

Lagrangian stability of a class of second-order periodic systems with nonlinear damping term

Jiang Shunjun^{1,2} Fang Fang³

(¹Department of Mathematics, Southeast University, Nanjing 211189, China)

(²College of Sciences, Nanjing University of Technology, Nanjing 211816, China)

(³Department of Basic Course, Nanjing Institute of Technology, Nanjing 211167, China)

Abstract: By the iteration of the KAM, the following second-order differential equation $(\Phi_p(x'))' + F(x, x', t) + \omega^p \Phi_p(x') + \alpha |x|^l + e(x, t) = 0$ is studied, where $\Phi_p(s) = |s|^{p-2}s$, $p > 1$, $\alpha > 0$ and $\omega > 0$ are positive constants, and l satisfies $-1 < \omega < p + 2$. Under some assumptions on the parities of $F(x, x', t)$ and $e(x, t)$, by a small twist theorem of reversible mapping, the existence of quasi-periodic solutions and boundedness of all the solutions are obtained.

Key words: reversible system; KAM theorem; boundedness of solutions

doi: 10.3969/j.issn.1003-7985.2012.01.022

The boundedness problem of solutions of the following nonlinear second-order periodic differential

$$x'' + g(x, t) = 0 \quad (1)$$

has been widely investigated by many authors. The first positive result of boundedness of solutions in the superlinear case (i. e., $\frac{g(x, t)}{x} \rightarrow \infty$ as $|x| \rightarrow \infty$) was due to

Morris^[1]. By means of the KAM theorem, Morris proved that every solution of the differential equation (1) is bounded if $g(x, t) = 2x^3 - p(t)$, where $p(t)$ is piecewise continuous and periodic. This result relies on the fact that the nonlinearity $2x^3$ can guarantee the twist condition of the KAM theorem. Later, Dieckerhoff et al.^[2-4] improved the Morris result for a large class of superlinear function $g(x, t)$.

The above differential equation essentially possesses the Hamiltonian structure. It is well known that the Hamiltonian structure and the reversible structure have many similar properties. Especially, they have a similar KAM theorem.

In Ref. [5], Kunze et al. considered the following

equation:

$$x'' + f(x, t)x' + n^2x + \phi(x) + p(x, t) = 0 \quad (2)$$

where $f(x, t)$, $\phi(x)$ are bounded and $f(x, t) = f(x, t + 1)$, $p(x, t) = p(x, t + 1)$. The idea is just the same as that in Ref. [1]. Ref. [6] studied (1) when $f(x, t)$, $\phi(x)$ and $p(x)$ are unbounded. Under some assumptions of $f(x, t)$, $\phi(x)$ and $p(x)$, the differential equation (2) has a reversible structure. By a KAM theorem for reversible mapping, Liu^[6] obtained the boundedness of all the solutions and his proof was based on a small twist theorem in Ref. [7]. Kunze et al.^[5-7] first found that the integrable part of the Pioncaré map is a twist map, and then show that there exist many invariant curves, which guarantees the boundedness of the solutions of Eq. (2).

Motivated by Refs.[1-3], this paper considers the following P-Laplace equation:

$$(\Phi_p(x'))' + F(x, x', t) + \omega^p \Phi_p(x') + \alpha |x|^l + e(x, t) = 0 \quad (3)$$

where $\Phi_p(s) = |s|^{p-2}s$ with $p > 1$, $\alpha > 0$ and $\omega > 0$ are positive constants and l satisfies $-1 < \omega < p + 2$. We want to generalize the result in Ref. [6] to a class of P-Laplace type differential equations of the form (3). When the functions F and e have some parities, the differential system (3) still has a reversible structure.

