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Dual pairs of tilting pairs

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Abstract: A module pair (C, T) over an Artin algebra Λ is called a tilting pair if both C and T are selforthogonal modules and the conditions $T \in \mathrm{add}\hat{C}$ and $C \in \mathrm{add}\check{T}$ hold. The duality on a tilting pair is investigated to discuss the condition under which the dual of a tilting pair is also a tilting pair. A necessary and sufficient condition of $(D(T), D(C))$ being an n-tilting pair over an Artin algebra for an n-tilting pair (C, T) is given. And, a necessary and sufficient condition of (T^*, C^*) being an n-tilting pair over a selfinjective Artin algebra for an n-tilting pair (C, T) is also given.

Key words: selforthogonal module; selfinjective; dual module; tilting pair

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In 1958, Morita[1] established the theory of the Morita duality, as a generalization of the duality of vector spaces over division rings. Azumaya[2] also investigated the theory of the Morita duality. The notion of tilting pairs was introduced by Miyashita[3] for constructing tilting modules with a left tilting series of ideals of an Artin algebra. A module pair (C, T) over an Artin algebra Λ is called a tilting pair if the following conditions hold: 1) C is selforthogonal; 2) T is selforthogonal; 3) $T \in \mathrm{add}\hat{C}$; 4) $C \in \mathrm{add}\check{T}$. T is an n-tilting module if and only if (Λ, T) is an n-tilting pair, and C is an n-cotilting module if and only if $(C, D(\Lambda))$ is an n-tilting pair. Hence, the tilting pair can be viewed as a generalization of the tilting module and the cotilting module. It is useful in the representation theory of Artin algebra. Tilting pairs are investigated by Wei et al[4–5]. The aim of this paper is to investigate the condition under which the dual of a tilting pair is also a tilting pair.

We prove that (C, T) is an n-tilting pair in mod Λ if and only if $(D(T), D(C))$ is an n-tilting pair in $(\mathrm{mod}\ \Lambda)^{\mathrm{op}}$ over an Artin algebra Λ, and (C, T) is an n-tilting pair in mod Λ if and only if (T^*, C^*) is an n-tilting pair in $(\mathrm{mod}\ \Lambda)^{\mathrm{op}}$ over a selfinjective Artin algebra Λ.

1 Preliminaries

Throughout this paper, Λ denotes an Artin algebra over a commutative Artin ring R; mod Λ denotes the category of finitely generated left Λ-modules; $(\mathrm{mod}\ \Lambda)^{\mathrm{op}}$ denotes the category of finitely generated right Λ-modules; $()^*$ denotes $\mathrm{Hom}_\Lambda(-, \Lambda)$ and D denotes the usual Matlis duality $\mathrm{Hom}_R(-, E(R/J(R)))$, where $J(R)$ is Jacobson radical of R and $E(M)$ is the injective envelope of a Λ-module M. For a Λ-module T, we denote add T the subcategory of mod Λ whose objects are all direct summands of the finite direct sum of copies of T.

For a subcategory $\mathscr{C}$ of mod Λ, we denote by $\hat{\mathscr{C}}$ the subcategory of mod Λ whose objects are the Λ-modules M, for which there is a finite exact sequence $0 \to C_n \to \ldots \to C_0 \to M \to 0$ with $C_i \in \mathscr{C}$, and denote by $\dim_{\mathscr{C}}(M)$ the least integer n such that there is an exact sequence $0 \to C_n \to \ldots \to C_0 \to M \to 0$ with $C_i \in \hat{\mathscr{C}}$, and $(\hat{\mathscr{C}})_n$ the subcategory of mod Λ consisting of all $M \in \hat{\mathscr{C}}$ with $\dim_{\mathscr{C}}(M) \leqslant n$. Dually we denote by $\check{\mathscr{C}}$ the subcategory of whose objects are the Λ-module M which admit a finite exact sequence $0 \to M \to C_0 \to \ldots \to C_n \to 0$ with $C_i \in \mathscr{C}$. Similarly, $\mathrm{codim}_{\mathscr{C}}(M)$ denotes the least integer n such that there is an exact sequence $0 \to M \to C_0 \to \ldots \to C_n \to 0$ with $C_i \in \mathscr{C}$, $(\check{\mathscr{C}})_n$ the subcategory of mod Λ consisting of all $M \in \check{\mathscr{C}}$ with $\mathrm{codim}_{\mathscr{C}}(M) \leqslant n$. We say that (C, T) is an n-tilting pair if (C, T) is a tilting pair such that $\dim_{\mathrm{add}\,\mathscr{C}}(T) \leqslant n$.

