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Travel time function for basic link considering signal control in network traffic model

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Abstract: In order to describe the travel time of signal-controlled roads, a travel time model for urban basic roads based on the cumulative curve is proposed. First, the traffic wave method is used to analyze the formation and dispersion of the vehicle queue. Cumulative curves for road entrances and exits are established. Based on the cumulative curves, the travel time of the one-lane road under stable flow input is derived. And then, the multi-lane road is decomposed into a series of single-lane links based on its topological characteristics. Hence, the travel time function for the basic road is obtained. The travel time is a function of road length, flow and control parameters. Numerical analyses show that the travel time depends on the supply-demand condition, and it has high sensitivity during peak hours.

Key words: travel time; traffic wave; queue length; signal control

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The successful wide scale deployment of advanced traveler information systems (ATIS) and advanced traffic management systems (ATMS) depends on the ability to obtain and subsequently disseminate information that accurately reflects network traffic conditions. Many different techniques for assessing traffic conditions are proposed[1].

Travel time is one of the most important traffic state indices and attracts much attention. There are many approaches to model it. One of them expresses the travel time of a link as a function of traffic flow (inflow and outflow) on that link. Carey et al.[2] extended it with a so-called whole-link model. The travel time is treated as a function of an estimate of the flow in the immediate neighborhood of the vehicle, averaged over the time the vehicle spends traversing the link. An equivalent form, as a function of link density, is used in Refs. [3 – 4]. The second approach uses the LWR model proposed by Lighthill et al[5-6]. It is widely accepted both theoretically and empirically[7-8]. Based on the simplified kinematic wave model[9], Kuwahara and Akamatsu[10] derived an analytical function of the instantaneous travel time to solve the dynamic user equilibrium. However, the travel time they derived does not represent the actual (or experienced) travel time unless traffic conditions remain constant. Finally, the theory of vertical queues suggests that the travel time can be decomposed into a free-flow travel time and a waiting time in a queue[11]. However, these travel-time functions typically do not consider other traffic flow and disregard the spillback effects.

Besides, the above methods are combined with new technologies to estimate travel time[1,12]. They focus on estimation performance rather than travel time modeling. The emphasis on new technology to measure travel time is partially due to a misunderstanding of how to interpret vehicle travel times. For example, Sun et al.[13] used conventional average velocity sampled at a detector station over fixed time periods as a base case in their analysis. They found that link travel times significantly differ from the quotient of local velocity and the link distance. This result is not surprising since the link travel time for a vehicle reflects traffic conditions averaged over a fixed distance and a variable amount of time, while the detector data only reflect traffic conditions averaged over a fixed time period at a single point in space.

If we examine the basic road in an urban network, we can see that its topology (see Fig. 1) is somewhat more complex than a single "line" which is adopted in current research. The basic road is composed of two sections: the upstream section and the channelized section (with traffic signals at the head). Furthermore, since flow rate and control parameters (i. e. red time and green time) are different for left-turn, through and right-turn flow, travel time also should be different, which implies that they must be evaluated respectively.

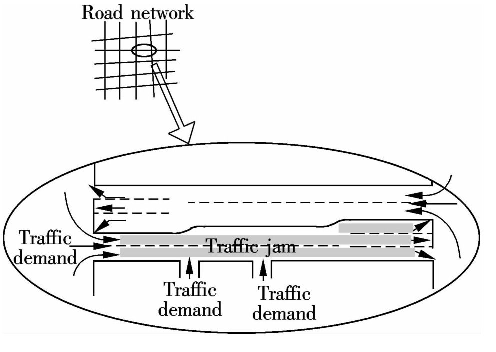

Fig. 1 Topology of basic road

Traditionally, the travel time is the sum of the free-flow travel time and the delay that is produced behind stop-line by signal control. However, if we check the velocity variation of a vehicle along a road (see Fig. 2), we find that delay can be attributed to two reasons: One is caused by signals and the other is caused by other vehicles (because of the decrease in velocity).

