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New criterion for delay-dependent absolute stability of Lurie system with interval time-varying delay

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Abstract: The delay-dependent absolute stability for a class of Lurie systems with interval time-varying delay is studied. By employing an augmented Lyapunov functional and combining a free-weighting matrix approach and the reciprocal convex technique, an improved stability condition is derived in terms of linear matrix inequalities (LMIs). By retaining some useful terms that are usually ignored in the derivative of the Lyapunov function, the proposed sufficient condition depends not only on the lower and upper bounds of both the delay and its derivative, but it also depends on their differences, which has wider application fields than those of present results. Moreover, a new type of equality expression is developed to handle the sector bounds of the nonlinear function, which achieves fewer LMIs in the derived condition, compared with those based on the convex representation. Therefore, the proposed method is less conservative than the existing ones. Simulation examples are given to demonstrate the validity of the approach.

Key words: Lurie system; reciprocal convex technique; absolute stability; interval time-varying delay; linear matrix inequality (LMI)

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Recently, there has been increasing interest in research on delay-dependent absolute stability conditions for Lurie systems with both constant delay[1] and time-varying one[2]. Yet, these references assumed the lower bound of time-delay to be zero. As we know, the lower bound of time delay is always greater than 0 in practical cases, which can be viewed as the case of interval time delay[3-6]. In order to effectively reduce the conservatism of derived results, some novel methods have been proposed[5-6]. In Ref. [5], a new stability criterion has been provided by choosing an augmented Lyapunov functional and estimating a tighter upper bound of its derivative. Its basic idea is to employ the convex combination technique based on the Jensen inequality lemma[7]. Though the convex combination technique has been verified to be efficient in many cases, it still needs some improvements. Later, Ref. [6] has introduced one lower bound lemma and it can tackle the stability of delay systems more efficiently than the one in Ref. [5]. Meanwhile, it has come to our attention that those above works have not considered the information on the lower bound of the delay derivative as it is available, which remains important and challenging.

Inspired by the above discussions, we make some great attempts to investigate the absolute stability for the Lurie system with interval time-varying delay. Through applying one lower bound lemma for the linear combination and choosing an improved Lyapunov functional, one less conservative delay-dependent condition is presented in terms of LMIs.

1 Problem Formulations and Preliminaries

Consider the system with interval time-varying delay and sector-bounded nonlinearity described by

$$\left.\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{x}(t-h(t))+\boldsymbol{E}\boldsymbol{f}(\boldsymbol{z}(t))\\ \boldsymbol{z}(t) &= \boldsymbol{C}\boldsymbol{x}(t)\\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t) \qquad t\in[-h_2,0]\end{aligned}\right\} \tag{1}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state; $\boldsymbol{z}(t)\in\mathbf{R}^m$ is the output; $\boldsymbol{\phi}(t)$ is the continuous vector-valued initial function; $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{E}$ are constant matrices with appropriate dimensions; $h(t)$ denotes an interval time-varying delay satisfying

$$h_1 \leqslant h(t) \leqslant h_2, \quad \mu_1 \leqslant \dot{h}(t)\mu_2 \tag{2}$$

The nonlinear function $\boldsymbol{f}(\boldsymbol{z}(t)) = [\boldsymbol{f}_1(\boldsymbol{z}_1(t)),\cdots,\boldsymbol{f}_m(\boldsymbol{z}_m(t))]^{\mathrm{T}}$ with $\boldsymbol{f}_j(\cdot)$ satisfying $\boldsymbol{f}_j(0)=0$ and

$$\boldsymbol{a}_j z_j^2 \leqslant z_j\boldsymbol{f}_j(z_j) \leqslant \boldsymbol{b}_j z_j^2 \qquad \forall z_j\neq 0,\ \text{for } j=1,2,\cdots,m \tag{3}$$

where $\boldsymbol{a}_j$, $\boldsymbol{b}_j$ are lower and upper sector bounds. Furthermore, comparing with the convex representation of sector bounds on $f_j(\cdot)$ in Ref. [8] and based on (3), we develop a new type of equality constraint as follows:

$$\boldsymbol{f}_j(z_j(t)) = (\boldsymbol{\Delta}_j+\varepsilon_j\bar{\boldsymbol{\Delta}}_j)\,z_j(t) \tag{4}$$

where $\boldsymbol{\Delta}_j = (\boldsymbol{a}_j+\boldsymbol{b}_j)/2$, $\boldsymbol{\Delta}_j = (\boldsymbol{b}_j-\boldsymbol{a}_j)/2$, $\varepsilon_j = (\boldsymbol{f}_j(z_j(t))-\boldsymbol{\Delta}_j z_j(t))/(\bar{\boldsymbol{\Delta}}_j z_j(t))$. Now, let $z_j(t) = \boldsymbol{c}_j^{\mathrm{T}}\boldsymbol{x}_j(t)$, $\boldsymbol{\Delta}=\mathrm{diag}\{\boldsymbol{\Delta}_1,\ldots,\boldsymbol{\Delta}_m\}$, $\bar{\boldsymbol{\Delta}}=\mathrm{diag}\{\bar{\boldsymbol{\Delta}}_1,\ldots,\bar{\boldsymbol{\Delta}}_m\}$, $\boldsymbol{\Sigma}=\mathrm{diag}\{\varepsilon_1,\cdots,\varepsilon_m\}$, where $\boldsymbol{c}_j^{\mathrm{T}}\in\mathbf{R}^n$ is the j-th row vector of $\boldsymbol{C}$. Then the nonlinearity $\boldsymbol{f}(z(t))$ can be rewritten as

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$$f(z(t)) = (\boldsymbol{\Delta} + \boldsymbol{\Sigma}\bar{\boldsymbol{\Delta}})\boldsymbol{C}\boldsymbol{x}(t) \tag{5}$$

Since $-1 \leqslant \varepsilon_j \leqslant 1$, the parameter $\boldsymbol{\Sigma}$ satisfies

$$\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{\Sigma} \leqslant \boldsymbol{I} \tag{6}$$

Lemma 1[6] Let the functions $f_1(t), \cdots, f_N(t): \mathbf{R}^m \to \mathbf{R}$ have the positive values in an open subset $\boldsymbol{D}$ of $\mathbf{R}^m$ and satisfy $\frac{1}{\alpha_1}f_1(t) + \cdots + \frac{1}{\alpha_N}f_N(t): \boldsymbol{D} \to \mathbf{R}$ with $\alpha_i > 0$ and $\sum_{i=1}^{N}\alpha_i = 1$. Then

$$\sum_i \frac{1}{\alpha_i}f_i(t) \geqslant \sum_i f_i(t) + \sum_{i \neq j} g_{i,j}(t)$$

$$\forall g_{i,j}(t): \mathbf{R}^m \to \mathbf{R},\ g_{i,j}(t) = g_{j,i}(t),\ \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geqslant 0$$

2 Main Results

In this section, by utilizing a novel augmented Lyapunov functional and the most improved techniques [5-6,9], we provide the following theorem. For simplicity, let $\mu_{12} = \mu_2 - \mu_1$ and $h_{12} = h_2 - h_1$.

Theorem 1 For given scalars h_1, h_2, μ_1, μ_2, system (1) is absolutely stable if there exist $n \times n$ positive definite matrices $\boldsymbol{P}, \boldsymbol{Q}_i, \boldsymbol{T}_i\ (i = 1,2), \boldsymbol{R}_j\ (j = 1,2,3)$, scalar $\delta > 0$ and any matrices $\boldsymbol{U}, \boldsymbol{S}, \boldsymbol{N}_1, \boldsymbol{N}_2$ such that

$$\begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{U} \\ * & \boldsymbol{R}_2 \end{bmatrix} \geqslant 0,\ \begin{bmatrix} \boldsymbol{R}_3 & \boldsymbol{S} \\ * & \boldsymbol{R}_3 \end{bmatrix} \geqslant 0 \tag{7}$$