For a subcategory or a singleton $\mathscr{C}$, we denote ${}^\perp\mathscr{C} = \bigcap_{i \geqslant 1} \mathrm{KerExt}_\Lambda^i(-, \mathscr{C})$ and $\mathscr{C}^\perp = \bigcap_{i \geqslant 1} \mathrm{KerExt}_\Lambda^i(\mathscr{C}, -)$.

For a selforthogonal Λ-module T, we denote by ${}_T\chi$ the subcategory of $T^\perp$ whose objects are the Λ-modules X such that there is an exact sequence

$$\ldots \to T_m \xrightarrow{f_m} T_{m-1} \to \ldots \to T_0 \xrightarrow{f_0} X \to 0$$

with $T_i \in \mathrm{add}\ T$ and $\mathrm{Ker}\, f_i \in T^\perp$ for all $i \geqslant 0$, and χ_T the subcategory of ${}^\perp T$ whose objects are the Λ-modules C such that there is an exact sequence

$$0 \to C \to T_0 \xrightarrow{f_0} \ldots \to T_n \xrightarrow{f_n} T_{n+1} \ldots$$

with $T_i \in \mathrm{add}\ T$ and $\mathrm{Im}\, f_i \in {}^\perp T$ for all $i \geqslant 0$.

For convenience, we often denote $\mathrm{Hom}(A, B)$ by (A, B) in some commutative diagrams.

2 Some Properties of Selforthogonal Modules

In this section we study the properties of left orthogonal modules of a selforthogonal module. Recall that a module $M \in \mathrm{mod}\ \Lambda$ is called selforthogonal if $\mathrm{Ext}_\Lambda^i(M, M) = 0$ for all $i \geqslant 1$.

Lemma 1 If $M \in \mathrm{mod}\ \Lambda$ is a selforthogonal module and $\Gamma = \mathrm{End}_\Lambda(M)^{\mathrm{op}}$, then

$$\mathrm{Hom}_\Lambda(C, A) \cong \mathrm{Hom}_\Gamma(\mathrm{Hom}_\Lambda(A, M), \mathrm{Hom}_\Lambda(C, M))$$

is an isomorphism for all $C \in \mathrm{mod}\ \Lambda$ and all $A \in \chi_M$.

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Proof 1) It is clear that $e_M = \mathrm{Hom}_\Lambda(-, M): \mathrm{Hom}_\Lambda(C, M) \to \mathrm{Hom}_\Gamma(e_M(M), e_M(C))$ is an isomorphism. It follows from the additivity of e_M that

$$e_M: \mathrm{Hom}_\Lambda(C, A) \to \mathrm{Hom}_\Gamma(e_M(A), e_M(C))$$

is an isomorphism for $C \in \mathrm{mod}\ \Lambda$ and $A \in \mathrm{add}\ M$.

2) Let $A \in \chi_M$. Then there is an exact sequence $0 \to A \to M_0 \xrightarrow{d} M_1 \to \ldots$, where $M_i \in \mathrm{add}\ M$, $\mathrm{Im}\ d \in \chi_M$ and $\mathrm{Coker}\ d \in \chi_M$. So the sequence

$$\mathrm{Hom}_\Lambda(M_1, M) \to \mathrm{Hom}_\Lambda(M_0, M) \to \mathrm{Hom}_\Lambda(A, M) \to 0$$

is exact. Now applying $\mathrm{Hom}_\Gamma(-, \mathrm{Hom}_\Lambda(C, M))$ to this sequence, we obtain the following commutative diagram with exact rows,

$$\begin{array}{ccccccc} 0 & \to & \mathrm{Hom}_\Lambda(C,A) & \to & \mathrm{Hom}_\Lambda(C,M_0) & \to & \mathrm{Hom}_\Lambda(C,M_1)\\ & & \downarrow \varphi_1 & & \downarrow \varphi_2 & & \downarrow \varphi_3 \\ 0 & \to & ((A,M),(C,M)) & \to & ((M_0,M),(C,M)) & \to & ((M_1,M),(C,M)) \end{array}$$

