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Deflection and stress of hollow CRCP slab under concentrated vehicle load

Chen Xiaobing Huang Xiaoming Ding Jianming Tong Jinhu

(School of Transportation, Southeast University, Nanjing 210096, China)

Abstract: Based on the equivalence principle of deflection and stress, the concentrated vehicle load which acts on the center of continuously reinforced concrete pavement (CRCP) is translated into the equivalent half-wave sine load by the Fourier transform. On the basis of this transform and the small deflection theory of elastic thin plates, the deflection and stress formulae of CRCP under the concentrated vehicle load with a hollow foundation are put forward. The sensitivity of parameters is analyzed. The results show that maximum deflection is directly proportional to the concentrated vehicle load and the slab width, and inversely proportional to the lateral bending stiffness and slab thickness. The effects of slab width and thickness are significant with regard to maximum deflection. Maximum stress is directly proportional to the concentrated vehicle load and the slab width as well as inversely proportional to slab thickness. The effect of slab thickness is significant with regard to maximum stress. According to the calculation results, the most effective measure to reduce maximum deflection and stress is to increase slab thickness.

Key words: concentrated vehicle load; equivalence principle; half-wave sine load; elastic thin plates; hollow continuously reinforced concrete pavement slab; deflection and stress formulae; slab thickness

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Continuously reinforced concrete pavement (CRCP) is a Portland cement concrete pavement structure with continuous longitudinal steel reinforcement with no preventive measures for transverse expansion or contraction joints[1-2]. The superiority of CRCP is shown under some special conditions (such as overloaded traffic, heavy traffic, uneven settlement and poor hydrogeological conditions).

It is believed that the CRCP will crack naturally[3]. Repetitive loading and thermal loading force the concrete to crack vertically. The closely spaced transverse and longitudinal cracks cause punchouts[4-5]. Field investigations of CRCP indicate that spalling is another cause of damage to CRCP[6]. Therefore, the CRCP punchouts and smoothness are proposed as the basic design parameters in the guide for mechanistic-empirical design[7]. This method comes from the given structure and load; therefore, the mechanistic design method is not proposed.

In this paper, the concentrated vertical load is translated into the equivalent half-wave sine load by the Fourier transform. According to the small deflection theory of elastic thin plates, the deflection and stress formulae of CRCP are put forward when CRCP on a hollow foundation is subjected to a concentrated vertical load. These analysis results are helpful in the study and design of CRCP on a hollow foundation under a concentrated vehicle load.

1 Basic Theory and Differential Equations

1.1 Basic theory

After construction, the early transverse cracks of the slab are caused by the combined effects of decreasing temperature and dryness shrinkage. The general crack spacing is within the range of 1 to 2.5 m[8]. Compared with the cross-section area of the slab, the area of steel reinforcement is very small (0.6% to 0.8%). As time goes by, the load transfer efficiency decreases. Therefore, it is safe to ignore the role of longitudinal reinforcement steel in the analysis. All the remaining conditions satisfy the Kiechhoff assumption of the elastic rectangular thin slab[1]. The small deflection theory of elastic thin plates is used in this analysis.

1.2 Differential equation of elastic thin plates

According to the basic assumptions of elastic thin plates and the balance of internal forces and loads, the deflection differential equation of elastic thin plates is calculated as [1-2]

$$D_x \nabla^2 \nabla^2 w = P(x) \tag{1}$$

where w is the deflection of the slab; P is the concentrated load in the center of the slab; ∇^2 is the Laplace operator, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$; D_x is the transverse bending rigidity of the slab. $D_x = E_{csx} h^3/12(1-\nu_c^2)$, where E_{csx} is the CRCP transverse elastic modulus[9]; h is the thickness of the slab; ν_c is the Poisson ratio of the slab. The concentrated force in the slab is shown in Fig. 1.

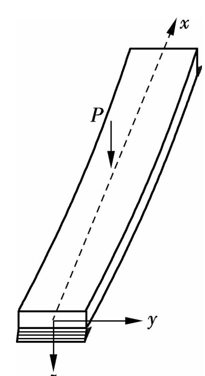

Fig. 1 Slab under the concentrated vehicle load

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Biographies: Chen Xiaobing (1973—), male, graduate, associate professor, xbchen@ seu. edu. cn; Huang Xiaoming (corresponding author), male, doctor, professor, huangxm@ seu. edu. cn.

