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Fast alternating direction method of multipliers for total-variation-based image restoration

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Abstract: A novel algorithm, i. e. the fast alternating direction method of multipliers (ADMM), is applied to solve the classical total-variation (TV)-based model for image reconstruction. First, the TV-based model is reformulated as a linear equality constrained problem where the objective function is separable. Then, by introducing the augmented Lagrangian function, the two variables are alternatively minimized by the Gauss-Seidel idea. Finally, the dual variable is updated. Because the approach makes full use of the special structure of the problem and decomposes the original problem into several low-dimensional sub-problems, the per iteration computational complexity of the approach is dominated by two fast Fourier transforms. Elementary experimental results indicate that the proposed approach is more stable and efficient compared with some state-of-the-art algorithms.

Key words: total variation; deconvolution; alternating direction method of multiplier

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Image restoration and reconstruction from blurry and noisy observation is known to be ill-posed. As we know that recovering original image from observation by directly solving the normal equation is unstable and produces useless results whenever noise exists. Since the condition number of related equation is quite large, the solution is highly sensitive to the noise level. To stabilize the recovery, one must utilize some prior information. Namely, we should add a regularization term to the data fidelity term.

Traditional regularization techniques include the Tikhonov regularization[1], the total variation (TV) regularization[2] and the Mumford-Shah regularization[3-4], etc. The Tikhonov regularization based model is relatively easy to solve; however, it tends to make images overly smoothed and often fails to adequately preserve sharp edges. In contrast, the TV-based model can successfully preserve image attributes[2, 5-7].

The key difficulty in the TV-based model is the presence of the nonsmooth TV term in the objective. By applying smoothing technique and using the classical gradient method, the solution can be achieved. However, the speed is very slow. Another well-known approach is the iterative shrinkage/thresholding (IST) algorithm [8-10], which applies the linearized gradient method to the TV model [11-12] by solving a series of denoising problems. In order to improve the speed, many efforts have been made, such as the two-step IST (TwIST) [5] and the fast iterative shrinkage-thresholding algorithm (FISTA) [13]. Although the FISTA performs better than IST and TwIST, it preserves the optimal convergence rate in theory. We note that all these approaches do not make full use of the special structure of the problem.

Recently, a fast TV deconvolution algorithm called FTVd has been proposed[14]. It first transformed the TV-based model into an equivalent constrained problem with equality constraints, then it applied the classical quadratic penalty method to solve the TV-based model. Experimental results show that the FTVd converges much faster than the algorithms mentioned before[14]. From optimization theory, the quadratic penalty approach well approximates the original TV model only when the penalty parameter becomes large, which results in numerical difficulties.

However, the classical augmented Lagrangian method (ALM) [15] can be applied to the TV-based model by avoiding the penalty parameter going to infinity. The disadvantage of the ALM is that the exact minimization of the augmented Lagrangian function with respect to two block variables is always expensive. In this paper, we propose the use of the alternating direction method of multiplier (ADMM)[16-19] for solving the problem, which can be referred as a splitting form of the ALM. Instead of obtaining an exact minimization with respect to two block variables at each iteration, the ADMM minimizes with respect to these variables separately in one round, then it updates the multiplier as the ALM. This approach also makes full use of the special structure of the problem and avoids the numerical unstable performance caused by the quadratic penalty approach.

1 Problem Description and Model

1.1 Problem description

Without loss of generality, we assume that the underlying image is in grayscale and has a square domain. Let $\bar{\boldsymbol{x}} \in \mathbf{R}^{n^2}$ be an original $n \times n$ image, $\boldsymbol{K} \in \mathbf{R}^{n^2 \times n^2}$ be a blurring (or convolution) operator, $\boldsymbol{\omega} \in \mathbf{R}^{n^2}$ be an additive noise, and $\boldsymbol{f} \in \mathbf{R}^{n^2}$ be an observation which satisfies the relationship $\boldsymbol{f} = \boldsymbol{K}\bar{\boldsymbol{x}} + \boldsymbol{\omega}$. Our objective is to recover $\bar{\boldsymbol{x}}$ from $\boldsymbol{f}$ with given $\boldsymbol{K}$, which is known as deconvolution or deblurring.

