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Reconstruction of impact force of mechanical press in time domain

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Abstract: To overcome the difficulty in directly measuring the impact force of a mechanical press, the inverse theory is employed to reconstruct the impact force from the corresponding response data in time domain. The nature of ill-posedness of impact force reconstruction is explored through singular value decomposition (SVD) and the Tikhonov regularization is utilized to deal with the ill-posedness, in which the optimal parameter is chosen in light of the L-curve criterion and the generalized cross-validation (GCV). The experimentally measured strain responses of upper and lower dies of the press are chosen as source data for impact force reconstruction, and the corresponding numerical results are compared with the experimental measurements, which verifies the effectiveness of the reconstruction method.

Key words: mechanical press; impact force; reconstruction; inverse problem; regularization

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The mechanical press has the characteristic of "less/non-chip finish" which allows it to produce parts closer to their final shapes to satisfy the requirements of clean and green manufacture. However, violent vibration and noise always follow the pressing process due to the impact force, which will bring about tremendous harm to the precision and lifetime of parts and the operators. Therefore, accurate measurement and proper control of the impact force are essential to the reduction of vibration and noise and the improvement of the performance of the mechanical press.

The conventional approach to measuring the impact force is to insert a force transducer between the colliding bodies. This method requires the transducer to be sufficiently small and soft so that it scarcely influences the colliding. In recent years, the reconstruction of the impact force has been widely studied to overcome the shortcoming of the former. The basic idea is to inversely estimate the impact force from the measured responses at the chosen points of the structure subjected to impact[1]. It is thus clear that the reconstruction of the impact force poses an ill-posed inverse problem, which complicates the solution process.

A number of researches on impact force inverse reconstruction have been performed in theory and application. Chang and Sun[2] estimated the transverse impact force on a composite laminate by solving the least-squares problem via the conjugate gradient method, the iteration number of which, however, was not discussed. By the use of the gradient projection method, Wu et al.[3-4] performed the numerical reconstruction of the impact force on the laminated plate and rod and demonstrated that their method could improve the precision of reconstruction through experiments. And they also claimed that too many iterations of the gradient projection method might yield unstable estimates. Liu et al.[5] proposed the enhanced least-squares and total least-squares methods to identify the dynamic force in frequency domain in which Morozov's discrepancy principle was employed to seek the optimal regularization parameter when both the transfer function matrix and the response data were contaminated by errors.

Besides, Jacquelin et al.[6] analyzed the nature of deconvolution for force reconstruction in terms of inverse problem and concluded that the unsatisfied Picard condition of the transfer matrix is principally responsible for the ill-posedness, which has been solved through SVD in their work. Furthermore, Jang et al.[7-8] employed both Landweber-Fridman and Tikhonov regularization methods for stabilization in reconstructing the impact load, damping and the restoring characteristics of nonlinear systems, and they reported that the stability and robustness of the inverse process mainly depends on the regularization of the ill-posed equation. As the time history (or frequency information) of an impact force cannot be generally obtained from response data with ill-posedness, Hansen made an in-depth study of the inverse problem theoretically[9-11].

As far as we know, no attempts have been reported to address the impact force reconstruction of complex mechanical equipment by inverse estimation. In our present work, the inverse theory is applied to reconstruct the impact force of the mechanical press with a structure of great complexity, and experiments are implemented to confirm the feasibility and validity of the inverse method.

1 Theoretical Formulation of Reconstruction

By virtue of dynamics, the response of a system subjected to an impact force can be considered to be linearly dependent on the impact force when the system is treated to be linearly elastic during the impact process. Thus, the response $e(t)$ at a certain point of the system can be expressed by the linear convolution of the impact force $p(t)$ and the impulse response function $g(t)$ as

$$e(t) = \int_0^t g(t-\tau)p(\tau)\,\mathrm{d}\tau \tag{1}$$

where the response can be the displacement, velocity, acceleration or strain which is taken herein, and it is assumed that $g(t) = p(t) = e(t) = 0$ for $t < 0$. Then, the impact force $p(t)$ can be reconstructed by the measurement of the response data with the knowledge of the impulse response function.