By 1), both φ_2 and φ_3 are isomorphisms. Hence, φ_1 is an isomorphism:

$$\mathrm{Hom}_\Lambda(C, A) \cong \mathrm{Hom}_\Gamma(\mathrm{Hom}_\Lambda(A, M), \mathrm{Hom}_\Lambda(C, M))$$

for all $C \in \mathrm{mod}\ \Lambda$ and all $A \in \chi_M$.

Furthermore, we investigate the relationship between the corresponding Ext-groups. We have the following result.

Proposition 1 If $M \in \mathrm{mod}\ \Lambda$ is a selforthogonal module and $\Gamma = \mathrm{End}_\Lambda(M)^{\mathrm{op}}$, then we have the isomorphism,

$$\mathrm{Ext}^i_\Lambda(C, A) \cong \mathrm{Ext}^i_\Gamma(\mathrm{Hom}_\Lambda(A, M), \mathrm{Hom}_\Lambda(C, M))$$

for all $C \in {}^\perp M$, $A \in \chi_M$ and all $i \geqslant 0$.

Proof Since $A \in \chi_M$, there exists an exact sequence

$$0 \to A \to M_0 \xrightarrow{d_0} M_1 \to \ldots \to M_n \xrightarrow{d_n} M_{n+1} \to \ldots$$

where $M_i \in \mathrm{add}\ M$ and $\mathrm{Im}\ d_i \in {}^\perp M$ for all $i \geqslant 0$. Then the exact sequence

$$\ldots \to (M_n, M) \to \ldots \to (M_1, M) \to (M_0, M) \to (A, M) \to 0$$

is a Γ-projective resolution of $\mathrm{Hom}_\Lambda(A, M)$. Now applying the functor $\mathrm{Hom}_\Gamma(-, \mathrm{Hom}_\Lambda(C, M))$ to this sequence, we obtain the following commutative diagram,

$$\begin{array}{ccccccc} 0 & \to & \mathrm{Hom}_\Lambda(C,A) & \to & \mathrm{Hom}_\Lambda(C,M_0) & \to & \mathrm{Hom}_\Lambda(C,M_1) \to \ldots\\ & & \downarrow \varphi_1 & & \downarrow \varphi_2 & & \downarrow \varphi_3 \\ 0 & \to & ((A,M),(C,M)) & \to & ((M_0,M),(C,M)) & \to & ((M_1,M),(C,M)) \to \ldots \end{array}$$

By Lemma 1 all $\varphi_1, \varphi_2, \varphi_3, \ldots$ are isomorphisms. The i-cohomology of the lower row is $\mathrm{Ext}^i_\Gamma(\mathrm{Hom}_\Lambda(A, M), \mathrm{Hom}_\Lambda(C, M))$. Thus it remains to show that the i-cohomology of the upper row, $\mathrm{KerHom}_\Lambda(C, d_i)/\mathrm{ImHom}_\Lambda(C, d_{i-1})$, is isomorphic to $\mathrm{Ext}^i_\Lambda(C, A)$. Since $C \in {}^\perp M$, applying the functor $\mathrm{Hom}_\Lambda(C, -)$ to the exact sequence,

$$0 \to \mathrm{Im}\ d_k \xrightarrow{\alpha_k} M_k \xrightarrow{\beta_{k+1}} \mathrm{Im}\ d_{k+1} \to 0 \qquad k \geqslant 0$$

yields the following exact sequence,

$$\mathrm{Ext}^i_\Lambda(C, M_k) = 0 \to \mathrm{Ext}^i_\Lambda(C, \mathrm{Im}\ d_{k+1}) \to \mathrm{Ext}^{i+1}_\Lambda(C, \mathrm{Im}\ d_k) \to 0$$

which gives rise to isomorphisms,

$$\mathrm{Ext}^i_\Lambda(C, \mathrm{Im}\ d_{k+1}) \cong \mathrm{Ext}^{i+1}_\Lambda(C, \mathrm{Im}\ d_k)$$

for all $i \geqslant 1$. So we have

$$\mathrm{Ext}^i_\Lambda(C, A) \cong \mathrm{Ext}^{i-1}_\Lambda(C, \mathrm{Im}\ d_0) \cong \ldots \cong \mathrm{Ext}^1_\Lambda(C, \mathrm{Im}\ d_{i-2}) \quad (1)$$