Note that the original image is recovered by solving the following model,

$$\min_{\boldsymbol{x}} \Phi_{\text{reg}}(\boldsymbol{x}) + \mu \Phi_{\text{fid}}(\boldsymbol{x}, \boldsymbol{f}) \tag{1}$$

In the objective function, $\Phi_{\text{reg}}(\boldsymbol{x})$ enforces certain prior constraints, $\Phi_{\text{fid}}(\boldsymbol{x})$ represents the data fidelity term, and μ is a positive parameter to balance the two terms.

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1.2 TV/L^2 model

We consider the image restoration problem from blurry and Gaussian noise with the TV-based model. In model (1), we take the discrete form of the TV as regularization, i.e.,

$$\mathrm{TV}(\boldsymbol{x}) = \sum_{i=1}^{n^2} \|\boldsymbol{D}_i\boldsymbol{x}\|_2 \tag{2}$$

where for each i, $\boldsymbol{D}_i\boldsymbol{x} \in \mathbf{R}^2$ represents the first-order finite difference of $\boldsymbol{x}$ at pixel i in both horizontal and vertical directions, the quantity $\|\boldsymbol{D}_i\boldsymbol{x}\|_2$ is the variation of $\boldsymbol{x}$ at pixel i, and the summation is taken over all pixels, which explains the name of TV. We note that the 2-norm can be replaced by the 1-norm in Eq. (2), which is called anisotropic discretization. In contrast, the TV is isotropic when the 2-norm is used. We emphasize that our approach is applicable for both the isotropic and the anisotropic TV deconvolution problems. For simplicity, we will only treat the isotropic case in detail because the treatment for the anisotropic case is completely analogous.

When the image is corrupted by Gaussian noise, the fidelity term in model (1) is usually taken as $\|\boldsymbol{Kx}-\boldsymbol{f}\|_2^2$. Combining TV regularization with this fidelity, we obtain the widely studied TV/L^2 model,

$$\min_x \sum_{i=1}^{n^2} \|\boldsymbol{D}_i\boldsymbol{x}\|_2 + \frac{\mu}{2}\|\boldsymbol{Kx}-\boldsymbol{f}\|_2^2 \tag{3}$$

2 ADMM for TV/L^2

2.1 A general framework of ADMM

The basic idea of the ADMM was proposed by Gabay and Mercier[16] and it was extensively studied by He et al.[19] in optimization and variational analysis. Consider the following problem,

$$\min_u \theta_1(\boldsymbol{u}) + \theta_2(\boldsymbol{Au}) \tag{4}$$

where $\theta_1(\cdot)$ and $\theta_2(\cdot)$ are convex functionals and $\boldsymbol{A}$ is a continuous linear operator. By introducing an auxiliary variable $\boldsymbol{v}$, problem (4) can be equivalently transformed to

$$\min_{u,v}\{\theta_1(\boldsymbol{u}) + \theta_2(\boldsymbol{v}) : \boldsymbol{Au} = \boldsymbol{v}\} \tag{5}$$

which decouples the difficulties relative to the functionals $\theta_1(\cdot)$ and $\theta_2(\cdot)$ from the possible ill-conditioning effects of the linear operator $\boldsymbol{A}$. The scheme of the ADMM is to apply alternating minimization to the augmented Lagrangian function of (5),

$$L_A(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\lambda}) = \theta_1(\boldsymbol{u}) + \theta_2(\boldsymbol{v}) - \boldsymbol{\lambda}^{\mathrm{T}}(\boldsymbol{Au}-\boldsymbol{v}) + \frac{\beta}{2}\|\boldsymbol{Au}-\boldsymbol{v}\|^2 \tag{6}$$

