Journal of Southeast University (English Edition) Vol. 28, No. 2, pp. 195 - 202 June 2012 ISSN 1003—7985

Flexural behaviors of steel reinforced ECC/concrete composite beams

Dong Luoting[1] Pan Jinlong[1] Yuan Fang[1] Leung CKY[2]

([1]Key Laboratory of Concrete and Pre-Stressed Concrete Structures of Ministry of Education, Southeast University, Nanjing 210096, China)

([2]Department of Civil and Environmental Engineering, Hong Kong University of Science and Technology, Hong Kong 999077, China)

Abstract: An engineered cementitious composite (ECC) is introduced to partially substitute concrete in the tension zone of a reinforced concrete beam to form an ECC/reinforced concrete (RC) composite beam, which can increase the ductility and crack resisting ability of the beam. Based on the assumption of the plane remaining plane and the simplified constitutive models of materials, the stress and strain distributions along the depth of the composite beam in different loading stages are comprehensively investigated to obtain calculation methods of the load-carrying capacities for different stages. Also, a simplified formula for the ultimate load carrying capacity is proposed according to the Chinese code for the design of concrete structures. The relationship between the moment and curvature for the composite beam is also proposed together with a simplified calculation method for ductility of the ECC/RC composite beam. Finally, the calculation method is demonstrated with the test results of a composite beam. Comparison results show that the calculation results have good consistency with the test results, proving that the proposed calculation methods are reliable with a certain theoretical significance and reference value.

Key words: engineered cementitious composites (ECC); reinforced concrete; composite beam; flexural properties; load carrying capacity

doi: 10.3969/j.issn.1003 - 7985.2012.02.012

Nowadays, concrete is one of the most widely used construction materials in civil engineering. Cracking and durability problems are the greatest challenges for the application of concrete for civil infrastructures[1]. For the durability problem, corrosion of the steel reinforcement is the most important factor affecting the safety and the service period of reinforced concrete structures. Existing experimental results show that corrosion of the steel reinforcement is a very complex process, and the corrosion rate is mostly related to the invasion speed of the harmful mediums in the environment. For concrete structures under the same conditions and with the same thickness of the protection layer, wider cracks always lead to more severe corrosion of the steel reinforcement. Hence, it is vital to control the width of cracks in real RC structures. In a corrosive environment, the load is often limited to a low level for obtaining reduced crack width, and the stresses in the steel reinforcement also maintain a low level, resulting in waste of materials. On the other hand, concrete is a typical brittle material and it is easy to crack under tension, which is not good for the seismic resistance performance of RC structures[2].

In 1992, Li and Leung[3] proposed a design method to obtain a class of advanced cementitious composites with super-high ductility. Due to its designability, this kind of cementitious composite is also called engineered cementitious composites (ECC). Existing experimental results show that the ECC has super-high toughness and energy absorption ability compared with normal concrete[4]. The ECC and the concrete have similar ranges of tensile (4 to 6 MPa) and compressive strengths (30 to 80 MPa), while they have distinct differences in tensile deformation behaviors. For the conventional concrete, it fails in a brittle manner once its tensile strength is reached. However, for an ECC plate under uniaxial tension, after first cracking, the tensile load capacity continues to increase with strain-hardening behavior accompanied by multiple cracks along the plate. For each individual crack, the crack tends to open steadily up to a certain crack width, and increased loading will result in the formation of an additional crack. With the same cracking mechanism, cracking of the ECC member can reach a saturated state with small crack spacing, which is determined by the stress transfer capacity of the fibers in the matrix. With increased loading, a random single crack localizes and softening behavior follows. Typically, mechanical softening starts at a tensile strain of 3% to 5%, with a crack spacing of 3 to 6 mm and a crack width of about 60 μm[5]. The application of the ECC provides a new way to solve the cracking and durability problems for the conven-

Received 2011-12-27.

Biographies: Dong Luoting (1986—), male, graduate; Pan Jinlong (corresponding author), male, doctor, professor, jinlongp@gmail.com.