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To solve the continuity equation (1) numerically by computer, a discrete process should be introduced and the discrete convolution equation necessitates the two fundamental principles: 1) It should be a proper approximation of the continuity equation; 2) It should be appropriate for deconvolution solving. This leads to the system of linear algebraic equation:

$$\boldsymbol{Gp} = \boldsymbol{e} \tag{2}$$

where $\boldsymbol{p}$ and $\boldsymbol{e}$ are the vectors composed of the discrete values of impact force $p(t)$ and response data $e(t)$, respectively; the coefficient matrix $\boldsymbol{G}$ is composed of the impulse response function $g(t)$ and has the Toeplitz-circulant format as follows:

$$\boldsymbol{G} = \begin{bmatrix} g(t_0) & g(t_{n-1}) & \cdots & g(t_2) & g(t_1) \\ g(t_1) & g(t_0) & \cdots & g(t_3) & g(t_2) \\ \vdots & \vdots & & \vdots & \vdots \\ g(t_{n-2}) & g(t_{n-3}) & \cdots & g(t_0) & g(t_{n-1}) \\ g(t_{n-1}) & g(t_{n-2}) & \cdots & g(t_1) & g(t_0) \end{bmatrix} \tag{3}$$

For the sake of simplification, the size of $\boldsymbol{G}$ is set to $n \times n$, and the lengths of the vectors $\boldsymbol{p}$ and $\boldsymbol{e}$ are set to n. It is well known that Eq. (2) is an ill-posed problem and the conventional least-squares solution cannot avoid giving worthless results in many situations of reconstruction of the impact force. Therefore, numerical regularization should be applied to stabilize the problem and minimize the error. Singular valued decomposition as a useful numerical tool is used to expose the essence of the ill-posedness of force reconstruction and the Tikhonov regularization method is employed to regularize the problem, in which the regularization parameter is selected in light of the L-curve and GCV criteria.

2 Ill-Posedness and Regularization

2.1 Least-squares solution and ill-posedness analysis

Examining the algebraic equation (2) reveals that the least-squares method is based on minimization of the residual norm $E = \|\boldsymbol{e} - \boldsymbol{Gp}\|^2$; therefore, assuming the first partial derivative of the residual norm to be zero with respect to $\boldsymbol{p}$ yields the normal equation:

$$\boldsymbol{G}^{\mathrm{T}}\boldsymbol{Gp} = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{e} \tag{4}$$

Then, the least-squares solution can be obtained as

$$\boldsymbol{p} = \boldsymbol{G}^{+}\boldsymbol{e} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{e} \tag{5}$$

where $\boldsymbol{G}^{+} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G})^{-1}\boldsymbol{G}^{\mathrm{T}}$ represents the Moore-Penrose pseudo of $\boldsymbol{G}$.

As ordinary least-squares methods tend to result in solutions poorly approximating actual ones as a result of the ill-posedness of Eq. (2), the SVD technique widely used as a serviceable numerical tool for ill-posed problem analysis is also employed to examine the ill-posedness of the reconstruction of the impact force.

We arrange the characteristic values of transfer matrix $\boldsymbol{G}$ in a descending order as $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_n$, and the singular values of $\boldsymbol{G}$ are defined as

$$\sigma_i = +\sqrt{\lambda_i} \qquad i = 1, 2, \ldots, n \tag{6}$$

Then, the SVD of $\boldsymbol{G}$ can be expressed as

$$\boldsymbol{G} = \boldsymbol{U\Sigma V}^{\mathrm{T}} = \sum_{i=1}^{n} \boldsymbol{u}_i \sigma_i \boldsymbol{v}_i^{\mathrm{T}} \tag{7}$$

where $\boldsymbol{U} = [\boldsymbol{u}_1 \quad \boldsymbol{u}_2 \quad \ldots \quad \boldsymbol{u}_n]$ and $\boldsymbol{V} = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \ldots \quad \boldsymbol{v}_n]$ are unitary matrices ($\boldsymbol{U}^{\mathrm{T}} = \boldsymbol{U}^{-1}$ and $\boldsymbol{V}^{\mathrm{T}} = \boldsymbol{V}^{-1}$); $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are the column vectors of $\boldsymbol{U}$ and $\boldsymbol{V}$; and $\boldsymbol{\Sigma}$ is a diagonal matrix possessing the non-negative diagonal elements (the singular values of $\boldsymbol{G}$) in a non-ascending order:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix} \tag{8}$$