and thus derives the following iterative scheme

$$\left.\begin{aligned} \boldsymbol{u}^{k+1} &= \arg\min_u L_A(\boldsymbol{u},\boldsymbol{v}^k,\boldsymbol{\lambda}^k) \\ \boldsymbol{v}^{k+1} &= \arg\min_v L_A(\boldsymbol{u}^{k+1},\boldsymbol{v},\boldsymbol{\lambda}^k) \\ \boldsymbol{\lambda}^{k+1} &= \boldsymbol{\lambda}^k - \gamma\beta(\boldsymbol{Au}^{k+1}-\boldsymbol{v}^{k+1}) \end{aligned}\right\} \tag{7}$$

where $\gamma \in \left(0, \frac{\sqrt{5}+1}{2}\right)$ and $\beta > 0$ to guarantee the convergence[18]. In the following, we apply the iterative scheme (7) to the TV/L^2 problem (3).

2.2 Applying ADMM to TV/L^2

In order to apply the ADMM to TV/L^2, we first transform (3) to an equivalent constrained problem as

$$\min_{x,y}\left\{\sum_i \|\boldsymbol{y}_i\| + \frac{\mu}{2}\|\boldsymbol{Kx}-\boldsymbol{f}\|^2 : \boldsymbol{y}_i = \boldsymbol{D}_i\boldsymbol{x},\ i=1,2,\ldots,n^2\right\} \tag{8}$$

where for each i, $\boldsymbol{y}_i \in \mathbf{R}^2$ is an auxiliary vector. For convenience, we let $\boldsymbol{y} = (\boldsymbol{y}^{(1)};\ \boldsymbol{y}^{(2)}) \in \mathbf{R}^{2n^2}$, where $\boldsymbol{y}^{(1)}$ and $\boldsymbol{y}^{(2)}$ are vectors of length n^2 satisfying $((\boldsymbol{y}^{(1)})_i; (\boldsymbol{y}^{(2)})_i) = \boldsymbol{y}_i \in \mathbf{R}^2$ for $i = 1, 2, \ldots, n^2$. Let $\Gamma_A(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda})$ be the augmented Lagrangian function of (8) and it is defined as

$$\Gamma_A(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}) = \sum_i\left(\|\boldsymbol{y}_i\| - \boldsymbol{\lambda}_i^{\mathrm{T}}(\boldsymbol{y}_i - \boldsymbol{D}_i\boldsymbol{x}) + \frac{\beta}{2}\|\boldsymbol{y}_i - \boldsymbol{D}_i\boldsymbol{x}\|^2\right) + \frac{\mu}{2}\|\boldsymbol{Kx}-\boldsymbol{f}\|^2$$

Starting from $\boldsymbol{x} = \boldsymbol{x}^k$ and $\boldsymbol{\lambda} = \boldsymbol{\lambda}^k$, the ADMM is applied to (8) and it yields the following iterative scheme,

$$\begin{aligned} \boldsymbol{y}^{k+1} &= \arg\min_y \Gamma_A(\boldsymbol{x}^k,\boldsymbol{y},\boldsymbol{\lambda}^k) \\ \boldsymbol{x}^{k+1} &= \arg\min_x \Gamma_A(\boldsymbol{x},\boldsymbol{y}^{k+1},\boldsymbol{\lambda}^k) \\ \boldsymbol{\lambda}^{k+1} &= \boldsymbol{\lambda}^k - \gamma\beta(\boldsymbol{y}^{k+1} - \boldsymbol{Dx}^{k+1}) \end{aligned}$$

It is easy to show that the minimization of $\Gamma_A(\boldsymbol{x}^k, \boldsymbol{y}, \boldsymbol{\lambda}^k)$ with respect to $\boldsymbol{y}$ is equivalent to n^2 two-dimensional problems of the form,

$$\min_{\boldsymbol{y}_i\in\mathbf{R}^2} \|\boldsymbol{y}_i\| + \frac{\beta}{2}\left\|\boldsymbol{y}_i - \left(\boldsymbol{D}_i\boldsymbol{x}^k + \frac{1}{\beta}(\boldsymbol{\lambda}^k)_i\right)\right\|^2 \qquad i=1,2,\ldots,n^2 \tag{9}$$