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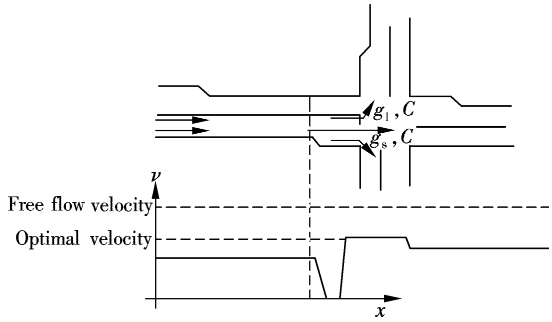

Fig. 2 Velocity profile along a link

In this paper, the travel time model for the basic link is proposed. Our ultimate objective is the properties of travel time. The method of the traffic wave is used. First, the queue dynamics of a simple link is analyzed. Based on the analysis, the travel time of a simple link is constructed. Then, the basic link is decomposed into a series of sub links and, hence, the travel for a basic link can be derived. Finally, properties of the travel time function are studied.

1 Preliminary Dynamics of Traffic Queue

The queue dynamics under signal control was clearly described in Ref. [14]. Here, we just simply describe the results. Consider a one-lane road controlled by signals without endogenous flow as shown in Fig. 3. Assume that the fundamental diagram of traffic flow is a parabolic function as shown in Fig. 4. The upstream flow is q (point A in Fig. 4), so $q = -\frac{q_m}{k_m^2}k(k-k_j)$. Density k can be obtained,

$$k = \frac{k_j - \sqrt{k_j^2 - 4\frac{k_m^2}{q_m}q}}{2} \tag{1}$$

where q_m denotes the maximum flow; k_j denotes the congestion density; and k_m denotes the optimal density. Formation and dispersion of a queue behind the stop line at the signal controlled road is shown in Fig. 5(a). At the beginning of red time, the stopping wave (line OB in Fig. 5(a) which represents the queue back) propagates upstream with velocity u_0 and a queue forms. When the effective green time begins, a starting wave (line AB in Fig. 5(a)) emerges and also propagates upstream with greater speed u_1. After t', the starting wave catches up with the stopping wave and the queue has dispersed. A new wave u_2 forms. It takes t'' for wave u_2 to run through the stop line. If the effective green time g is greater than $t'+t''$, then the traffic state will reproduce cycle by cycle. However, if g is smaller than $t'+t''$, the wave propagation profile is different (see Fig. 5(b)) from cycle to cycle. At first, the stopping wave spreads with speed u_1, which makes the queue back of this cycle further from the stop line than the former cycle. So the queue length will be longer and longer. It is an unstable condition which represents oversaturation.

Based on the analysis above, some formulae can be obtained as follows:

$$u_1 = \frac{q_m}{k_j - k_m},\ u_0 = \frac{q}{k_j - k},\ u_2 = \frac{q_m - q}{k_m - k} \tag{2}$$

$$u_0(r+t') = u_1 t' \Rightarrow t' = \frac{u_0 r}{u_1 - u_0} \tag{3}$$

$$l_{max} = u_1 t' = \frac{u_1 u_0 r}{u_1 - u_0} \tag{4}$$

$$t'' = \frac{L_{max}}{u_2} \tag{5}$$

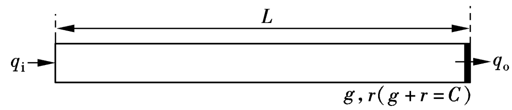

Fig. 3 A single one-way link

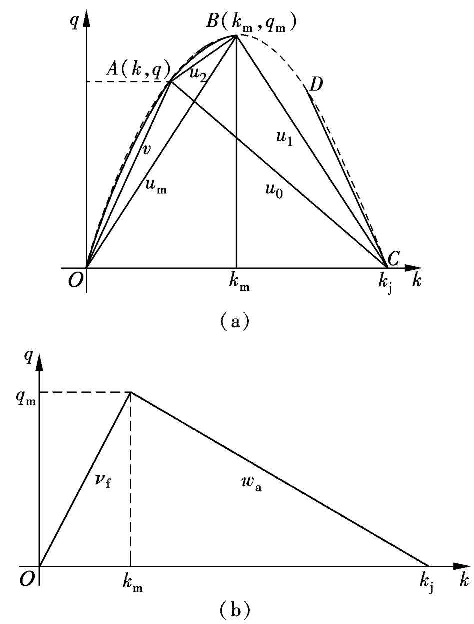

Fig. 4 Flow-density relationship. (a) Parabolic fundamental diagram; (b) Triangular fundamental diagram