Two typical features of the ill-posed problem of impact force reconstruction are commonly found[10]: 1) The singular values σ_i of transfer matrix $\boldsymbol{G}$ decay gradually to zero with no particular gap in the spectrum and an increase in the dimensions of $\boldsymbol{G}$ will increase the number or small singular values; 2) The left and right singular vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ tend to have more sign changes in their elements as the index i increases (or σ_i decreases). In terms of the SVD, the Moore-Penrose pseudo of $\boldsymbol{G}$ can be denoted as $\boldsymbol{G}_{\mathrm{SVD}}^{+} = \boldsymbol{V\Sigma U}^{\mathrm{T}} = \sum_{i=1}^{n} \frac{\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{v}_i}{\sigma_i}$, and the least-squares solution expressed in Eq. (5) can be rewritten as

$$\boldsymbol{p} = \boldsymbol{G}_{\mathrm{SVD}}^{+}\boldsymbol{e} = \sum_{i=1}^{n} \frac{\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{e}}{\sigma_i}\boldsymbol{v}_i \tag{9}$$

which clearly illustrates that if the transfer matrix $\boldsymbol{G}$ has small singular values σ_i and the corresponding coefficients $|\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{e}|$ do not decay to zero as fast as σ_i, the least-squares solution becomes dominated by the small singular values σ_i, indicating that any perturbation in response data $\boldsymbol{e}$ will cause prodigious changes in the reconstructed force. In other words, the slight error or noise in $\boldsymbol{e}$ will be propagated and amplified by the small singular values σ_i, which makes the solution unstable. As a consequence, the solution in Eq. (9) appears nearly random due to many sign changes in the elements of the right-hand singular vectors $\boldsymbol{v}_i$.

2.2 Regularization

Generally speaking, solving Eq. (2) directly by Eq. (9) may produce meaningless results on account of the disperse spectrum of the transfer matrix $\boldsymbol{G}$. Therefore, regularization needs to be adopted to stabilize the problem for a feasible solution. The Tikhonov regularization can yield a stable approximate solution of Eq. (2) by adding a smoothing norm $\|\boldsymbol{\Gamma p}\|_2^2$ and seeking a proper tradeoff between the residual and smoothing norms, thus transforming the problem into a new minimum one:

$$\min J_{\alpha(p)} = \|\boldsymbol{e} - \boldsymbol{Gp}\|_2^2 + \alpha\|\boldsymbol{\Gamma p}\|_2^2 \tag{10}$$

where $\|\cdot\|_2^2$ denotes the Euclidean norm of the matrices; α is the positive regularization parameter; and $\boldsymbol{\Gamma}$ is the Tikhonov matrix which is set as an identity matrix herein. Then we have the normal regularization equation:

$$(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \alpha\boldsymbol{I})\boldsymbol{p} = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{e} \tag{11}$$

And the regularization solution can be obtained as

$$\boldsymbol{p}_{\mathrm{REG}} = \boldsymbol{G}_{\mathrm{REG}}^{+}\boldsymbol{e} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \alpha\boldsymbol{I})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{e} \tag{12}$$

In the regularization process, the parameter α plays a key role in determining the quality of reconstruction results, balancing the norm of the residual $\|\boldsymbol{e} - \boldsymbol{Gp}\|_2^2$ and that of the regularized solution $\|\boldsymbol{p}\|_2^2$. If α is chosen as a small value, the spectrum characteristic of the transfer matrix $\boldsymbol{G}$ will not be ameliorated and ill-posedness may still exist. On the contrary, a large α will achieve a stable solution of the new problem but a poor approximation of the actual one. Hence, the optimal regularization parameter should balance both the norm of the residual and that of the regularized solution for a fair tradeoff between them. The L-curve and GCV criteria are robust methods for choosing the optimal regularization parameter[10], both being employed in the present study.