According to Ref. [14], the solution of (9) is explicitly given by the two-dimensional shrinkage,

$$\boldsymbol{y}_i^{k+1} = \max\left\{\left\|\boldsymbol{D}_i\boldsymbol{x}^k + \frac{1}{\beta}(\boldsymbol{\lambda}^k)_i\right\| - \frac{1}{\beta}, 0\right\}\frac{D_i\boldsymbol{x}^k + \frac{1}{\beta}(\boldsymbol{\lambda}^k)_i}{\left\|\boldsymbol{D}_i\boldsymbol{x}^k + \frac{1}{\beta}(\boldsymbol{\lambda}^k)_i\right\|} \qquad i=1,2,\ldots,n^2 \tag{10}$$

where $0\cdot(0/0) = 0$ is assumed. The computational cost of Eq. (10) is linear with respect to the problem size. When the 1-norm is used in the definition of TV, $\boldsymbol{y}_i^{k+1}$ is given by the simpler one-dimensional shrinkage,

$$\boldsymbol{y}_i^{k+1} = \max\left\{\left\|\boldsymbol{D}_i\boldsymbol{x}^k + \frac{1}{\beta}(\boldsymbol{\lambda}^k)_i\right\| - \frac{1}{\beta}, 0\right\}\circ\operatorname{sgn}\left(D_i\boldsymbol{x}^k + \frac{1}{\beta}(\boldsymbol{\lambda}^k)_i\right) \qquad i=1,2,\ldots,n^2$$

where "$\circ$" and "sgn" represent the point-wise product and the signum function, respectively, and all the operations are implemented component-wise. On the other hand, fixing $\boldsymbol{\lambda} = \boldsymbol{\lambda}^k$ and $\boldsymbol{y} = \boldsymbol{y}^{k+1}$ (recall that $\boldsymbol{y}^{k+1}$ is a reordering of $\boldsymbol{y}_i^{k+1}$, $i = 1, 2, \ldots, n^2$), the minimization of Γ_A with respect to $\boldsymbol{x}$ is a least-squares problem and the corresponding normal equations are

$$\left(\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}+\frac{\mu}{\beta}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}\right)\boldsymbol{x}=\boldsymbol{D}^{\mathrm{T}}\left(\boldsymbol{y}^{k+1}-\frac{1}{\beta}\boldsymbol{\lambda}^{k}\right)+\frac{\mu}{\beta}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{f} \quad (11)$$

where $\boldsymbol{D}=(\boldsymbol{D}^{(1)};\boldsymbol{D}^{(2)})\in\mathbf{R}^{2n^2\times n^2}$ is the global first-order finite difference operator. $\boldsymbol{D}^{(1)}$ and $\boldsymbol{D}^{(2)}$, which are n^2-by-n^2 matrices, represent first-order finite difference operators in horizontal and vertical directions, respectively. As used in Eq. (2), $\boldsymbol{D}_i\in\mathbf{R}^{2\times n^2}$ is a two-row matrix formed by stacking the i-th row of $\boldsymbol{D}^{(1)}$ onto that of $\boldsymbol{D}^{(2)}$. We follow the standard assumption of $N(\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K})\cap N(\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D})=0$, where $N(\cdot)$ represents the null space of a matrix, which ensures the nonsingularity of the coefficient matrix in Eq. (11). Under the periodic boundary conditions for x, both $\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}$ and $\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K}$ are block circulant matrices with circulant blocks [20], and thus are diagonalizable by the two-dimentional discrete fast Fourier transforms (FFT). As a result, Eq. (11) can be exactly solved by two FFTs (including one inverse FFT), each at a cost of $O(n^2\log(n))$. Finally, we update $\boldsymbol{\lambda}$ by

$$\boldsymbol{\lambda}^{k+1}=\boldsymbol{\lambda}^{k}-\gamma\beta(\boldsymbol{y}^{k+1}-\boldsymbol{D}\boldsymbol{x}^{k+1}) \quad (12)$$

Under certain reasonable technical assumptions, the convergence of the ADMM framework was established in Refs. [17 - 19].