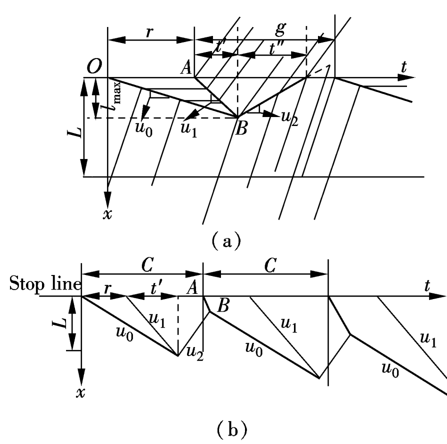

Fig. 5 Traffic wave propagation. (a) Unsaturated condition; (b) Over-saturated condition

When $g = t' + t''$ holds (this equation determines a specific split λ), a stable state forms (the decrease in λ will result in oversaturation), i. e.,

$$l_{max} = u_1 t' = \frac{u_1 u_0 r}{u_1 - u_0},\ g = \frac{l_{max}}{u_1} + \frac{l_{max}}{u_2} \tag{6}$$

2 Single-Lane Condition

Consider a simple one-way road controlled by signals displayed in Fig 3. Under the non-saturated condition, the traffic wave profile will reproduce cycle by cycle. Fig. 6(a) presents the wave dynamics within one cycle. Fig. 6(b) depicts the cumulative vehicle number of two locations: $x = 0$ and $x = L$. It can be expressed as $n(x, t)$ and means the cumulative vehicle number of location x at time t. The ideal or expected cumulative curve $n'(L, t)$ for the departing flow takes a shift from the cumulative curve of $x = 0$. So the lateral difference between $n(0, t)$ and $n(L, t)$ represents travel time while the lateral difference between $n'(L, t)$ and $n(L, t)$ represents delay. From Fig. 6(b), we can derive the travel time profile. The biggest delay is $r + L/v(q) - t_f$ and the smallest is $L/v(q) - t_f$. So the shape of the delay profile can be obtained (see Fig. 6(c)). t_f is the free-flow speed.

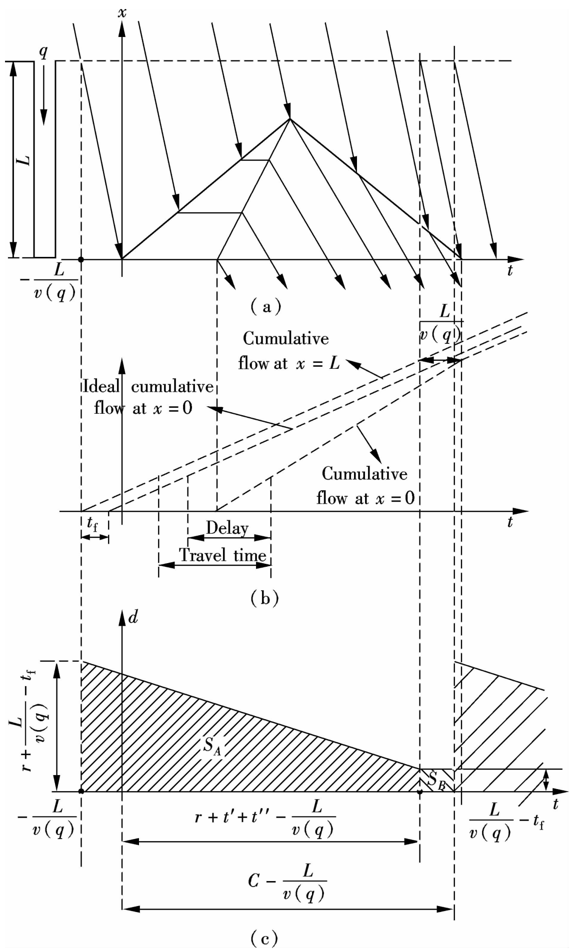

Fig. 6 Traffic wave and travel time derivation. (a) Wave profile; (b) Cumulative curve; (c) Delay pattern