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The moment and shearing force per unit width are given by

$$\left.\begin{aligned} M_x &= -D_x\left(\frac{\partial^2 w}{\partial x^2}+\nu_c\frac{\partial^2 w}{\partial y^2}\right)\\ M_y &= -D_x\left(\frac{\partial^2 w}{\partial y^2}+\nu_c\frac{\partial^2 w}{\partial x^2}\right)\\ M_{xy} &= -D_x(1-\nu_c)\frac{\partial^2 w}{\partial x\partial y}\end{aligned}\right\} \tag{2}$$

$$Q_x = -D_x\frac{\partial}{\partial x}\nabla^2 w \tag{3}$$

The stress of the slab is given by

$$\sigma_x = -\frac{E_{csx}z}{1-\nu_c{}^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu_c\frac{\partial^2 w}{\partial y^2}\right) \tag{4}$$

2 Load Transform

2.1 Method of load transform

With the Fourier transform, the concentrated vehicle load is expanded as a half-wave sine load with peak load p_0.

$$P(x) = p_0 \sin\frac{\pi x}{B} \tag{5}$$

where p_0 is the maximum value of the sine curve load; B is the slab width; x is the transverse distance from the edge of the slab ($0\leqslant x\leqslant B$). After the balanced load transform, the load pattern on the slab is shown in Fig. 2.

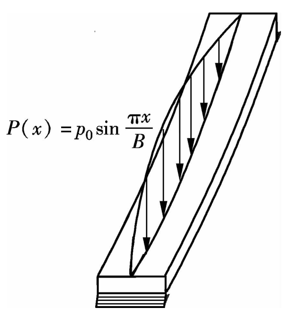

Fig. 2 Slab under the equivalent half-wave sine load

2.2 Equivalency of load transform

As $0\leqslant x\leqslant B$, Eq. (5) satisfies Dirichlet conditions[10]. Then, Eq. (5) is equal to P within $0\leqslant x\leqslant B$.

3 Deflection and Stress

3.1 Deflection in the center of span

Substituting the load transform of Eq. (5) into the elastic slab balance of Eq. (1), the new elastic thin plates' differential equation of CRCP on a hollow foundation under the concentrated vehicle load is written as

$$D_x\nabla^2\nabla^2 w = p_0\sin\frac{\pi x}{B} \tag{6}$$

The right side item of the basic differential Eq. (6) is $p_0\sin(\pi x/B)$, and the left side item is $D_x(\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4)$, which expresses the second-order and the fourth-order partial derivatives of w with respect to x and y. For the n-order derivative of $\sin x$, when n is an even number ($2k, k=0, 1, 2, \ldots$), $(\sin x)^{(2k)} = (-1)^k\sin x$; when n is an odd number ($2k+1$, $k=0, 1, 2, \ldots$), $(\sin x)^{(2k+1)} = (-1)^k\cos x$[11]. According to the character of these higher-order derivatives, w can be written as

$$w = (A_0x^4 + A_1x^3y + A_2x^2y^2 + A_3xy^3 + A_4y^4 + A_5)\sin\frac{\pi x}{B} \tag{7}$$

From the boundary conditions, when $x=0$, $M_x=0$, we have $A_3=0$. When $x=0$, $M_{xy}=0$, we have $A_4=0$. And when $y=l_d/2$, $M_y=0$, we have $A_0=A_1=A_2=0$, where l_d is the distance between adjacent cracks.

Then putting A_0, A_1, A_2, A_3 and A_4 into Eq. (7), w can be simply written as

$$w = A_5\sin\frac{\pi x}{B} \tag{8}$$

Substituting the deflection of Eq. (8) into the elastic thin plates differential of Eq. (6), we have

$$D_x w''''(x) = p_0\sin\frac{\pi x}{B} \tag{9}$$

After the successive integration of Eq. (9), we can obtain

$$D_x w''(x) = -\frac{p_0B^2}{\pi^2}\sin\frac{\pi x}{B} + C_1x + C_2 \tag{10}$$

$$D_x w(x) = \frac{p_0B^4}{\pi^4}\sin\frac{\pi x}{B} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4 \tag{11}$$

According to the boundary conditions, we can obtain integration constants. As $x=0$, $w(x)=0$, we have $C_4=0$. As $x=0$, $w''(x)=0$, we have $C_2=0$. As $x=B$, $w(B)=0$, we have $C_1B^3/6 + C_3B=0$, and with the condition $x=B$, $w''(B)=0$, we can simultaneously have $C_1=0$ and $C_3=0$.