$$\begin{bmatrix} \boldsymbol{\Phi}_{11} + \delta\boldsymbol{C}^{\mathrm{T}}(\bar{\boldsymbol{\Delta}})^2 C & \boldsymbol{R}_1 & \boldsymbol{U}^{\mathrm{T}} & \boldsymbol{\Phi}_{14} & \boldsymbol{\Phi}_{15} & \boldsymbol{N}_1 \\ * & \boldsymbol{\Phi}_{22} & \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{R}_3 - \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{\Phi}_{33} & \boldsymbol{\Phi}_{34} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & \boldsymbol{\Phi}_{44} + \mu_{12}\boldsymbol{T}_j & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{E} & \boldsymbol{0} \\ * & * & * & * & \boldsymbol{\Phi}_{55} & \boldsymbol{N}_2 \\ * & * & * & * & * & -\delta\boldsymbol{I} \end{bmatrix} < 0$$
$$j = 1,2 \tag{8}$$

where

$$\boldsymbol{\Phi}_{11} = \boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{A} + \sum_{i=1}^{2}(\boldsymbol{Q}_i - \boldsymbol{R}_i) + \boldsymbol{N}_1\boldsymbol{\Delta}\boldsymbol{C} + \boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Delta}\boldsymbol{N}_1^{\mathrm{T}}$$
$$\boldsymbol{\Phi}_{14} = \boldsymbol{P}\boldsymbol{B} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{B} - \boldsymbol{U}^{\mathrm{T}} + \boldsymbol{R}_2$$
$$\boldsymbol{\Phi}_{15} = \boldsymbol{P}\boldsymbol{E} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{E} - \boldsymbol{N}_1 + \boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Delta}\boldsymbol{N}_2^{\mathrm{T}}$$
$$\boldsymbol{\Phi}_{22} = \boldsymbol{T}_1 - \boldsymbol{Q}_1 - \boldsymbol{R}_1 - \boldsymbol{R}_3$$
$$\boldsymbol{\Phi}_{33} = -\boldsymbol{Q}_2 - \boldsymbol{T}_2 - \sum_{i=2}^{3}\boldsymbol{R}_i$$
$$\boldsymbol{\Phi}_{34} = \sum_{i=2}^{3}\boldsymbol{R}_i - \boldsymbol{U} - \boldsymbol{S}$$
$$\boldsymbol{\Phi}_{44} = -(1-\mu_1)\boldsymbol{T}_1 + (1-\mu_2)\boldsymbol{T}_2 + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{B} - 2\sum_{i=2}^{3}\boldsymbol{R}_i + \boldsymbol{U} + \boldsymbol{U}^{\mathrm{T}} + \boldsymbol{S} + \boldsymbol{S}^{\mathrm{T}}$$
$$\boldsymbol{\Phi}_{55} = \boldsymbol{E}^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{E} - \boldsymbol{N}_2 - \boldsymbol{N}_2^{\mathrm{T}}$$

in which $\boldsymbol{Y} = h_1^2\boldsymbol{R}_1 + h_2^2\boldsymbol{R}_2 + h_{12}^2\boldsymbol{R}_3$.

Proof Choose a Lyapunov functional candidate as follows:

$$V(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t) + \int_{t-h_1}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{x}(s)\mathrm{d}s + \int_{t-h_2}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_2\boldsymbol{x}(s)\mathrm{d}s + \int_{t-h(t)}^{t-h_1}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{T}_1\boldsymbol{x}(s)\mathrm{d}s +$$
$$\int_{t-h_2}^{t-h(t)}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{T}_2\boldsymbol{x}(s)\mathrm{d}s + \int_{-h_1}^{0}\int_{t+\theta}^{t}h_1\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta +$$
$$\int_{-h_2}^{0}\int_{t+\theta}^{t}h_2\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t}h_{12}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \tag{9}$$

where $\boldsymbol{x}_t = \boldsymbol{x}(t+\theta)$, $\theta \in [-h_2, 0]$. Then, taking the differential of $V(\boldsymbol{x}_t)$ yields