1 Main Result

Theorem 1 Suppose that e and F are of class C^6 in their arguments and 2π -periodic with respect to t such that

$$\begin{aligned} F(x, -y, -t) &= F(x, y, t), e(x, -t) = e(x, t) \\ F(-x, y, -t) &= -F(x, y, t), e(-x, -t) = -e(x, t) \end{aligned} \quad (4)$$

Moreover, suppose that there exists $\sigma < l$ such that

$$\left\{ \begin{aligned} \left| x^k y^m \frac{\partial^{k+m+s} F(x, y, t)}{\partial x^k \partial y^m \partial t^s} \right| &\leq C |x|^{\sigma+1} \\ \left| x^k \frac{\partial^{k+s} e(x, t)}{\partial x^k \partial t^s} \right| &\leq C |x|^{\sigma+1} \end{aligned} \right\} \quad (5)$$

when $\forall x \neq 0, \forall 0 \leq k, s, m \leq 6$. Then every solution of (3) is bounded.

Received 2011-04-18.

Biography: Jiang Shunjun (1979—), male, doctor, lecturer, jiangshunjun@yahoo.com.cn.

Foundation items: The National Natural Science Foundation of China (No. 11071038), the Natural Science Foundation of Jiangsu Province (No. BK2010420).

Citation: Jiang Shunjun, Fang Fang. Lagrangian stability of a class of second-order periodic systems with nonlinear damping term[J]. Journal of Southeast University (English Edition), 2012, 28(1): 130 – 134. [doi: 10.3969/j.issn.1003-7985.2012.01.022]

2 Proof of Theorem 1

The proof of Theorem 1 mainly consists of two steps. The first one is to find an equivalent system, which has a nearly integrable form of the reversible system. The second one is to show that the Poincaré map of the equivalent system satisfies the small twist theorem for reversible mapping.

2.1 Action-angle variables and coordination transformation

The symmetric properties (4) imply a reversible structure of the system (3).

Let $y = \Phi_p(x') = |x'|^{p-2}x'$. Then $x' = \Phi_q(y)$, where q satisfies $\frac{1}{p} + \frac{1}{q} = 1$. Hence the differential equation (3) is changed into the following planar system,

$$\left. \begin{aligned} x' &= \Phi_q(y) \\ y' &= -\omega^p \Phi_p(x) - \alpha |x|^l x - e(x, t) - F(x, \Phi_q(y), t) \end{aligned} \right\} \quad (6)$$

By (4), it is easy to see that the system (6) is reversible with respect to the involution $G: (x, y) \rightarrow (x, -y)$.

It is shown that the reversible system (6) is a form of small perturbation. For this purpose, the action-angle variables will be introduced by the homogeneous differential equation:

$$(\Phi_p(u'))' + \Phi_p(u) = 0$$

This equation is an integrable part of (3). One of the solutions for the above equation is the function \sin_p as defined below. Let the number π_p be defined by

$$\pi_p = 2 \int_0^{(p-1)^{1/p}} \frac{ds}{[1 - s^p/(p-1)]^{1/p}}$$

Define the function $\omega(t): [0, \frac{\pi_p}{2}] \rightarrow [0, (p-1)^{1/p}]$, implicitly by

$$\int_0^{\omega(t)} \frac{ds}{[1 - s^p/(p-1)]^{1/p}} = t$$

The function $\omega(t)$ will be extended to the whole real axis R as explained below, and the extension will be denoted by \sin_p . Define \sin_p on $[\frac{\pi_p}{2}, \pi_p]$ by $\sin_p(t) = \omega(\pi_p - t)$. Then define \sin_p on $[-\pi_p, 0]$ such that \sin_p is an odd function. Finally, we extend \sin_p to R by $2\pi_p$ -periodicity.

It is easy to verify that $\sin_p(\omega\theta)$ satisfies

$$(\Phi_p(u'))' + \omega^p \Phi_p(u) = 0$$

with the initial condition $(x(0), x'(0)) = (0, \omega)$. Define a transformation $\Theta: (x, y) \mapsto (r, \theta)$ by

$$x = r^{2/p} \sin_p \omega \theta, \quad y = r^{2/q} \Phi_p(\sin_p' \omega \theta)$$

It is easy to see that $\frac{\partial(x, y)}{\partial(r, \theta)} = -\frac{2}{q} \omega^p r$. Since the Jacobian matrix of Θ is nonsingular for r , so the transformation Θ is a local homeomorphism at each point (r, θ) of the set $R^+ \left[0, \frac{2\pi}{\omega}\right)$.