On the other hand, applying the functor $\mathrm{Hom}_\Lambda(C, -)$ to the commutative diagram with exact row and column,

$$\begin{tikzcd} & & & 0 \arrow[d] & \\ 0 \arrow[r] & \mathrm{Im}\,d_{i-2} \arrow[r, "\alpha_{i-2}"] & M_{i-1} \arrow[r, "\beta_{i-1}"] \arrow[dr, "d_{i-1}"'] & \mathrm{Im}\,d_{i-1} \arrow[r] \arrow[d, "\alpha_{i-1}"] & 0 \\ & & & M_i \arrow[d, "d_i"] & \\ & & & M_{i+1} & \end{tikzcd}$$

we obtain the following commutative diagram with exact row and column,

$$\begin{tikzcd} & 0 \arrow[d] & & \\ (C,M_{i-1}) \arrow[r, "{(C,\beta_{i-1})}"] \arrow[dr, "{(C,d_{i-1})}"'] & (C,\mathrm{Im}\,d_{i-1}) \arrow[r] \arrow[d, "{(C,\alpha_{i-1})}"] & \mathrm{Ext}^1_\Lambda(C,\mathrm{Im}\,d_{i-2}) \arrow[r] & 0 \\ & (C,M_i) \arrow[d, "{(C,d_i)}"] & & \\ & (C,M_{i+1}) & & \end{tikzcd}$$

It follows from the above diagram that

$$\begin{aligned} &\mathrm{KerHom}_\Lambda(C, d_i)/\mathrm{ImHom}_\Lambda(C, d_{i-1}) \cong \\ &\quad \mathrm{ImHom}_\Lambda(C, \alpha_{i-1})/\mathrm{Im}(\mathrm{Hom}_\Lambda(C, \alpha_{i-1}) \circ \mathrm{Hom}_\Lambda(C, \beta_{i-1})) \cong \\ &\quad \mathrm{Hom}_\Lambda(C, \mathrm{Im}\ d_{i-1})/\mathrm{ImHom}_\Lambda(C, \beta_{i-1}) \cong \\ &\quad \mathrm{Ext}^1_\Lambda(C, \mathrm{Im}\ d_{i-2}) \end{aligned}$$

Thus by formula (1), we have

$$\mathrm{KerHom}_\Lambda(C, d_i)/\mathrm{ImHom}_\Lambda(C, d_{i-1}) \cong \mathrm{Ext}^i_\Lambda(C, A)$$

Therefore, $\mathrm{Ext}^i_\Gamma(\mathrm{Hom}_\Lambda(A, M), \mathrm{Hom}_\Lambda(C, M)) \cong \mathrm{Ext}^i_\Lambda(C, A)$.

Corollary 1 If $M \in \mathrm{mod}\ \Lambda$ is a cotilting module and $\Gamma = \mathrm{End}_\Lambda(M)^{\mathrm{op}}$, then we have the isomorphism,

$$\mathrm{Ext}^i_\Lambda(C, A) \cong \mathrm{Ext}^i_\Gamma(\mathrm{Hom}_\Lambda(A, M), \mathrm{Hom}_\Lambda(C, M))$$

for all $A, C \in {}^\perp M$ and all $i \geqslant 0$.

Proof Since $M \in \bmod \Lambda$ is a cotilting module, we have $\chi_M = {}^{\perp}M$ by Ref. [6]. Thus, from Proposition 1, the desired result follows.

3 Dual Pairs of Tilting Pairs

In this section, a necessary and sufficient condition of $(D(T), D(C))$ and (T^*, C^*) being an n-tilting pair is obtained for a given n-tilting pair (C, T).

The following lemma is useful for our discussion.

Lemma 2[7] The functor $\mathrm{Hom}_R(-, \mathrm{E}(R/J(R))) = D$ and $\mathrm{Hom}_R(-, D(\Lambda))$ from $\bmod \Lambda$ to $(\bmod \Lambda)^{\mathrm{op}}$ are isomorphic.