The overall delay is the sum of the shaded area:

$$d = S_A + S_B \tag{7}$$

where S_A, with a trapezoid shape, and S_B, with a rectangle shape, are areas shown in Fig. 6(c),

$$S_A = \frac{\left(r + \frac{L}{v(q)} - t_f + \frac{L}{v(q)} - t_f\right)\left(r + t' + t'' - \frac{L}{v(q)} + \frac{L}{v(q)}\right)}{2} = \frac{\left(r + \frac{2L}{v(q)} - 2t_f\right)(r + t' + t'')}{2} \tag{8}$$

$$S_B = \left(\frac{L}{v(q)} - t_f\right)\left[\left(C - \frac{L}{v(q)}\right) - \left(r + t' + t'' - \frac{L}{v(q)}\right)\right] = \left(\frac{L}{v(q)} - t_f\right)(g - t' - t'') \tag{9}$$

So we have

$$d = S_A + S_B = \frac{\left(r + \frac{2L}{v(q)} - 2t_f\right)(r + t' + t'')}{2} + \left(\frac{L}{v(q)} - t_f\right)(g - t' - t'') \tag{10}$$

The average delay is

$$\bar{d} = \frac{d}{qC} = \frac{\left(r + \frac{2L}{v(q)} - 2t_f\right)(r + t' + t'')}{2qC} + \frac{\left(\frac{L}{v(q)} - t_f\right)(g - t' - t'')}{qC} \tag{11}$$

The ultimate average travel time is the sum of free-flow travel time and delay:

$$\bar{T} = t_f + \bar{d} = t_f + \frac{\left(r + \frac{2L}{v(q)} - 2t_f\right)(r + t' + t'')}{2qC} + \frac{\left(\frac{L}{v(q)} - t_f\right)(g - t' - t'')}{qC} = f_1(L, q, r, g) \tag{12}$$

3 Travel Time Function with Decomposition Method

Problems discussed above mainly focus on the ideal scenario of one-way links while the case in reality shows more topological complexity. We define the basic road as shown in Fig. 7(a) that always appears in an urban road network. It is a road controlled by signals with a channelized section of length l_d. Overall upstream traffic demand, q_i is assumed to be uniformly distributed between two lanes, i. e., $q_{i1} = q_{i2}$. The proportion of left-turn flow, right-turn flow and through flow is p_{ol}, p_{os} and p_{or} ($p_{or} = 1 - p_{ol} - p_{os}$), respectively. The symbol expression method for signal parameters is the same, i. e., g_{ol}, g_{os}, r_{ol}, and r_{os}. So we can obtain

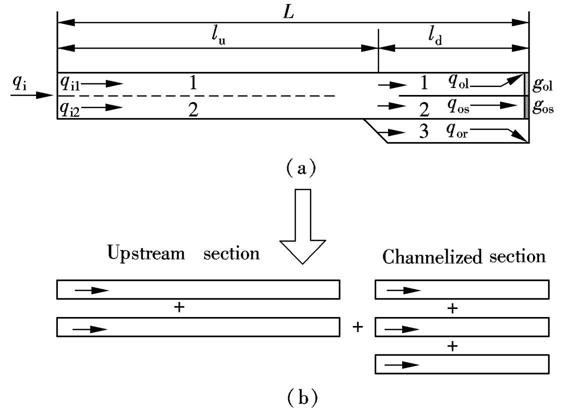

Fig. 7 Sketch map. (a) Basic road; (b) Decomposition

$$q_i = q_{ol} + q_{os} + q_{or},\ q_{ol} = q_i p_{ol},\ q_{os} = q_i p_{os},\ q_{or} = q_i p_{or} \tag{13}$$

We decompose the basic road as shown in Fig. 7(b). The basic road is divided into two sections: the upstream section and the channelized section. Both include some single one-way links. The channelized sections are controlled by the signal (except the right-turn lane). So this problem is simplified.