$$\begin{aligned}\dot{V}(\boldsymbol{x}_t) = {} & 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t) + \boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{Q}_1 + \boldsymbol{Q}_2)\boldsymbol{x}(t) - \\ & \boldsymbol{x}^{\mathrm{T}}(t-h_1)(\boldsymbol{Q}_1 - \boldsymbol{T}_1)\boldsymbol{x}(t-h_1) - \\ & \boldsymbol{x}^{\mathrm{T}}(t-h_2)(\boldsymbol{Q}_2 + \boldsymbol{T}_2)\boldsymbol{x}(t-h_2) - \\ & (1-\dot{h}(t))\boldsymbol{x}^{\mathrm{T}}(t-h(t))\boldsymbol{T}_1\boldsymbol{x}(t-h(t)) + \\ & (1-\dot{h}(t))\boldsymbol{x}^{\mathrm{T}}(t-h(t))\boldsymbol{T}_2\boldsymbol{x}(t-h(t)) + \\ & \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Y}\dot{\boldsymbol{x}}(t) - \int_{t-h_1}^{t}h_1\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s - \\ & \int_{t-h_2}^{t}h_2\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s - \int_{t-h_2}^{t-h_1}h_{12}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s\end{aligned} \tag{10}$$

Furthermore, letting $\alpha_1 = (h_2 - h(t))/h_2$, $\beta_1 = h(t)/h_2$ and according to the Jensen inequality lemma [7] and Lemma 1 we have

$$-\int_{t-h_1}^{t}h_1\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s \leqslant -[\boldsymbol{x}(t) - \boldsymbol{x}(t-h_1)]^{\mathrm{T}}\boldsymbol{R}_1[\boldsymbol{x}(t) - \boldsymbol{x}(t-h_1)] \tag{11}$$

$$\begin{aligned} & -\int_{t-h_2}^{t}h_2\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s \leqslant -\frac{1}{\alpha_1}[\boldsymbol{x}(t-h(t)) - \\ & \boldsymbol{x}(t-h_2)]^{\mathrm{T}}\boldsymbol{R}_2[\boldsymbol{x}(t-h(t)) - \boldsymbol{x}(t-h_2)] - \\ & \frac{1}{\beta_1}[\boldsymbol{x}(t) - \boldsymbol{x}(t-h(t))]^{\mathrm{T}}\boldsymbol{R}_2[\boldsymbol{x}(t) - \boldsymbol{x}(t-h(t))] \leqslant \\ & -\begin{bmatrix} \boldsymbol{x}(t-h(t)) - \boldsymbol{x}(t-h_2) \\ \boldsymbol{x}(t) - \boldsymbol{x}(t-h(t)) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \boldsymbol{R}_2 & \boldsymbol{U} \\ \boldsymbol{U}^{\mathrm{T}} & \boldsymbol{R}_2 \end{bmatrix} \times \\ & \begin{bmatrix} \boldsymbol{x}(t-h(t)) - \boldsymbol{x}(t-h_2) \\ \boldsymbol{x}(t) - \boldsymbol{x}(t-h(t)) \end{bmatrix}\end{aligned} \tag{12}$$

Similarly,

$$\begin{aligned} & -\int_{t-h_2}^{t-h_1}h_{12}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s \leqslant \\ & -\begin{bmatrix} \boldsymbol{x}(t-h(t)) - \boldsymbol{x}(t-h_2) \\ \boldsymbol{x}(t-h_1) - \boldsymbol{x}(t-h(t)) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \boldsymbol{R}_3 & \boldsymbol{S} \\ \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{R}_3 \end{bmatrix} \times \\ & \begin{bmatrix} \boldsymbol{x}(t-h(t)) - \boldsymbol{x}(t-h_2) \\ \boldsymbol{x}(t-h_1) - \boldsymbol{x}(t-h(t)) \end{bmatrix}\end{aligned} \tag{13}$$