Now, we can obtain the following result.

Theorem 1 Assume that $C, T \in \bmod \Lambda$. Then (C, T) is an n-tilting pair in $\bmod \Lambda$ if and only if $(D(T), D(C))$ is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$.

Proof $(\Rightarrow)$ Assume that (C, T) is an n-tilting pair. Since $C \in (\mathrm{add}\check{\ } T)_n$, there exists an exact sequence,

$$0 \to C \to T_0 \to \ldots \to T_{n-1} \to T_n \to 0 \qquad (2)$$

with $T_i \in \mathrm{add}\, T$. Note that $D(\Lambda)$ is a finitely generated two-sided injective cogenerator[8]. By Lemma 2, the functor $D \cong \mathrm{Hom}_R(-, (\Lambda))$ is exact. Applying functor D to sequence (2), we obtain the following exact sequence,

$$0 \to D(T_n) \to D(T_{n-1}) \to \ldots \to D(T_0) \to D(C) \to 0 \qquad (3)$$

Since $T_i \in \mathrm{add}\, T$, we have $D(T_i) \in \mathrm{add}\, D(T)$. It follows that $D(C) \in (\widehat{\mathrm{add}}\, D(T))_n$ from (3). Similarly, $T \in (\widehat{\mathrm{add}}\, C)_n$ yields $D(T) \in (\check{\mathrm{add}}\, D(C))_n$.

It remains to show that both $D(T)$ and $D(C)$ are selforthogonal. Note that $D(\Lambda)$ is a finitely generated two-sided injective cogenerator. For $M = D(\Lambda)$, we have $\chi_M = {}^{\perp}M = \bmod \Lambda$. So $\Gamma = \mathrm{End}_\Lambda(M)^{\mathrm{op}} \cong D^2(\Lambda)^{\mathrm{op}} \cong \Lambda^{\mathrm{op}}$ by Lemma 2. Now, by Lemma 2 and Corollary 1, we obtain

$$\begin{aligned}&\mathrm{Ext}^i_{\Lambda^{\mathrm{op}}}(D(T), D(T)) \cong \\ &\quad \mathrm{Ext}^i_{\Lambda^{\mathrm{op}}}((T, D(\Lambda)), (T, D(\Lambda))) \cong \\ &\quad \mathrm{Ext}^i_\Lambda((T, T) = 0\end{aligned}$$

for all $i \geqslant 1$. Similarly, we have $\mathrm{Ext}^i_{\Lambda^{\mathrm{op}}}(D(C), D(C)) = 0$ for all $i \geqslant 1$.

$(\Leftarrow)$ Assume that $(D(T), D(C))$ is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$. Note that D is a duality between $\bmod \Lambda$ and $(\bmod \Lambda)^{\mathrm{op}}$. By the if-part of this theorem, we have that $(D^2(C), D^2(T))$ is an n-tilting pair in $\bmod \Lambda$ and so (C, T) is an n-tilting pair in $\bmod \Lambda$.

An Artin algebra Λ is said to be symmetric if $\Lambda \cong D(\Lambda)$ as a two-sided Λ-module[7,9]. When Λ is a symmetric Artin algebra, by Lemma 2, we have $D \cong ()^*$ and then (C, T) is an n-tilting pair in $\bmod \Lambda$ if and only if (T^*, C^*) is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$. For a general case, we have the following result.

Proposition 2 Assume that $C, T \in \bmod \Lambda$. If (C, T) is an n-tilting pair in $\bmod \Lambda$ and $C, T \in \chi_\Lambda$, then (T^*, C^*) is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$.

Proof First, we show that both T^* and C^* are selforthogonal. In fact, set $M = \Lambda$ in Proposition 1, we have $\Gamma = \mathrm{End}_\Lambda(\Lambda)^{\mathrm{op}} \cong \Lambda^{\mathrm{op}}$. Since $T \in \chi_\Lambda$ and T is selforthogonal, we have, by Proposition 1, that

$$\mathrm{Ext}^i_{\Lambda^{\mathrm{op}}}(T^*, T^*) \cong \mathrm{Ext}^i_\Lambda((T, T) = 0$$

for all $i \geqslant 1$. Similarly, $\mathrm{Ext}^i_{\Lambda^{\mathrm{op}}}(C^*, C^*) = 0$ for all $i \geqslant 1$.