Delay also can be divided into two parts: upstream delay and channelized delay, so we have

$$\bar{d}_l = \bar{d}_u + \bar{d}_{dl} \tag{14}$$

where $\bar{d}_l$ denotes left-turn flow delay; $\bar{d}_u$ is the delay at the upstream section (It should be identical for three directions); $\bar{d}_{dl}$ is the delay for left-turn flow at the channelized section.

$$\bar{d}_u = \frac{l_u}{v_f} - \frac{l_u}{v(q_i/2)} \tag{15}$$

By Eq. (12), we can obtain

$$\bar{d}_{dl} = f_1(l_d, q_{ol}, r_{ol}, g_{ol}) \tag{16}$$

And the delay for left-turn flow can be derived,

$$\bar{d}_l = \bar{d}_u + \bar{d}_{dl} = \left(\frac{l_u}{v(q_i/2)} - \frac{l_u}{v_f}\right) + f_1(l_d, q_{ol}, r_{ol}, g_{ol}) \tag{17}$$

Similarly, the delay for through flow is

$$\bar{d}_s = \bar{d}_u + \bar{d}_{ds} = \frac{l_u}{v(q_i/2)} - \frac{l_u}{v_f} + f_1(l_d, q_{os}, r_{os}, g_{os}) \tag{18}$$

For right-turn flow, the delay is caused by other flows rather than signals, so we have

$$\bar{d}_r = \left(\frac{l_u}{v(q_i/2)} - \frac{l_u}{v_f}\right) + \left(\frac{l_d}{v(q_{os})} - \frac{l_d}{v_f}\right) \tag{19}$$

Hence, the travel time for three directions can be derived:

$$\bar{T}_l = \frac{l_u}{v(q_i/2)} + f_1(l_d, q_{ol}, r_{ol}, g_{ol}) = f_l(l_u, l_d, q_i, q_{ol}, r_{ol}, g_{ol}) \tag{20}$$

$$\bar{T}_s = \frac{l_u}{v(q_i/2)} + f_1(l_d, q_{os}, r_{os}, g_{os}) = f_s(l_u, l_d, q_i, q_{os}, r_{os}, g_{os}) \tag{21}$$

$$\bar{T}_r = \frac{l_u}{v(q_i/2)} + \frac{l_d}{v(q_{or})} = f_r(l_u, l_d, q_i, q_{or}) \tag{22}$$

4 Properties

As known to all that the traffic state is determined by traffic demand and supply (sometimes stochastic factors also work). The relationship among these three entities is shown in Fig. 8.

The travel time is also confined by supply and demand. From the above analysis, we can see that travel time can be formulated as the function of four parameters: link length, link structure (such as channelization length, number of lanes etc.), control parameters, and the flow rate (of three directions). The former three can be seen as traffic supply because they influence the overall capacity. The latter one can be seen as demand. In the following section, we discuss how much these factors can influence the travel time.

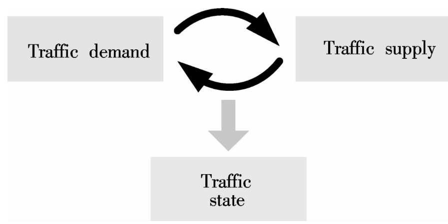

Fig. 8 Formation of traffic state

4.1 Relationship with supply structure

4.1.1 Link length

Rearrange the travel time function for the one-way link into the following form:

$$\bar{T} = t_f + \frac{\left(r + \frac{2L}{v(q)} - 2t_f\right)(r + t' + t'')}{2qC} + \frac{\left(\frac{L}{v(q)} - t_f\right)(g - t' - t'')}{qC} = \left[\frac{1/v - 1/v_f}{q} + \frac{1}{v_f}\right]L + \ldots \tag{23}$$

$$\frac{\mathrm{d}\bar{T}}{\mathrm{d}L} = \frac{1/v - 1/v_f}{q} + \frac{1}{v_f} \tag{24}$$

We can see that when other parameters are fixed, the travel time is in a linear relationship with link length. The slope is determined by the flow rate and the zero flow velocity.