In the following, we present the framework of the ADMM for solving TV/L^2 problem (3).

Algorithm 1 ADMM

Input $\boldsymbol{f}$, $\boldsymbol{K}$, $\mu>0$, $\beta>0$ and $\boldsymbol{\lambda}^0$.

Initialize $\boldsymbol{x}=\boldsymbol{f}$ and $\boldsymbol{\lambda}=\boldsymbol{\lambda}^0$.

While “not converged”, do

1) Compute $\boldsymbol{y}^{k+1}$ according to (10) for given $(\boldsymbol{x}^k,\boldsymbol{\lambda}^k)$.

2) Compute $\boldsymbol{x}^{k+1}$ via solving (11).

3) Update $\boldsymbol{\lambda}^{k+1}$ via (12).

End do

For simplicity, we terminate Algorithm 1 by relative change in $\boldsymbol{x}$ in all of our experiments, i. e.,

$$\frac{\|\boldsymbol{x}^{k+1}-\boldsymbol{x}^{k}\|}{\max\{\|\boldsymbol{x}^{k}\|,1\}}<\varepsilon \quad (13)$$

where $\varepsilon>0$ is a given tolerance.

3 Numerical Simulation

In this section, we present numerical results to compare the ADMM with the FTVd, which has been mentioned before and shown to be highly efficient for solving the TV-based model. We tested on the image Cameraman (256 × 256). All the codes were written by Matlab 7.12 (R2011a) and were run on a ThinkPad notebook with the Intel Core i5-2140M CPU at 2.3 GHz and 4 GB of memory.

We measured the quality of restoration by the signal-to-noise ratio (SNR), which is measured in decibels (dB) and defined by $\mathrm{SNR}(\boldsymbol{x})=10\log_{10}\frac{\|\bar{\boldsymbol{x}}-\tilde{\boldsymbol{x}}\|^2}{\|\bar{\boldsymbol{x}}-\boldsymbol{x}\|^2}$, where $\bar{\boldsymbol{x}}$ is the original image and $\tilde{\boldsymbol{x}}$ is the mean intensity value of $\bar{\boldsymbol{x}}$. In our experiments, we used $\mu=0.05/\mathrm{std}^2$ as recommended in Ref. [14], where std is the standard deviation of the additive Gaussian noise $\boldsymbol{\omega}$.

Set $\gamma=1.618$, $\beta=10$ in Algorithm 1 and $\varepsilon=3\times10^{-3}$ in (13). The FTVd is stopped by default setting. Both the algorithms start at the blurry and noisy image.

First, let Cameraman go through the average blurry with a kernel size of 15 and Gaussian noise with mean zero and std $=10^{-3}$. The blurry and noisy images and the restored ones are presented in Fig. 1. The SNR values of blurry, recovered by the FTVd and the ADMM are 7.00, 15.47 and 15.52 dB, respectively. The cost time of the FTVd and the ADMM are 1.25 and 0.32 s, respectively. The history of the SNR and the objective function value with respect to the iteration number are also illustrated in Fig. 2. As observed, the ADMM obtains images of comparable quality, the same as the FTVd, with less time. The per iteration computation of both methods is dominated by two FFTs. Therefore, the running time consumed by the two algorithms is proportional to the iteration numbers. From Fig. 2, the ADMM can reach a reasonable SNR in merely several iterations and decrease the objective function rapidly, while the FTVd generally takes more iterations to reach a similar quality solution. Furthermore, the ADMM reaches smaller function values than the FTVd throughout the whole iteration process, see Fig. 2(b).

To verify the robustness of the proposed approach, we test on different levels of average blur under the same Gaussian noise. Let the kernel size change at 3 to 31 with odd values. In Fig. 3, we list the recovered SNR of the two methods and consumed time at different levels of blurry in Figs. 3(a) and (b), respectively.

From the above comparison results, it is safe to conclude that the ADMM is more efficient than the FTVd.