Foundation items: The National Natural Science Foundation of China (No. 50808043), the National Basic Research Program of China (973 Program) (No. 2009CB623200), Foundation of Jiangsu Key Laboratory of Construction Materials, Program for Special Talents in Six Fields of Jiangsu Province (No. 2011-JZ-010).

Citation: Dong Luoting, Pan Jinlong, Yuan Fang, et al. Flexural behaviors of steel reinforced ECC/concrete composite beams[J]. Journal of Southeast University (English Edition), 2012, 28(2): 195 - 202. [doi: 10.3969/j.issn.1003 - 7985.2012.02.012]

tional concrete structures[6].

Maalej and Li[7] proposed to substitute concrete with PE-ECC around the tensile steel reinforcement in the tension side of the concrete beam. The test results show that the ultimate load capacity is 10% higher than that of the control concrete beam, and the flexural crack width is much smaller than that in the concrete beam. With the application of the ECC around the tensile steel reinforcement, the crack width can be controlled to 0.05 mm at the normal use limit state, and 0.2 mm at the ultimate limit state. The test results prove that substituting concrete with the ECC around the tensile steel reinforcement can effectively control opening of cracks and avoid or delay corrosion of the steel reinforcement in the structures.

In this paper, calculation methods are proposed to calculate the load carrying capacities of the steel reinforced ECC/concrete composite beam at different stages. Since the mechanical properties of the ECC material are distinctly different from those of the concrete material, the mechanical parameters of the ECC material should be obtained from existing experimental results. With the parameters, the design method for the ECC/concrete composite beam is proposed. Finally, the calculated moment-curvature relationship is compared with the experimental results to verify the validity.

1 Basic Assumptions for ECC/Concrete Composite Beam

Fig. 1 shows the cross section of an ECC/concrete composite beam by substituting concrete with the ECC around the tensile steel reinforcement. For the composite beam, the positions of cracks and the stress state of the ECC are related to the thickness of the ECC layer, the amount of the steel reinforcement, the distribution and the magnitude of external loading and so on. Since the cost of the ECC is more than 4 times that of the normal concrete[8], the thickness of the ECC layer is only controlled around the tensile steel reinforcement and the protection layer. According to the Chinese code for design of concrete structures (GB 50010—2010)[9], it is necessary to provide some assumptions for calculating the load carrying capacity of the ECC/concrete composite beam, and the assumptions can be summarized as follows:

1) The cross section of the composite beam remains plane under external loading;

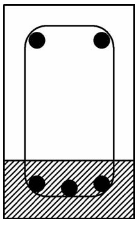

Fig. 1 Cross section of steel reinforced ECC/concrete composite beam

2) There is no relative sliding between the steel reinforcement and the ECC material;

3) The tensile strength of the concrete is ignored. However, the strain hardening behavior of the ECC material is fully considered when the ECC is under tension.

2 Stress-Strain Behaviors of Materials

According to the uniaxial tensile test results of our group[10] and the model proposed by Kanda et al[11], a double line model is proposed for the stress-strain behavior of the ECC (see Fig. 2), which is given by

$$\sigma_{\text{t-ecc}} = \begin{cases} k_1 \varepsilon_{\text{t-ecc}} & 0 \leqslant \varepsilon_{\text{t-ecc}} \leqslant \varepsilon_{\text{tc}} \\ \sigma_{\text{tc}} + k_2(\varepsilon_{\text{t-ecc}} - \varepsilon_{\text{tc}}) & \varepsilon_{\text{tc}} \leqslant \varepsilon_{\text{t-ecc}} \leqslant \varepsilon_{\text{tu}} \end{cases} \tag{1}$$

where $k_1 = \dfrac{\sigma_{\text{tc}}}{\varepsilon_{\text{tc}}}$, $k_2 = \dfrac{\sigma_{\text{tu}} - \sigma_{\text{tc}}}{\varepsilon_{\text{tu}} - \varepsilon_{\text{tc}}}$ define the slopes of the two lines and they can be obtained by fitting the test results; σ_{tc} and ε_{tc} are the initial cracking strength and strain of the ECC; σ_{tu} and ε_{tu} are the ultimate strength and strain of the ECC, respectively.