Secondly, since $C \in (\check{\mathrm{add}}\, T)_n$, there exists an exact sequence,

$$0 \to C \to T_0 \to \ldots \to T_{n-1} \to T_n \to 0$$

with $T_i \in \mathrm{add}\, T$. Note that $T \in \chi_\Lambda \subseteq {}^{\perp}\Lambda$. Applying the functor $()^*$ to the sequence above, we have the following exact sequence,

$$0 \to T_n^* \to T_{n-1}^* \to \ldots \to T_0^* \to C^* \to 0$$

Since $T_i \in \mathrm{add}\, T$, $T_i^* \in \mathrm{add}\,(T^*)$. Hence $C^* \in (\widehat{\mathrm{add}}\, T^*)_n$.

Finally, note that $C, T \in \chi_\Lambda \subseteq {}^{\perp}\Lambda$. It easily follows from $T \in (\widehat{\mathrm{add}}\, C)_n$ that $T^* \in (\widehat{\mathrm{add}}\, C^*)_n$.

Therefore, the dual pair (T^*, C^*) is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$.

The rest of this section is devoted to discussing the case of a selfinjective Artin algebra for the dual pair (T^*, C^*). An Artin algebra Λ is said to be selfinjective if it is injective as a Λ-module[7,9]. From Ref. [7], we know that Λ^{op} is also a selfinjective Artin algebra if Λ is a selfinjective Artin algebra.

Lemma 3 Assume that Λ is selfinjective. Then $\mathrm{Hom}_\Lambda(-, \Lambda): \chi_\Lambda \to \chi_{\Lambda^{\mathrm{op}}}$ is a duality with dual inverse $\mathrm{Hom}_{\Lambda^{\mathrm{op}}}(-, \Lambda^{\mathrm{op}}): \chi_{\Lambda^{\mathrm{op}}} \to \chi_\Lambda$.

Proof Since the Artin algebra Λ is selfinjective, it is easy to see that $\chi_\Lambda = \bmod \Lambda = {}_\Lambda\chi$. Note that Artin algebra Λ^{op} is also selfinjective. We have $\chi_{\Lambda^{\mathrm{op}}} = (\bmod \Lambda)^{\mathrm{op}} = {}_{\Lambda^{\mathrm{op}}}\chi$. Now, according to Ref. [7], $\mathrm{Hom}_\Lambda(-, \Lambda): \chi_\Lambda \to \chi_{\Lambda^{\mathrm{op}}}$ is a duality.

Theorem 2 Assume that Λ is selfinjective and $C, T \in \bmod \Lambda$. Then (C, T) is an n-tilting pair in $\bmod \Lambda$ if and only if (T^*, C^*) is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$.

Proof $(\Rightarrow)$ Since Λ is selfinjective, we have $\chi_\Lambda = \bmod \Lambda$. Then by Proposition 2, (T^*, C^*) is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$.

$(\Leftarrow)$ Conversely, suppose that (T^*, C^*) is an n-tilting pair in $(\bmod \Lambda)^{\mathrm{op}}$. By Proposition 2, (C^{**}, T^{**}) is an n-tilting pair in $\bmod \Lambda$. Then by Lemma 3, we have $C^{**} \cong C$. and $T^{**} \cong T$. Hence (C, T) is an n-tilting pair in $\bmod \Lambda$.

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倾斜对的对偶对

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摘要:在一个 Artin 代数 Λ 上，如果模 C 和 T 都是自正交模且满足条件 $T\in \mathrm{add}\hat{}\, C$ 和 $C\in \mathrm{add}\check{}\, T$，则模对$(C,T)$称为倾斜对. 讨论了倾斜对的对偶性,探讨在何种条件下一个倾斜对的对偶仍然是一个倾斜对. 给出了一个 Artin 代数上的模对$(D(T),D(C))$成为一个 n-倾斜对的充分必要条件. 同时也给出了一个自内射 Artin 代数上的模对(T^*,C^*)成为一个 n-倾斜对的充分必要条件.
关键词:自正交模；自内射性；对偶模；倾斜对
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