Under the transformation Θ , the system (6) is changed to

$$\left. \begin{aligned} r' &= f_1(t, \theta, r) = N_1(t, \theta, r) + P_1(t, \theta, r) \\ \theta' &= 1 + f_2(t, \theta, r) = 1 + N_2(t, \theta, r) + P_2(t, \theta, r) \end{aligned} \right\} \quad (7)$$

where

$$N_1(t, \theta, r) = -\alpha \frac{q}{2} \frac{1}{\omega^{(p-1)}} r^{4/p-1+2/p} \sin_p' \theta | \sin_p' \theta | \sin_p' \theta$$

$$\begin{aligned} P_1(t, \theta, r) &= -\frac{q}{2} \frac{1}{\omega^{(p-1)}} r^{1-2/q} \sin_p' \theta F \cdot \\ &\quad \left(r^{2/p} \sin_p \theta, r^{2/q} \Phi_p(\omega \sin_p' \theta), t \right) - \\ &\quad \frac{q}{2} \frac{1}{\omega^{(p-1)}} r^{1-2/q} \sin_p' \theta e(r^{2/p} \sin_p \theta, t) \end{aligned}$$

$$N_2(t, \theta, r) = \alpha \frac{q}{p} \frac{1}{\omega^p} r^{4/p-2+2/p} | \sin_p' \theta | \sin_p^2 \theta$$

$$\begin{aligned} P_2(t, \theta, r) &= \frac{q}{p} \frac{1}{\omega^p} r^{-2/q} \sin_p \theta F \left(r^{2/p} \sin_p \theta, r^{2/q} \Phi_p(\omega \sin_p' \theta), t \right) + \\ &\quad \frac{q}{p} \frac{1}{\omega^p} r^{-2/q} \sin_p \theta e(r^{2/p} \sin_p \theta, t) \end{aligned}$$

It is easily verified that $f_1(-t, -\theta, r) = -f_1(t, \theta, r)$ and $f_2(-t, -\theta, r) = f_2(t, \theta, r)$, so the system (7) is reversible with respect to the involution $G: (r, \theta) \rightarrow (r, -\theta)$.

To estimate $f_1(t, \theta, r)$, $f_2(t, \theta, r)$, the following lemma will be introduced.

Lemma 1 Let $F(t, \theta, r) = F(r^{2/p} \sin_p \theta, r^{2/q} \Phi_p(\omega \sin_p' \theta), t)$, $e(t, \theta, r) = F(r^{2/p} \sin_p \theta, t)$. If $F(t, \theta, r)$ and $e(t, \theta, r)$ satisfy (5), then

$$\left\{ \begin{aligned} \left| r^k \frac{\partial^{k+s} F(t, \theta, r)}{\partial r^k \partial t^s} \right| &\leq c r^{2/p(\sigma+1)} \\ \left| r^k \frac{\partial^{k+s} e(t, \theta, r)}{\partial r^k \partial t^s} \right| &\leq c r^{2/p(\sigma+1)} \end{aligned} \right\} \quad (8)$$

for $\forall \theta \in \mathbf{R}$, $k+s \leq 6$.

Proof The lemma can be proved by direct calculation, so the detail is omitted.

To simply describe the estimates in Lemma 1, the function space $M_m(\Psi)$ can be referred to Ref. [6].

Definition 1 Let $n = (n_1, n_2) \in \mathbf{N}^2$. We say $f \in M_n(\Psi)$, if $0 < j \leq n_1$, $0 < s \leq n_2$, there exist $r_0 > 0$ and $c > 0$ such that

$$r^j |D_r^j D_\theta^s f(t, \theta, r)| \leq c \Psi(r), \quad \forall r \geq r_0; \quad \forall (t, \theta) \in S^1 \times S^1$$

By the definition of function space $M_m(\Psi)$, it is easy to obtain that

$$f_1(t, \theta, r) \in M_{(5,5)}(r^{\beta+1}), \quad f_2(t, \theta, r) \in M_{(5,5)}(r^\beta) \quad (9)$$

where $\beta = 2(2 - p + l)/p$.