On the other hand, using the uncertainty of $\boldsymbol{f}(z(t))$ in Eq. (5), the following equation is true:

$$\begin{aligned} 0 = {} & 2[\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{x}(t) + \boldsymbol{N}_2^{\mathrm{T}}\boldsymbol{f}(z(t))]^{\mathrm{T}}[-\boldsymbol{f}(z(t)) + \\ & (\boldsymbol{\Delta} + \boldsymbol{\Sigma}\bar{\boldsymbol{\Delta}})\boldsymbol{C}\boldsymbol{x}(t)]\end{aligned} \tag{14}$$

Then, let $\boldsymbol{\chi}(t) = \mathrm{col}\{\boldsymbol{x}(t), \boldsymbol{x}(t-h_1), \boldsymbol{x}(t-h_2), \boldsymbol{x}(t-h(t)), \boldsymbol{f}(z(t))\}$. Therefore, combining (10) to (14), along the trajectories of system (1), it is straightforward to show that

$$\dot{V}(\boldsymbol{x}_t) \leqslant \boldsymbol{\chi}^{\mathrm{T}}(t)[\boldsymbol{\Phi} + (\dot{h}(t) - \mu_1)\boldsymbol{\Xi}\boldsymbol{T}_1\boldsymbol{\Xi}^{\mathrm{T}} + (\mu_2 - \dot{h}(t))\boldsymbol{\Xi}\boldsymbol{T}_2\boldsymbol{\Xi}^{\mathrm{T}} + \hat{\boldsymbol{M}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{N}}^{\mathrm{T}} + \hat{\boldsymbol{N}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{M}}^{\mathrm{T}}]\boldsymbol{\chi}(t) \equiv \boldsymbol{\chi}^{\mathrm{T}}(t)\boldsymbol{\Gamma}\boldsymbol{\chi}(t) \tag{15}$$

where

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{R}_1 & \boldsymbol{U}^{\mathrm{T}} & \boldsymbol{\Phi}_{14} & \boldsymbol{\Phi}_{15} \\ * & \boldsymbol{\Phi}_{22} & \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{R}_3 - \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{0} \\ * & * & \boldsymbol{\Phi}_{33} & \boldsymbol{\Phi}_{34} & \boldsymbol{0} \\ * & * & * & \boldsymbol{\Phi}_{44} & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{YE} \\ * & * & * & * & \boldsymbol{E}^{\mathrm{T}}\boldsymbol{YE} \end{bmatrix}$$

$$\hat{\boldsymbol{M}} = \begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{N}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \quad \hat{\boldsymbol{N}} = \begin{bmatrix} \bar{\boldsymbol{\Delta}}\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{\Xi} = [\boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{I} \quad \boldsymbol{0}]^{\mathrm{T}}, \ \hat{\boldsymbol{\Sigma}} = \mathrm{diag}\{\boldsymbol{\Sigma}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}\}$$

It can be seen from (15) that $\boldsymbol{\Gamma} < 0$ is the sufficient condition to ensure that $\dot{V}(\boldsymbol{x}_t) < 0$ for $\boldsymbol{\chi}(t) \neq 0$, which implies the asymptotic stability of system (1). Note that $\hat{\boldsymbol{\Sigma}}$ satisfies (6), therefore combining the convex combination technique[5,10], we can deduce that $\boldsymbol{\Gamma} < 0$ is equivalent to the following inequality for $\delta > 0$:

$$\boldsymbol{\Gamma}^{(i)} = \boldsymbol{\Phi} + \boldsymbol{\Xi}(\mu_{12}\boldsymbol{T}_i)\boldsymbol{\Xi}^{\mathrm{T}} + \delta^{-1}\boldsymbol{M}\boldsymbol{M}^{\mathrm{T}} + \delta\boldsymbol{N}\boldsymbol{N}^{\mathrm{T}} < 0 \tag{16}$$

where

$$\boldsymbol{M} = [\boldsymbol{N}_1^{\mathrm{T}} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{N}_2^{\mathrm{T}}]^{\mathrm{T}}$$
$$\boldsymbol{N} = [\bar{\boldsymbol{\Delta}}\boldsymbol{C} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0}]^{\mathrm{T}}$$

Finally, by Schur's complement, $\boldsymbol{\Gamma}^{(i)} < 0$ can be guaranteed by (8), and it completes the proof.