According to the equivalence principle of the load transform, we have

$$P = \int_0^B p_0\sin\frac{\pi x}{B}\mathrm{d}x = \frac{2B}{\pi}p_0 \tag{12}$$

$$p_0 = \frac{P\pi}{2B} \tag{13}$$

By putting the coefficients $C_1=C_2=C_3=C_4=0$ and Eq. (13) into Eq.(11), the deflection formula is written as

$$w(x) = \frac{PB^3}{2\pi^3D_x}\sin\frac{\pi x}{B} \tag{14}$$

As $x=B/2$, maximum deflection is obtained from

$$w_{x\max} = \frac{PB^3}{2\pi^3D_x} = \frac{6PB^3(1-\nu_c^2)}{\pi^3E_{csx}h^3} \tag{15}$$

3.2 Equivalence verification of load transform

For the same piece of the slab, the width B and bending rigidity D_x are constants. According to the nature of the derivative of trigonometric functions, the following formula is established.

$$\frac{w_1(x)}{w_2(x)} = \frac{w_1''(x)}{w_2''(x)} = \frac{w_1''''(x)}{w_2''''(x)} = \frac{P_1(x)}{P_2(x)} = \text{constant} \qquad (16)$$

From Eq. (16), we can see that the rules for the change of loads, deflections and internal forces are consistent. It shows that the concentrated vehicle load on the slab can be replaced by the half-wave sine load through the Fourier transform. In this way, the analysis of the slab deflection and stress is reasonable.

3.3 Bending moment and shearing force

Now, putting the deflection Eq.(14) into Eq.(2) and Eq.(3), we can obtain the bending moment and the shearing force of the unit width as follows:

$$M_x = \frac{PB}{2\pi}\sin\frac{\pi x}{B}, \quad Q_x = \frac{P}{2}\cos\frac{\pi x}{B}$$

3.4 Stress of concrete slab

According to Saint-Venant's principle[12], when deflection Eq.(14) is put into Eq.(4), the stress expression of the slab is written as

$$\sigma_x = \frac{12M_x}{h^3}z = \frac{6PBz}{\pi h^3}\sin\frac{\pi x}{B}$$

When $z = h/2$ and $x = B/2$, the maximum stress is obtained from

$$\sigma_{x\max} = \frac{3PB}{\pi h^2}$$

4 Effect Analysis of Various Parameters on Maximum Deflection and Maximum Stress

4.1 Effect of parameters on maximum deflection

As shown in Fig. 3, the slab width has a significant effect on the maximum bending deflection of the slab. Maximum deflection rapidly increases with the increase in slab width, while it only gradually increases with the increase in the load. When the slab width is greater than or equal to 10 m, the load has a significant effect on maximum deflection.

Similarly, the effect of the slab thickness on maximum deflection of the slab is as significant as the slab width, but

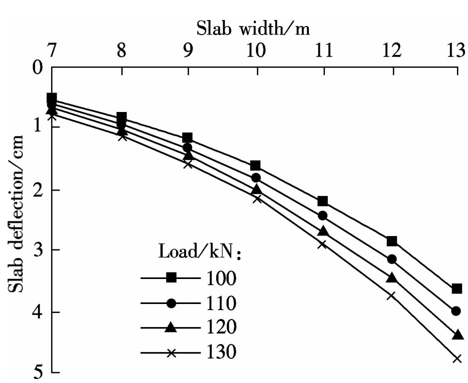

Fig. 3 Effect of slab width on maximum deflection

with an opposite tendency. Maximum deflection rapidly decreases with the increase in slab thickness while it gradually increases with the increase in the load. When the slab thickness is greater than or equal to 26 m, the load has no significant effect on maximum deflection, as shown in Fig. 4.

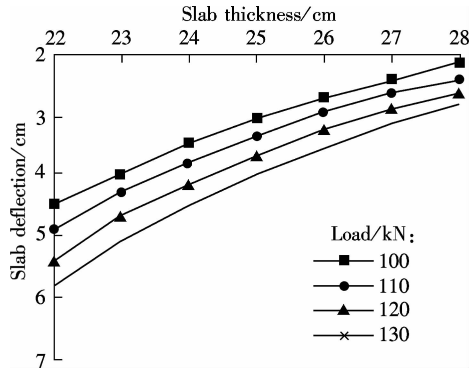

Fig. 4 Effect of slab thickness on maximum bending deflection

4.2 Effect of parameters on maximum stress

Fig. 5 shows the effect of the slab width on maximum stress. Maximum stress rapidly increases with the increase in slab width and gradually increases with the increase in the load.