The L-curve is to present a log-log plot of the norm regularized solution $\|\boldsymbol{p}\|_2^2$ vs. the corresponding residual norm $\|\boldsymbol{e} - \boldsymbol{Gp}\|_2^2$, in which one can easily obtain the compromise between the minimizations of these two quantities. The L-curve indicates that the appropriate regularization parameter α is located at the corner of the curve and can be obtained by calculating the maximum curvature. Besides, visually from the L-curve, one can easily perceive how α affects the characteristics of the regularized solution.

Generalized cross-validation (GCV) is based on the idea that if an arbitrary element e_i of the response $\boldsymbol{e}$ is left out, the corresponding regularized solution should predict the observation well. The GCV function[10] is defined as

$$W(\alpha) = \frac{\|\boldsymbol{Gp}_{\mathrm{REG}} - \boldsymbol{e}\|_2^2}{(\mathrm{trace}(\boldsymbol{I} - \boldsymbol{GG}_{\mathrm{REG}}^{+}))^2} \tag{13}$$

where $\boldsymbol{G}_{\mathrm{REG}}^{+} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \alpha\boldsymbol{I})^{-1}\boldsymbol{G}^{\mathrm{T}}$, $\mathrm{trace}(\boldsymbol{I} - \boldsymbol{GG}_{\mathrm{REG}}^{+}) = \sum_{i=1}^{n}(1 - a_{ii}(\alpha))$ and $a_{ii}(\alpha)$ stand for the diagonal elements of $\boldsymbol{GG}_{\mathrm{REG}}^{+}$. The optimal α can be determined by minimizing the function.

3 Reconstruction of Impact Force

3.1 Model description

Studied herein is the VH16 mechanical press for forging and stamping, as shown in Fig. 1. The primary motion transmission mechanism is a crank-slider structure which converts the rotary motion of the crank shaft driven by the motor into the straight reciprocating motion to generate the impact force for forging.

Obviously, the impact force has a direct effect on the precision of parts and is followed by impact vibration and noise. Therefore, the accurate measurement or estimation of the impact force is the key to improving the performance and precision of the mechanical press. In this section, the inverse theory is applied to the reconstruction of the impact force under the pile driving condition. The schematic diagram of the test setup is presented in Fig. 2.

Fig. 1 Physical model of VH16 mechanical press

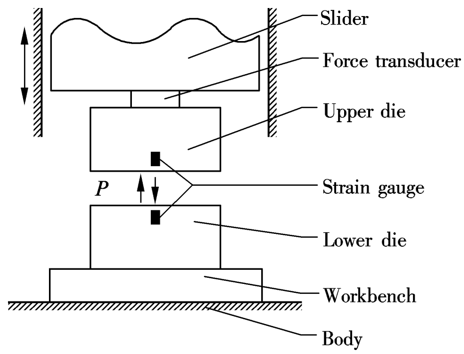

Fig. 2 Schematic diagram of test setup

Two strain gauges adhering to the upper and lower dies are assigned for recording the vertical strain responses. For minimizing the influence of colliding, the piezoelectric force transducer is fixed between the slide and the upper die for direct impact force collection. As shown in Fig. 3, the impact force and strain response data, which are transformed into voltage signals and amplified, are collected by the multi-channel data acquisition unit and analyzed by a PC. The sampling time is set to 0.2 ms for both strain response and impact force.

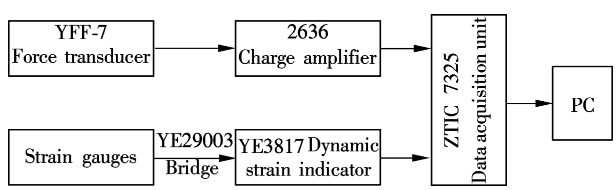

Fig. 3 Schematic diagram of data collecting

3.2 Numerical experiment

In many cases, the failure to satisfy the Picard condition may account for the ill-posedness of the impact force reconstruction in Eq. (2). The Picard condition is met when the coefficients $|\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{e}|$ in Eq. (9) decay to zero faster than the corresponding singular values σ_i in the Picard plot. The strain response of the lower die is chosen for estimating the impact force. And the Picard plot shown in Fig. 4 clearly indicates that the singular values σ_i decay to zero more quickly than $|\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{e}|$. As a result, the small error or noise in strain response data will be propagated and amplified in the time history of the impact force by the small singular values σ_i. Apparently, it is an ill-posed problem that requires regularization.