Since $-1 < l < p - 2$, we obtain $\beta < 0$. So $|f_2| \leq r^\beta \ll 1$ for sufficiently large r . When $r \gg 1$, the system (7) is equivalent to the following system:

$$\left. \begin{aligned} \frac{dr}{d\theta} &= f_1(t, \theta, r) (1 + f_1(t, \theta, r))^{-1} \\ \frac{dt}{d\theta} &= (1 + f_2(t, \theta, r))^{-1} \end{aligned} \right\} \quad (10)$$

It is easily verified that $f_1(-t, -\theta, r) = -f_1(t, \theta, r)$, $f_2(-t, -\theta, r) = f_2(t, \theta, r)$. Hence system (10) is reversible with respect to the involution $G: (r, t) \rightarrow (r, -t)$. The Poincaré mapping which can be a small perturbation of integrable reversible mapping will be proved. For this purpose, (10) is written as a small perturbation of an integrable reversible system. Write the system (10) in the form

$$\left. \begin{aligned} \frac{dr}{d\theta} &= f_1(t, \theta, r) + h_1(t, \theta, r) = \\ &N_1(t, \theta, r) + (P_1(t, \theta, r) + h_1(t, \theta, r)) \\ \frac{dt}{d\theta} &= 1 - f_2(t, \theta, r) + h_2(t, \theta, r) = \\ &1 - N_2(t, \theta, r) + (-P_2(t, \theta, r) + h_2(t, \theta, r)) \end{aligned} \right\} \quad (11)$$

where $h_1(t, \theta, r) = -\frac{f_1 f_2}{1 + f_2}$, $h_2(t, \theta, r) = -\frac{f_2^2}{1 + f_2}$ with $f_1(t, \theta, r)$, $f_2(t, \theta, r)$ defined in (7). It follows that $h_1(-t, -\theta, r) = -h_1(t, \theta, r)$, $h_2(-t, -\theta, r) = h_2(t, \theta, r)$, so (11) is also reversible with respect to the involution $G(r, t) \rightarrow (r, -t)$. By direct computation, it is easy to see that

$$h_1(t, \theta, r) \in M_{(5,5)}(r^{2\beta+1}), \quad h_2(t, \theta, r) \in M_{(5,5)}(r^{2\beta}) \quad (12)$$

Now the system (11) becomes

$$\left. \begin{aligned} \frac{d\lambda}{d\theta} &= N_1(t, \theta, r) + g_1(t, \theta, r) \\ \frac{dt}{d\theta} &= 1 - N_2(t, \theta, r) + g_2(t, \theta, r) \end{aligned} \right\} \quad (13)$$

With $g_1(t, \theta, r) = P_1(t, \theta, r) + h_1(t, \theta, r)$, $g_2(t, \theta, r) = -P_2(t, \theta, r) + h_2(t, \theta, r)$. By (9), we can easily obtain that

$$g_1(t, \theta, r) \in M_{(5,5)}(r^{\beta+1-\tilde{\sigma}}), \quad g_2(t, \theta, r) \in M_{(5,5)}(r^{\beta-\tilde{\sigma}}) \quad (14)$$

where $\tilde{\sigma} = \min \left\{ -\beta, -\frac{2}{p}(\sigma - l) \right\} > 0$ with $\sigma < l < p - 2$, $-1 < l$.

At the present stage of proof, it is not possible to employ a theorem on the existence of invariant curves for the Poincaré map of (3), as this map is not sufficiently close to a small twist map. Hence we need to transform (13) further.

Lemma 2 There exists a transformation of the form

$$t = t, \quad \lambda = r + S(r, \theta)$$

So the system (13) is changed to

$$\left. \begin{aligned} \frac{d\lambda}{d\theta} &= \tilde{g}_1(t, \theta, r) \\ \frac{dt}{d\theta} &= 1 - N_2(t, \theta, r) + \tilde{g}_2(t, \theta, r) \end{aligned} \right\} \quad (15)$$

with $\tilde{g}_1(t, \theta, r) \in M_{(5,5)}(r^{\beta+1-\tilde{\sigma}})$ and $\tilde{g}_2(t, \theta, r) \in M_{(5,5)}(r^{\beta-\tilde{\sigma}})$. Moreover, the system (13) is reversible with respect to the involution $G(\lambda, -t) \rightarrow (\lambda, t)$.