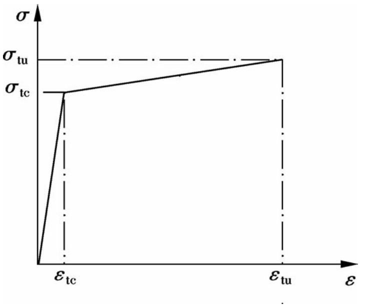

Fig. 2 Simplified stress-strain relationship for ECC material

According to the Chinese code for design of concrete structures[9], an ideal elastoplastic stress-strain relationship is assumed for the steel reinforcement (see Fig. 3) and it can be given by

$$\sigma_{\text{s}} = \begin{cases} E_{\text{s}}\varepsilon_{\text{s}} & 0 \leqslant \varepsilon_{\text{s}} \leqslant \varepsilon_{\text{y}} \\ f_{\text{y}} & \varepsilon_{\text{y}} \leqslant \varepsilon_{\text{s}} \leqslant \varepsilon_{\text{su}} \end{cases} \tag{2}$$

where f_{y}, ε_{y}, ε_{su} are yielding strength, yielding strain and ultimate strain of the steel reinforcement, respectively.

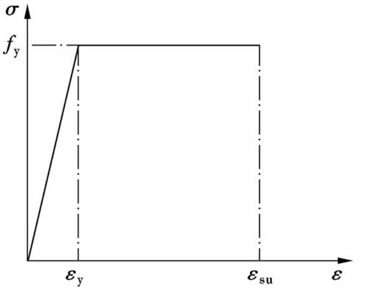

Fig. 3 Stress-strain relationship of steel reinforcement

The stress-strain relationship for the concrete is shown in Fig. 4 and it can be given by

$$\sigma_t = \begin{cases} k_3 \varepsilon_t & 0 \leqslant \varepsilon_t \leqslant \varepsilon_{\text{tu-con}} \\ 0 & \varepsilon_t > \varepsilon_{\text{tu-con}} \end{cases} \tag{3a}$$

$$\sigma_c = \begin{cases} f_c \left[2\dfrac{\varepsilon_c}{\varepsilon_0} - \left(\dfrac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] & 0 \leqslant \varepsilon_c \leqslant \varepsilon_0 \\ f_c & \varepsilon_0 < \varepsilon_c \leqslant \varepsilon_{cu} \end{cases} \tag{3b}$$

where $k_3 = \dfrac{f_t}{\varepsilon_{\text{tu-con}}}$ is the slope of the stress-strain curve of the concrete in uniaxial tension; f_t, $\varepsilon_{\text{tu-con}}$ are the ultimate tensile strength and the strain of the concrete, respectively; f_c, ε_0, ε_c are the maximum compressive strength, initial cracking strain and ultimate strain of the concrete, respectively.

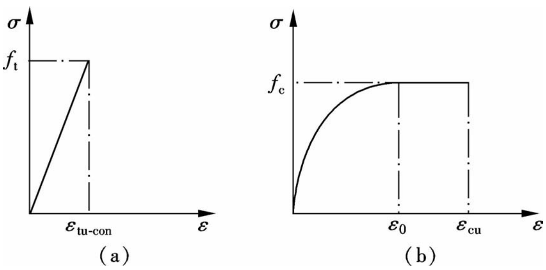

Fig. 4 Stress-strain relationship of concrete. (a) In tension; (b) In compression

3 Failure Characterization of ECC/Concrete Composite Beam in Flexure Conditions

Just as with the conventional reinforced concrete beam, the loading process for the ECC/concrete composite beam can be divided into three stages: the elastic stage, the cracking stage and the ultimate failure stage. In the following, the stress and strain states in each stage will be comprehensively analyzed.