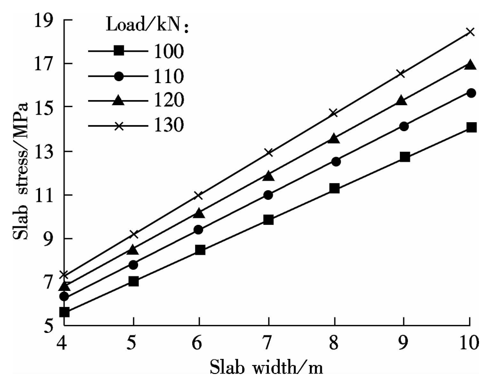

Fig. 5 Effect of slab width on maximum stress

It is clear from Fig. 6 that the effect of the slab thickness on maximum stress is also significant. Maximum stress decreases rapidly with the increase in the slab thickness while it increases gradually with the increase in the load. When the slab thickness is greater than or equal to 28 cm, maximum stress is less than the slab bending strength (5 MPa) under a standard axle load (100 kN). When slab thickness is less than or equal to 27 cm, maximum stress is greater than the

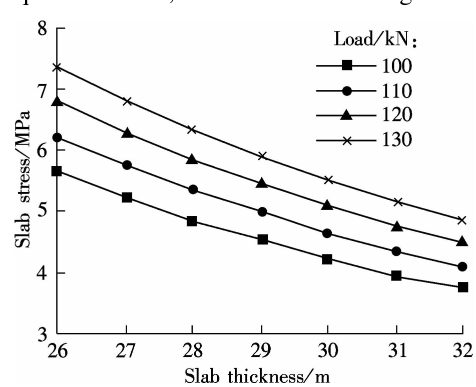

Fig. 6 Effect of slab thickness on maximum stress

slab bending strength under a standard axle load and cracking occurs in the bottom layer of the slab. As time goes by, the crack extends to the top of the slab, and a punchout occurs.

5 Conclusions

1) Based on the equivalence principle of deflection and stress, the concentrated vehicle load which acts on the center of CRCP can be translated into the equivalent half-wave sine load by the Fourier transform. The formulae and results given are reliable.

2) Maximum deflection is directly proportional to the load magnitude and the slab width. Contrarily, it is inversely proportional to the transverse elastic modulus and the slab thickness, where the slab width has a significant effect.

3) Maximum stress is directly proportional to the load magnitude and the slab width. Contrarily, it is inversely proportional to the slab thickness, which has a significant effect on maximum stress. Therefore, it should be possible to increase slab thickness to reduce maximum deflection and maximum stress.

4) The method can be used to research CRCP deflection and the stress on soft soil base, loess foundation, liquefied sand and mining subgrade when the CRCP slab is subjected to a concentrated vehicle load.

5) When CRCP is under several loads which are not in the center of the slab, according to the equivalence principle, the loads can be replaced with concentrated loads in the center and two bending moments in combination. So we can first obtain the deflection and the stress under concentrated loads using the method proposed in this paper. Secondly, the deflection and the stress caused by bending moments can be aggregated according to the superposition principle [13]. In this way, we can obtain the deflection and the stress of the CRCP slab under complex loads.

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车辆集中荷载作用下脱空 CRCP 板的挠度与应力

陈小兵　黄晓明　丁建明　童金虎

(东南大学交通学院,南京 210096)

摘要:基于挠度与应力等效原则,将作用于连续配筋混凝土路面(CRCP)中央的车辆集中荷载通过傅里叶级数展开为等效的半波正弦荷载.在荷载转换和小挠度弹性薄板理论的基础上,提出了车辆集中荷载作用下脱空地基上 CRCP 的挠度和应力计算公式,分析了参数的敏感性.结果表明:板的最大挠度与车辆集中荷载、板的宽度成正比,与板的横向弯曲刚度、板的厚度成反比,其中板的宽度与厚度影响显著.板的最大应力与车辆集中荷载、板的宽度成正比,与板的厚度成反比,其中板的厚度影响显著.根据计算结果,降低最大挠度和应力最有效的方法是增加板的厚度.

关键词:车辆集中荷载;等效原理;半波正弦荷载;弹性薄板;脱空的 CRCP 板;挠度与应力公式;板的厚度

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