Proof Let

$$S(r, \theta) = \int_0^\theta N_1(t, \theta, r) d\theta = \frac{q}{2} \frac{\alpha}{\omega^{p-1}} \frac{1}{l+2} |\sin_p^{l+2} \theta| r^{\beta+1}$$

Then $S(r, \theta) = S(r, \theta + 2\pi_p)$, $S(r, -\theta) = S(r, \theta)$. It is easy to see that $S(r, \theta) \in M_{(5,5)}(r^{\beta+1})$. Hence the map $(r, \theta) \rightarrow (\lambda, t)$ with $\lambda = r + S(r, \theta)$ is diffeomorphism for $r \gg 1$. Thus, there is a function $L = L(\lambda, \theta)$ such that $r = \lambda + L(\lambda, \theta)$ and $L(\lambda, \theta + 2\pi_p) = L(r, \theta)$, $L(\lambda, -\theta) = L(\lambda, \theta)$ and $L(r, \theta) \in M_{(5,5)}(\lambda^{\beta+1})$.

Under this transformation, the system (13) is changed to (15) with

$$\begin{aligned} \tilde{g}_1(t, \theta, \lambda) &= \tilde{g}_1(t, \theta, \lambda + L) \\ \tilde{g}_2(t, \theta, \lambda) &= N_2(t, \theta, \lambda) - N_2(t, \theta, \lambda + L) + g_2(t, \theta, \lambda + L) \end{aligned}$$

By (14) and direct computation, we have

$$\tilde{g}_1(t, \theta, r) \in M_{(5,5)}(r^{\beta+1-\tilde{\sigma}}), \quad \tilde{g}_2(t, \theta, r) \in M_{(5,5)}(r^{\beta-\tilde{\sigma}})$$

Since $L(\lambda, -\theta) = L(\lambda, \theta)$, the system (15) is reversible in θ with respect to involution $G(\lambda, -t) \rightarrow (\lambda, t)$. Thus, Lemma 2 is proved.

Now we make an average on the nonlinear term $N_2(t, \theta, r)$ in the second equation of (15).

Lemma 3 There exists a transformation of the form

$$\lambda = \lambda, \quad \tau = t + \tilde{S}(\lambda, \theta)$$

So the system (15) is changed to

$$\left. \begin{aligned} \frac{d\lambda}{d\theta} &= H_1(\lambda, \tau, \theta) \\ \frac{dt}{d\theta} &= 1 - [N_2] + H_2(\lambda, \tau, \theta) \end{aligned} \right\} \quad (16)$$

where $[N_2] = \tilde{\alpha} \lambda^\beta$ with $\tilde{\alpha} = \frac{1}{2\pi_p} \frac{q}{p} \frac{\alpha}{\omega^p} \int_0^{2\pi_p/\omega} |\sin_p^l \theta|^{l+2} d\theta$.

$H_1(\lambda, \tau, \theta)$, $H_2(\lambda, \tau, \theta)$ satisfy

$$\left\{ \begin{array}{l} \left| \lambda^k \frac{\partial^{k+s}}{\partial \lambda^k \partial \tau^s} H_1(\lambda, \tau, \theta) \right| \leq C \lambda^{\beta+1-\bar{\sigma}} \\ \left| \lambda^{k+1} \frac{\partial^{k+s}}{\partial \lambda^k \partial \tau^s} H_2(\lambda, \tau, \theta) \right| \leq C \lambda^{\beta+1-\bar{\sigma}} \end{array} \right\} \quad (17)$$

Moreover, the system (16) is reversible with respect to the involution $G: (\lambda, \tau) \rightarrow (\lambda, -\tau)$.

Proof The proof is similar to Lemma 2, so we omit the details.

2.2 Poincaré map and invariant curves

Below a small parameter will be introduced such that the system (6) is written as a form of small perturbation of an integrable system.

