交通拥堵等多种因素的影响,道路使用者会根据自身习惯、偏好及掌握的信息选择不同的出行路径. 综合考虑了路径选择过程中确定性因素和随机性因素对时间感知价值的影响,建立了交通分配的变分不等式模型,设计了求解该模型的延迟加载算法,并叙述了该算法的求解过程. 算例结果表明:相对于全有全无交通分配,该模型和算法可以对道路网络中的交通流量进行更优化的分配. 分析研究结果可以为实际的交通网络规划与管理提供借鉴.

关键词:交通分配;道路网络;固定收费;延迟加载算法

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