4.1.2 Link structure

Take the left-turn flow travel time as an example. It can be formulated as the function of upstream section length, channelization section length and flow structure as well as signal parameters, i.e., $f_l(l_u, l_d, q_i, q_{ol}, r_{ol}, g_{ol})$. So

$$\frac{\mathrm{d}\bar{T}_l}{\mathrm{d}l_u} = \frac{1}{v(q_i/2)} \tag{25}$$

$$\frac{\mathrm{d}\bar{T}_l}{\mathrm{d}l_d} = \frac{1}{q_{ol}v(q_{ol})} + \frac{1}{v_f} \tag{26}$$

4.1.3 Control parameters

Based on Eq. (12), Fig. 9 and Fig. 10 present the relationship between travel time and signal parameters. Given

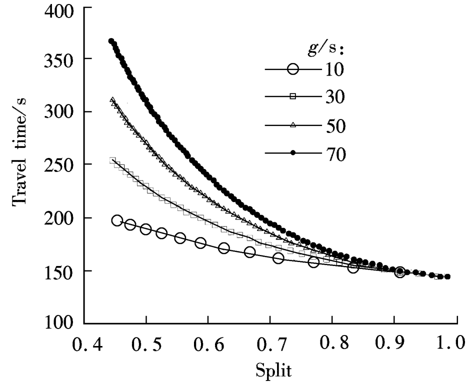

Fig. 9 Travel time vs. split with arriving flow rate of 800 veh/h and $l = 400$ m

split, the travel time will increase with green time. Furthermore, the slope which depicts the sensitivity of travel time to control parameters also shows an increasing trend. This implies that when the traffic state is bad, the travel time fluctuates more than that of the normal state.

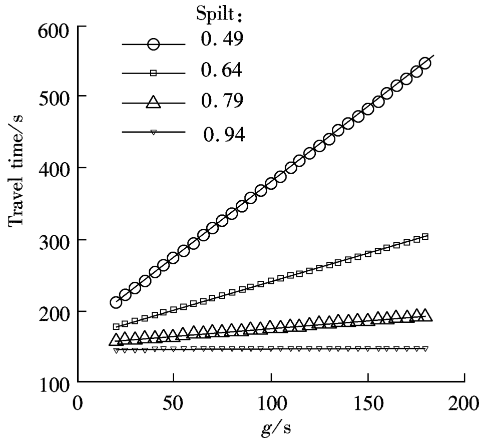

Fig. 10 Travel time vs. green time with arriving flow rate of 800 veh/h

4.2 Relationship with demand

4.2.1 Flow rate

The flow rate can be seen as traffic demand of an intersection and it is an important factor in travel time formation.

Fig. 11 gives the dynamic relationship of the flow rate and the travel time when other parameters are fixed. We can see that when the flow rate increases, the travel time first decreases and then increases. Because from Fig. 6, $S_A > r^2/2$, and $S_B > 0$, then $d > r^2/2$. So $\overline{d} \to \infty$ as $q \to 0$.