Let $[N_2] = \varepsilon \rho$. Since $[N_2] = \bar{\alpha} \lambda^\beta \rightarrow 0$ as $\lambda \rightarrow +\infty$, then $\lambda \rightarrow +\infty \Leftrightarrow \varepsilon \rightarrow 0^+$.

Now, define a transformation by $\lambda = \left(\frac{\varepsilon \rho}{\bar{\alpha}} \right)^{1/\beta}$, $\tau = \tau$, then the system (16) has the form

$$\left\{ \begin{array}{l} \frac{d\rho}{d\theta} = g_1(\rho, \tau, \theta, \varepsilon) \\ \frac{d\tau}{d\theta} = 1 - \varepsilon \rho + g_2(\rho, \tau, \theta, \varepsilon) \end{array} \right\} \quad (18)$$

where

$$\left\{ \begin{array}{l} g_1(\rho, \tau, \theta, \varepsilon) = \varepsilon^{-1} \frac{d[N_2]}{d\lambda} H_1(\lambda(\varepsilon, \rho), \tau, \theta) \\ g_2(\rho, \tau, \theta, \varepsilon) = H_2(\lambda(\varepsilon, \rho), \tau, \theta) \end{array} \right\} \quad (19)$$

Lemma 4 The perturbations g_1 and g_2 satisfy the following estimates:

$$\left| \frac{\partial^{k+s}}{\partial \rho^k \partial \tau^s} g_1 \right| \leq c \varepsilon^{1+\sigma_0}, \quad \left| \frac{\partial^{k+s}}{\partial \rho^k \partial \tau^s} g_2 \right| \leq c \varepsilon^{1+\sigma_0} \quad (20)$$

where $\sigma_0 = -\frac{\bar{\delta}}{\beta} > 0$.

Proof By (17), (19) and $\lambda = \left(\frac{\varepsilon \rho}{\bar{\alpha}} \right)^{1/\beta}$, it follows that

$$|g_1| = \left| \frac{[N]'}{\varepsilon} \tilde{H}_1 \right| \leq c \left| \varepsilon^{-1} \lambda^{\beta+1} \tilde{H}_1 \right| \leq c \varepsilon^{-1} \lambda^{\beta+1} \lambda^{\beta+1-\bar{\sigma}} \leq c \lambda^{2\beta-1} \leq c \varepsilon^{1+\sigma_0}$$

In the same way, $g_2 = |\tilde{H}_2| \leq c \lambda^{\beta-\bar{\sigma}} \leq c \varepsilon^{1+\sigma_0}$.

The estimates (20) for $k+s \geq 1$ easily follow from (17). Thus Lemma 4 is proved.

From Lemma 2, Lemma 3 and (19), we have

$$\begin{aligned} g_1(\rho, -\tau, -\theta, \varepsilon) &= -g_1(\rho, \tau, \theta, \varepsilon) \\ g_2(\rho, -\tau, -\theta, \varepsilon) &= g_2(\rho, \tau, \theta, \varepsilon) \end{aligned}$$

Then the system (18) is reversible in θ with respect to the involution $G: (\rho, \tau) \rightarrow (\rho, -\tau)$.

Denote the Poincaré map of (18) by P , then P is also reversible with the same involution $G: (\rho, \tau) \rightarrow (\rho, -\tau)$ and has the form

$$\left. \begin{array}{l} \tau_1 = \tau + 2\pi_p - 2\varepsilon \pi_p \rho + \bar{g}_1(\rho, \tau, \varepsilon) \\ \rho_2 = \rho + \bar{g}_2(\rho, \tau, \varepsilon) \end{array} \right\} \quad (21)$$

where $\tau \in S^1$ and $\rho \in [1, 2]$. Moreover, $\bar{g}_1(\rho, \tau, \varepsilon)$, $\bar{g}_2(\rho, \tau, \varepsilon)$ satisfy

$$\left| \frac{\partial^{k+s}}{\partial \rho^k \partial \tau^s} \bar{g}_1 \right| \leq c \varepsilon^{1+\sigma_0}, \quad \left| \frac{\partial^{k+s}}{\partial \rho^k \partial \tau^s} \bar{g}_2 \right| \leq c \varepsilon^{1+\sigma_0} \quad (22)$$

To prove Theorem 1, we need a small twist theorem for reversible mapping. We will use the theorem in Ref. [7] to prove the main result.