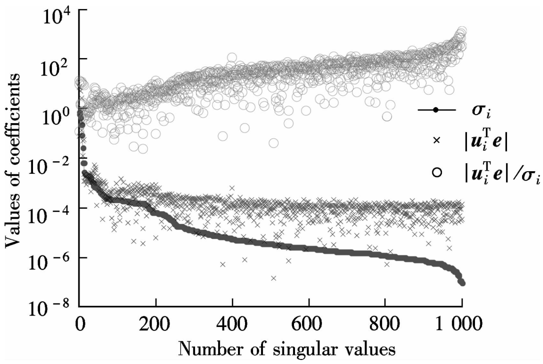

Fig. 4 Picard plot for ill-posedness analysis

To obtain the regularization solution expressed by Eq. (12), the regularization parameter α needs to be determined by adopting the L-curve and GCV criteria. As illustrated in Fig. 5, the L-curve consists of the horizontal and vertical parts separated by a distinct corner, which are sensitive to the residual norm $\| e - Gp \|_2^2$ and the regularized solution norm $\|p\|_2^2$. It is evident that an overestimate of α is likely to generate a stable solution with a regularization error that is too large, while an underestimate of α can hardly improve the spectrum characteristics leaving the instability unsettled. Therefore, by seeking the point with maximum curvature, the curve corner is selected to balance the norms of residual and regularized solutions. The plot of the GCV function is demonstrated in Fig. 6, which is used for determining the optimal regularization parameter α by searching for the minimum value of the curve.

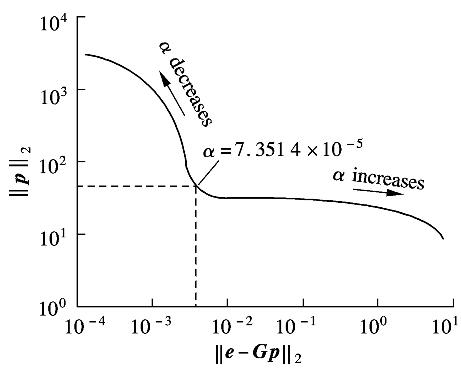

Fig. 5 L-curve for regularization parameter

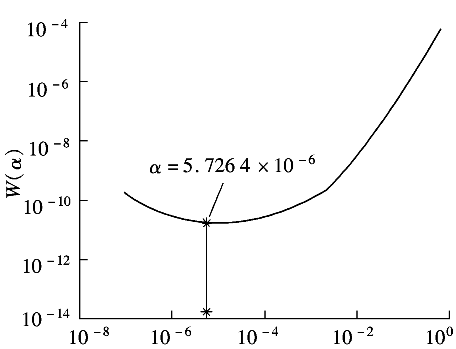

Fig. 6 GCV function for regularization parameter

The optimal regularization parameters chosen for Tikhonov regularization are 7.3514×10^{-5} (L-curve) and 5.7264×10^{-6} (GCV). Using these optimal parameters, stable results reconstructed from the measured strain response are obtained and compared as well with the directly measured impact force, as illustrated in Fig. 7.

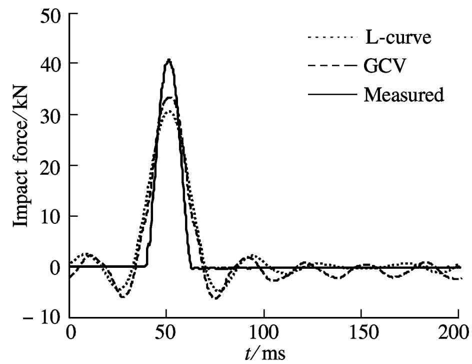

Fig. 7 Results of impact force reconstruction

As indicated in Fig. 7, the reconstructed results basically match with the directly measured force, which confirms the validity of applying the inverse theory to the impact force estimation of the mechanical press. It is also clearly illustrated that the estimated solution using the parameter determined by the L-curve has a smoother appearance but with a larger error in the peak value, while the GCV approach yields a regularization parameter that makes a solution more approximate to the experimentally measured force.