3.1 Elastic stage

When the external moment is small, the deflection is linear with the external load, and the strain is linearly distributed along the cross section. The central axis is located at the centroid of the equivalent cross section. In this stage, the stresses in the concrete, the ECC and the steel reinforcement increase with the increase in external loading. All the materials are in an elastic stage. The concrete shows a triangular stress distribution in the compression zone, and the ECC also shows a linear tensile stress distribution. Since the elastics modulus of the ECC is much smaller than that of the concrete, significant interfacial stresses occur along the concrete-to-ECC interface, and the interfacial bond can be guaranteed by adding shear connections between the ECC and the concrete. At the end of this stage, the fiber strain of the outermost side of the ECC/concrete composite beam reaches the initial cracking strain of the ECC. With the initial cracking strain ε_{tc} of the ECC, the initial cracking moment of the ECC/concrete composite beam can be calculated. Fig. 5 shows the stress and strain distributions of the ECC/concrete composite beam in an elastic stage.

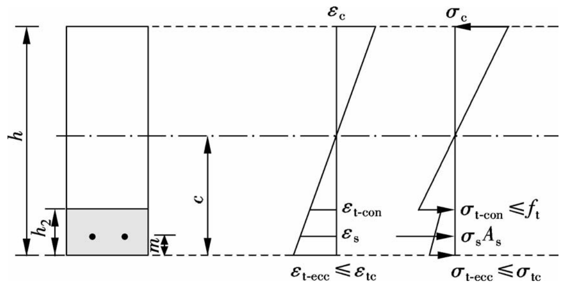

Fig. 5 Stress and strain distributions in elastic stage

3.2 Work stage with cracks

With increased loading, the maximum strain in the ECC is greater than its initial cracking strain, and cracking of the beam occurs, resulting in reduced stiffness of the composite beam. The composite beam starts to enter the second stage, i. e., the work stage with cracks. In this stage, the first crack occurs at the outside of the ECC layer, and more and more tiny cracks occur with small cracks spacing near the first crack due to the bridging effect of fibers in the ECC layer. After cracking, the stress in the ECC increases slowly with the strain. With increased loading, the ECC layer tends to enter the strain hardening stage from the outermost layer to the ECC-to-concrete interface, which is shown in Fig. 6(a).

For the ECC material, the initial cracking strain is greater than that of the normal concrete[12]. When the thickness of the ECC layer is beyond two times that of the protection layer, the concrete in the tension zone starts to crack after the outermost layer of the ECC reaches its initial cracking strain, as shown in Fig. 6(b). With increased loading, cracks in the concrete tend to propagate from the ECC-to-concrete interface to the central axis of the beam. In this stage, the flexural stiffness of the beam decreases with progressive cracking of the concrete and ECC layer, and the central axis of the beam shifts upward. The concrete in the compression zone tends to enter the nonlinear stage (see Figs. 6(c) and (d)).

Since the ultimate strain of the ECC is much greater than the yielding strain of the steel reinforcement[12], the ECC around the steel reinforcement remains in the strain hardening stage at the end of the work stage with cracking, at which the steel reinforcement starts to enter the yielding stage. The stress distributions in the work stage with cracks exhibit the stress state of the composite beam in the serviceability limit stage. The calculations of the yielding moment of the cross section M_y, deflection and crack width are based on this stage.

3.3 Ultimate failure stage

Theoretically, the final failure of the composite beam can be classified into tension failure and compression fail-