Case 1 $2\pi_p$ is rational. Let $I = -l_1 = 2\pi_p \rho$, it is easy to see that

$$l_1(\rho, \tau) \in C^6(A), \quad l_1(\rho, \tau) = 2\pi_p \rho < 0, \quad \frac{\partial l_1(\rho, \tau)}{\partial \rho} < 0$$

$$I(\rho, \tau) \in C^6(A), \quad \frac{\partial I}{\partial \rho}(\rho, \tau) > 0, \quad l_2(\rho, \tau) = 0$$

And

$$l_1(\rho, \tau) \frac{\partial I}{\partial \tau}(\rho, \tau) + l_2(\rho, \tau) \frac{\partial I}{\partial \sigma}(\rho, \tau) = 0$$

Since I only depends on ρ and $\frac{\partial I(\rho, \tau)}{\partial \rho} > 0$, all the conditions in Theorem 2 in Ref. [7] hold.

Case 2 $2\pi_p$ is irrational. Since $\int_0^{2\pi_p} \frac{\partial l_1(\rho, \tau)}{\partial \rho} d\tau = -(2\pi_p)^2 < 0$, all the assumptions in Theorem 1 in Ref. [7] hold.

Thus, in both of the two cases, the Poincaré map P always has invariant curves for ε being sufficiently small. Since $\varepsilon \ll 1 \Leftrightarrow \lambda \gg 1$, we know that for any $\lambda \gg 1$, there is an invariant curve of the Poincaré map, which guarantees the boundedness of solutions of the system (7). Hence all the solutions of (3) are bounded. This completes the proof of theorem 1.

References

- [1] Morris G R. A case of boundedness of Littlewood's problem on oscillatory differential equations [J]. *Bulletin of the Australian Mathematical Society*, 1976, **14**(1): 71–93.
- [2] Dieckerhoff R, Zehnder E. Boundedness of solutions via the twist theorem [J]. *Annali della Scuola Normale Superiore di Pisa-Classe di Scienze*, 1987, **14**(4): 79–95.
- [3] Levi M. Quasiperiodic motions in superquadratic time-periodic potential [J]. *Communications in Mathematical Physics*, 1991, **143**(1): 43–83.
- [4] Liu B. Boundedness for solutions of nonlinear Hill's equations with periodic forcing terms via Moser's twist theorem [J]. *Journal of Differential Equations*, 1989,

79(2):304–315.

[5] Kunze M, Kupper T, Liu B. Boundedness and unboundedness of solutions for reversible scillatorsat resonance [J]. *Nonlinearity*, 2001, **14**(5):1105–1122.

[6] Liu B. Quasi-periodic solutions of semilinear Liénard reversible oscillators [J]. *Discrete and Continuous Dynamical Systems*, 2005, **12**(1):137–160.

[7] Liu B, Song J. Invariant curves of reversible mappings with small twist [J]. *Acta Mathematica Sinica*, 2004, **20**(1):15–24.

一类带有非线性阻尼项的二阶周期系统的 Lagrangian 稳定性

江舜君^{1,2} 方 芳³

(¹ 东南大学数学系, 南京 211189)
(² 南京工业大学理学院, 南京 211816)
(³ 南京工程学院基础部, 南京 211167)

摘要: 用 KAM 迭代方法研究了下列二阶微分方程: $(\Phi_p(x'))' + F(x, x', t) + \omega^p \Phi_p(x') + \alpha |x|^l + e(x, t) = 0$, 其中, $\Phi_p(s) = |s|^{p-2}s, p > 1, \alpha > 0, \omega > 0$ 为正常数, l 满足 $-1 < \omega < p + 2$. 当 $F(x, x', t)$ 与 $e(x, t)$ 的导数满足一定条件时, 利用可逆映射的小扭转定理得到拟周期解的存在性与所有解的有界性.

关键词: 可逆系统; KAM 定理; 解的有界性

中图分类号: O193