Fig. 1 Blurry and recovered results. (a) Blurry; (b) Recovered result from FTVd; (c) Recovered result from ADMM

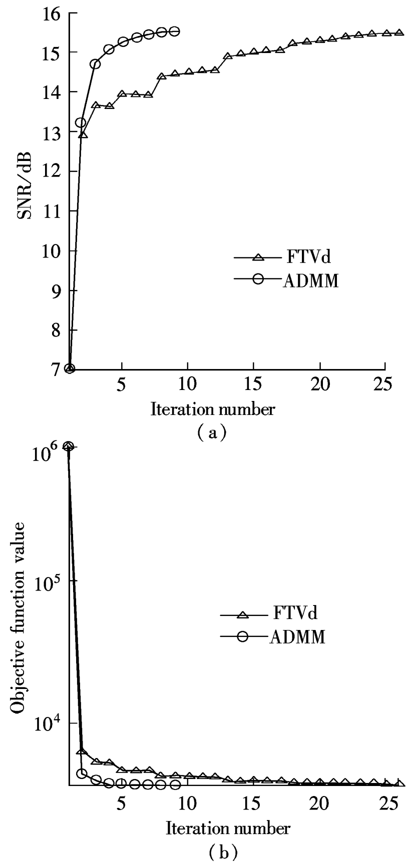

Fig. 2 History of SNR and objective function value. (a) SNR from FTVd and ADMM; (b) Objective function value from FTVd and ADMM

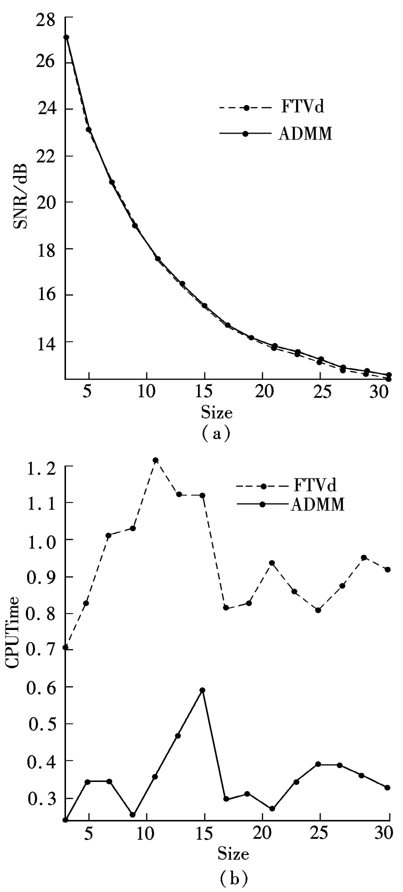

Fig. 3 Comparison results of recovered SNR and cost time with different kernel sizes. (a) Recovered SNR from FTVd and ADMM; (b) Consuming time from FTVd and ADMM

4 Conclusion

Based on the classical augmented Lagrangian function and an alternating minimization framework, we propose the use of the alternating direction method of multiplier (ADMM) for recovering images from blurry and Gaussian noise. Elementary comparison results indicate that the ADMM is more stable and efficient and, in particular, faster compared with the FTVd. And its excellent performance depends on fewer fine tunings of parameters than the FTVd. The fast convergence of the ADMM is due to the advantage of iterative updates of multipliers instead of increasing penalty parameters to a large value.

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快速交替方向乘子法求解基于全变分的图像重建问题

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摘要:采用一种快速的新型算法,即交替方向乘子法求解图像重建的全变分模型. 首先,对全变分模型进行等价变形,使之转化成带有等式约束的可分的凸优化问题. 然后,通过引入增广拉格朗日函数,并采用 Gauss-Seidel 迭代的思想,对问题中 2 块变量交替极小化,最后更新乘子. 因为该方法充分利用了问题的特殊结构,将原问题分解成一系列容易求解的低维子问题, 所以每步的计算工作量主要是由 2 次快速傅立叶变换决定. 初步的数值结果表明所提出的快速方法比一些经典的方法更加稳定、有效.

关键词:全变分;反卷积;交替方向乘子法
中图分类号:O221.2