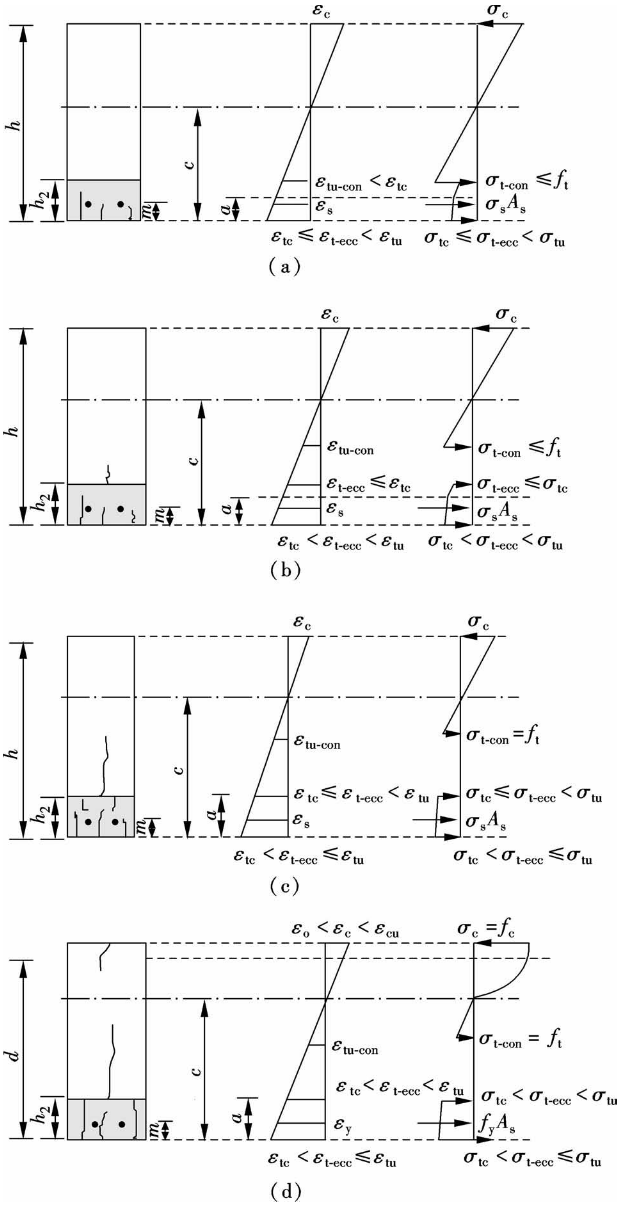

Fig. 6 Stress distributions of composite beam in work stage (a) Cracking of ECC layer; (b) Cracking in both ECC and concrete; (c) Further extension of cracks in both ECC and concrete; (d) Yielding of steel reinforcement

ure. For the tension failure, the steel reinforcement reaches the ultimate strain before the concrete in the compression zone reaches its ultimate compression strain. This failure mode is a typical brittle failure, which should be avoided in designing concrete structures. Meanwhile, the tension failure is also accompanied by significant deformation of the beam, which indicates that the composite beam cannot sustain further loading. In the following section, the compression failure of the ECC/concrete composite beam will be comprehensively analyzed.

With further increased loading, the tensile steel reinforcement enters a yielding stage, and the composite beam starts to get into the third stage, i.e., the ultimate failure stage. In this stage, the strains increase more quickly than the stresses due to the yielding of the tensile steel reinforcement. The ECC around the tensile steel reinforcement is still in a strain hardening stage, and the ECC continues to sustain tensile stresses together with tensile steel reinforcement. Cracks along the composite beam continue to propagate towards the upper side of the beam, accompanied by the degradation of flexural stiffness and the increase in the deformation of the beam. With further increased loading, the height of the compression zone becomes smaller and smaller until the fiber at the edge of the compression zone reaches the ultimate compression strain of the concrete, as shown in Fig. 7. Finally, the ECC/concrete composite beam exhibits concrete crushing failure with a sufficient development of the steel yielding behavior and the ECC strain hardening behavior. Since the ultimate strain of the ECC is much greater than that of the steel reinforcement defined in the Chinese code for design of concrete structures[9], strain softening of the ECC layer will not occur before the rupture of the steel reinforcement. The ultimate load carrying capacity of the ECC/concrete composite beam can be calculated based on the ultimate limited state.