Remark 1 Theorem 1 can be applied to the case that takes into account the lower and upper bounds of delay and its derivative. On the other hand, if we denote $\boldsymbol{T}_1 = 0$ (respectively, $\boldsymbol{T}_2 = 0$) in $V(\boldsymbol{x}_t)$, our results can be true when only μ_1 (respectively, μ_2) is available. Furthermore, when $\mu_i (i = 1, 2)$ are unknown, or $h(t)$ is not differentiable, Theorem 1 still holds by denoting $\boldsymbol{T}_i = 0 (i = 1, 2)$. Moreover, the number of matrix variables in Theorem 1 is much smaller than present ones, which can save much computation cost.

3 Numerical Example

Example 1 When $h(t) = h$, consider system (1) with

$$\boldsymbol{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \ \boldsymbol{E} = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}$$
$$\boldsymbol{C} = [0.6 \quad 0.8], \ a_1 = 0, \ b_1 = 0.5$$

where $\mu_i (i = 1, 2)$ are unknown. Then by utilizing Theorem 1 and Remark 1, the maximum upper bounds on delay (MUBDs) h_2 are shown in Tab. 1. It is easy to see that our results provide less conservatism than the ones in Refs. [1, 3].

Example 2 Consider the Lurie system (1)-(4) with the following parameters:

$$\boldsymbol{A} = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}, \ \boldsymbol{E} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$$
$$\boldsymbol{C} = [0.6 \quad 0.8], a_1 = 0.2, b_1 = 0.5$$

By applying Theorem 1, we can derive the MUBDs h_2 with given lower bound h_1 and $\mu_2 = 0.9$, which are listed in Tab. 2. It can be seen that the available lower bound of the delay derivative plays an important role in reducing the conservatism.

Tab. 1 MUBDs for various h_1 in Example 1

Methods	h_1				
	0	0.2	0.5	0.8	1
Ref. [1]	2.449 8				
Ref. [3]	1.211 3	1.246 0	1.313 1	1.403 9	1.480 5
Our results	4.845 1	4.845 1	4.845 1	4.845 1	4.845 1

Tab. 2 MUBDs for $\mu_2 = 0.9$ in Example 2

h_1	μ_1					Unknown
	0.8	0.7	0.6	0.5	0	μ_1
0.2	1.451 5	1.433 8	1.429 9	1.429 9	1.429 9	1.391 7
0.5	1.592 9	1.535 9	1.514 5	1.510 4	1.510 4	1.412 8
1	2.071 8	1.952 5	1.839 3	1.726 4	1.544 6	1.444 6

4 Conclusion

In this paper, by constructing a novel augmented Lyapunov functional which contains much more information of the delay than previous ones, a new stability criterion with significantly reduced conservatism is derived in terms of LMIs. Two numerical examples are given to demonstrate the effectiveness of the presented criterion and their improvements over the existent methods. Finally, it should be worth noting that the idea and the method presented in this paper are widely applicable in many cases.

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区间变时滞 Lurie 系统的绝对稳定新准则

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摘要:研究了一类具有区间变时滞的 Lurie 系统时滞相关绝对稳定性问题. 利用增广 Lyapunov 泛函,结合自由权矩阵方法和反凸组合技术,提出了一种新的基于线性矩阵不等式的稳定性条件. 通过保留 Lyapunov 泛函导数中常被忽略的有用信息,使得到的稳定性充分条件不仅依赖于时滞的上下界和时滞导函数的上下界,还依赖于它们的差,与已有文献的结果相比,具有更广泛的应用背景. 此外,在处理非线性函数的扇形域时,给出了非线性函数的一种新的等式描述,和已有文献中的凸描述相比,要验证的线性矩阵不等式较少,因此具有更低的保守性. 仿真实例表明了所提方法的有效性.

关键词: Lurie 系统;反凸组合技术;绝对稳定;区间变时滞;线性矩阵不等式(LMI)
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