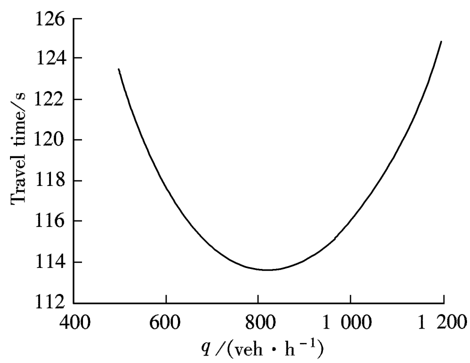

Fig. 11 Travel time vs. q with $r = 40$, $g = 80$, $l = 400$ m

4.2.2 Flow rate structure

From Eqs. (20) to (22), it can be concluded that travel time for a specific direction such as left-turn flow also relates to other flows. Take left turn flow as an example, the related term in the travel time is $\dfrac{l_u}{v(q_i/2)}$. Fig. 12 presents the term profile with respect to the dynamic flow rate of q_{os}.

To explore the inner influences of other direction flows, we take derivates,

$$\frac{\mathrm{d}\,\overline{T}_l}{\mathrm{d}q_{os}} = \frac{\mathrm{d}\,\dfrac{l_u}{v(q_i/2)}}{\mathrm{d}q_{os}} = -\frac{l_u}{2(v(q_i/2))^2} v'\left(\frac{q_i}{2}\right) \tag{27}$$

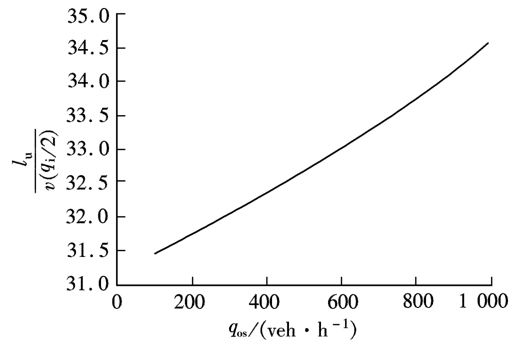

Fig. 12 $l_u/v(q_i/2)$ vs. q_{os} under the condition of $q_{ol} = 500$ veh/h, $q_{or} = 200$ veh/h, $l_u = 320$ m

$$v'(q) = \frac{\mathrm{d}\,\dfrac{v_f + \sqrt{v_f^2 - 4(v_f/k_j)q}}{2}}{\mathrm{d}q} = -\frac{v_f}{k_j\sqrt{v_f^2 - 4(v_f/k_j)q}} \tag{28}$$

$$\frac{\mathrm{d}\,\overline{T}_l}{\mathrm{d}q_{os}} = \frac{l_u v_f}{2k_j\left(v\left(\dfrac{q_i}{2}\right)\right)^2\sqrt{v_f^2 - 2\dfrac{q_i v_f}{k_j}}} \tag{29}$$

We can see that the travel time for different flows is not identical, which should be evaluated respectively.

5 Conclusion

The travel time function that is suitable for network analysis with a time scale such as 10 min is studied. The function describes the relationship between travel time and traffic demand as well as supply. Properties of the travel time model are studied in this supply-demand framework. We find that travel time is very sensitive to traffic state, especially when the saturation tends to be 1. This model outperforms other models in the situation that road geometry, flow structure and signal parameters are all accounted for.

However, traffic dynamics may be complicated when the demand exceeds supply. Under this condition, the vehicle queue becomes longer and longer. Vehicle trajectory together with traffic wave changes cycle by cycle. This situation should be taken into account. Furthermore, the decomposition method may cause some inaccuracy which should be overcome by attaching a revision coefficient. These will be our future work.

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网络交通模型中考虑信号控制的行程时间函数模型

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摘要:为了描述信号控制路段的行程时间,基于累计曲线法提出了城市基本道路的行程时间模型. 首先利用交通波方法分析了路段排队形成及消散过程;然后构建路段入口和出口的累计曲线,通过累计曲线得到了单车道、稳定流量输入条件下的形成时间函数;最后考虑路段的拓扑结构特性,将多车道路段分解成多个单车道的组合,并得出了基本道路的行程时间模型. 该行程时间函数是路段长度、流量以及信号控制参数的函数. 数值分析显示,行程时间取决于路段交通供给−需求情况,且在高峰情况下更加敏感.

关键词:行程时间;交通波;排队长度;信号控制
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