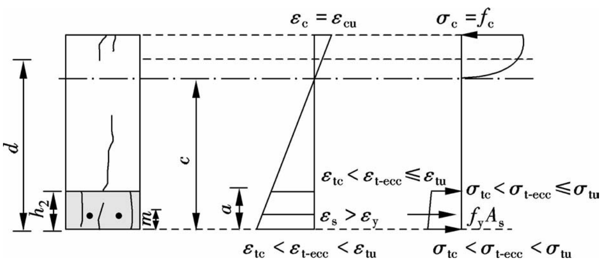

Fig. 7 Stress and strain distributions of ECC/concrete composite beam in ultimate failure stage

4 Calculation Method of Load Carrying Capacity of ECC/Concrete Composite Beam

Based on the above mentioned assumptions and stress-strain behaviors of the materials, the load carrying capacities of the ECC/concrete composite beam in different stages can be derived, including initial cracking moment M_{cr}, yielding moment M_y and ultimate failure moment M_u. Finally, a simplified calculation method of the ultimate failure moment is proposed for the steel reinforced ECC/concrete composite beam.

4.1 Derivation of initial cracking moment of ECC/concrete composite beam

For the steel reinforced ECC/concrete composite beam, the fiber at the bottom of the ECC layer reaches an initial cracking strain ε_{tc} and it defines the moment when the initial cracking moment of the composite beam is reached. The stress distribution for this moment is shown in Fig. 5. Assuming that $\varepsilon_{t\text{-}ecc}$ is the maximum tensile strain of the ECC layer, the strains along the cross section are given by

$$\varepsilon(x)=\begin{cases}\dfrac{c-x}{c}\varepsilon_{\text{t-ecc}} & 0\leqslant x\leqslant c\\ \dfrac{x-c}{c}\varepsilon_{\text{t-ecc}} & c\leqslant x\leqslant h\end{cases}\tag{4}$$

And, the stresses along the cross section are given by

$$\sigma(x)=\begin{cases}k_1\varepsilon(x) & 0\leqslant x\leqslant h_2\\ \dfrac{f_t}{\varepsilon_{\text{tu-con}}}\varepsilon(x) & h_2\leqslant x\leqslant c\\ f_c\left[2\dfrac{\varepsilon(x)}{\varepsilon_0}-\left(\dfrac{\varepsilon(x)}{\varepsilon_0}\right)^2\right] & c\leqslant x\leqslant h\end{cases}\tag{5}$$

For the tensile steel reinforcement, for an arbitrary point with $x=m$, the strain is $\varepsilon_s=\dfrac{c-m}{c}\varepsilon_{\text{t-ecc}}$ and the stress is $\sigma_s=\dfrac{c-m}{c}E_s\varepsilon_{\text{t-ecc}}$.

According to the force equilibrium of the cross section, the following equation can be obtained:

$$\int_0^{h_2}b\sigma(x)\,\mathrm{d}x+\int_{h_2}^{c}b\sigma(x)\,\mathrm{d}x+\sigma_sA_s-\int_c^h b\sigma(x)\,\mathrm{d}x=0\tag{6}$$

Substituting the strain expressions into Eq. (6), we have

$$\begin{aligned}&c\left(-\frac{k_1h_2^2}{2}+\frac{k_3h_2^2}{2}-\frac{E_sA_sm}{b}-\frac{f_ch^2\varepsilon_{\text{t-ecc}}}{\varepsilon_0^2}\right)+\\&c^2\left(k_1h_2-k_3h_2+\frac{E_sA_s}{b}+\frac{2f_ch}{\varepsilon_0}+\frac{f_ch\varepsilon_{\text{t-ecc}}}{\varepsilon_0^2}\right)+\\&c^3\left(\frac{k_3}{2}-\frac{f_c}{\varepsilon_0}-\frac{f_c\varepsilon_{\text{t-ecc}}}{3\varepsilon_0^2}\right)+\frac{f_c\varepsilon_{\text{t-ecc}}h^3}{3\varepsilon_0^2}=0\end{aligned}\tag{7}$$

The height of central axis c_{cr} can be determined by assuming $\varepsilon_{\text{t-ecc}}$ equal to ε_{tc}. Meanwhile, according to the moment equilibrium of the cross section, we have

$$M=\int_c^h b\sigma(x)x\mathrm{d}x-\int_0^{h_2}b\sigma(x)x\mathrm{d}x-\int_{h_2}^{c}b\sigma(x)x\mathrm{d}x-\sigma_sA_sm\tag{8}$$