5% random noise, as an inevitable error and noise in experiments and reality, is added to the strain response data to analyze the influence on the reconstruction process. The Picard plot for the problem with noise, given in Fig. 8, displays a remarkable phenomenon that the singular values σ_i have a much faster rate of decay than the coefficients $|u_i^T e|$ with respect to the plot in Fig. 4 without noise. Consequently, the corresponding ratio $|u_i^T e|/\sigma_i$ tends to increase notably, by which the noise in the strain response data will be wildly amplified and propagated to the reconstructed impact force. It is thus clear that the noise in the response data intensifies the ill-posedness of impact force reconstruction.

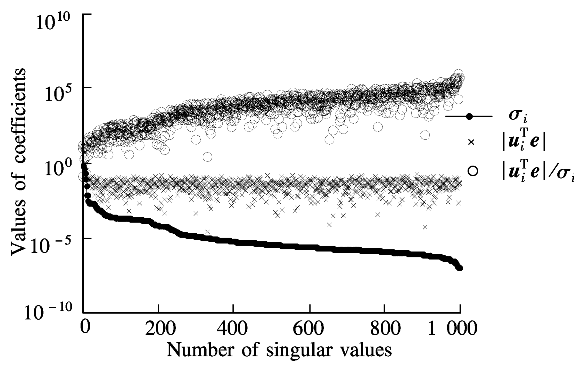

Fig. 8 Picard plot with 5% noise

The optimum regularization parameter is set to 1.7264×10^{-4} via the GCV method in view of the fact that the Tikhonov regularization functions well in deriving a stable solution from the noise response in Fig. 9. It is obvious that noise

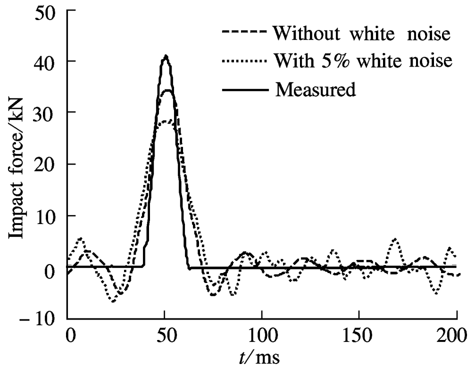

Fig. 9 Results of impact force reconstruction with 5% noise

can bring a more oscillatory feature to the reconstructed result. Under the noise condition, the numerical regularization can still help to obtain a stable solution but with little contribution to the reconstruction precision.

4 Conclusion

The inverse theory is successfully applied in the mechanical press to reconstruct the impact force from the strain response, and the validity of the approach is confirmed by our experiment. To overcome the ill-posedness exposed by the SVD approach, numerical regularization is adopted, whereby the optimal regularization parameter is determined by L-curve criterion and the GCV method. The reconstructed result related to the L-curve appears a bit smoother but with a poorer approximate precision of the peak value of the impact force than that obtained by the GCV method. Furthermore, the influence of contaminated response data on the reconstruction is examined by artificially adding 5% noise to the strain response. Though the proposed method has yielded a stable solution to the problem, it contributes little to the reconstruction precision. And this makes our future research focus on modifying the numerical regularization method for optimal impact force reconstruction with higher precision.

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压力机冲击力的时域重构分析

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摘要:针对压力机冲击力难以直接测量问题,运用反演理论在时域范畴内根据冲击力作用下的响应信号对冲击力进行逆向重构. 通过奇异值分解技术对冲击力重构问题的病态机理进行深入分析,采用 Tikhonov 正则化方法对病态问题进行数值处理以获得稳定解. 其中,最优正则化参数分别运用 L 曲线准则和广义交叉验证(GCV)准则选取. 选取实验测量的压力机上下模的应变响应作为重构信号,并将由此获得的冲击力数值重构结果与实验直接测量结果进行比较,验证了运用反演理论对压力机冲击力进行重构的有效性.

关键词:压力机;冲击力;重构;反问题;正则化

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