By assuming $\varepsilon_{\text{t-ecc}}$ equal to ε_{tc}, the initial cracking moment of the composite beam can be given by

$$\begin{aligned}M_{cr}=&bf_c\left[\frac{\varepsilon_{tc}}{\varepsilon_0}\left(\frac{2h^3}{3c_{cr}}-h^2+\frac{c_{cr}^2}{3}\right)-\frac{\varepsilon_{tc}^2}{\varepsilon_0^2}\left(\frac{h^4}{4c_{cr}^2}-\frac{2h^3}{3c_{cr}}+\frac{h^2}{2}-\frac{c_{cr}^2}{12}\right)\right]-\\&b\varepsilon_{tc}\left[k_1\left(\frac{h_2^2}{2}-\frac{h_2^3}{3c_{cr}}\right)+k_3\left(\frac{c_{cr}^2}{6}-\frac{h_2^2}{2}+\frac{h_2^3}{3c_{cr}}\right)+\frac{c_{cr}-m}{bc_{cr}}E_sA_sm\right]\end{aligned}\tag{9}$$

4.2 Derivation of yielding moment of ECC/concrete composite beam

The yielding moment of the composite beam is reached when the tensile steel reinforcement just reaches yielding strength. At this moment, the maximum strain of the concrete in the compression zone has not reached the ultimate strain ε_{cu} of concrete, and the stress distribution of the cross section is shown in Fig. 6(d).

The strain distribution of the cross section can also be given by Eq. (4). When the steel reinforcement reaches yielding strength, the strain ε_s is equal to ε_y, and the maximum strain $\varepsilon_{\text{t-ecc}}$ in the ECC layer is $\dfrac{c}{c-m}\varepsilon_y$. The stress distribution of the cross section is given by

$$\sigma(x)=\begin{cases}\sigma_{tc}+k(\varepsilon(x)-\varepsilon_{tc}) & 0\leqslant x\leqslant h_2\\ 0 & h_2<x\leqslant c\\ f_c\left[2\dfrac{\varepsilon(x)}{\varepsilon_0}-\left(\dfrac{\varepsilon(x)}{\varepsilon_0}\right)^2\right] & c<x\leqslant d\\ f_c & d<x\leqslant h\end{cases}\tag{10}$$

According to the force equilibrium, we can obtain

$$\begin{aligned}&\int_0^{h_2}b\sigma(x)\,\mathrm{d}x+\int_{h_2}^{c}b\sigma(x)\,\mathrm{d}x+f_yA_s-\\&\int_c^d b\sigma(x)\,\mathrm{d}x-\int_d^h b\sigma(x)\,\mathrm{d}x=0\end{aligned}\tag{11}$$

where $d=c+c\varepsilon_0/\varepsilon_{\text{t-ecc}}$.

Substituting the stress expressions into Eq. (11), we have

$$\begin{aligned}&c^2\left(\frac{f_c}{3}\frac{\varepsilon_0}{\varepsilon_{\text{t-ecc}}}+f_c\right)-\left(k_2\frac{\varepsilon_{\text{t-ecc}}h_2^2}{2}+\frac{f_yA_smc}{b(c-m)}\right)+\\&c\left(\varepsilon_{tc}+k_2h_2(\varepsilon_{\text{t-ecc}}-\varepsilon_{tc})+\frac{f_yA_sc}{b(c-m)}-f_ch\right)=0\end{aligned}\tag{12}$$

With $\varepsilon_{\text{t-ecc}}=\dfrac{c}{c-m}\varepsilon_y$, the height of the central axis c_y is given by

$$\begin{aligned}&c_y^2(c_y-m)f_c\left(\frac{\varepsilon_0}{3\varepsilon_y}+1\right)-c_y^2\varepsilon_y-\\&c_y(c_y-m)\left(\frac{mf_c\varepsilon_0}{3\varepsilon_y}+\varepsilon_{tc}-k_2h_2\varepsilon_{tc}-f_ch\right)=0\end{aligned}\tag{13}$$

According to the moment equilibrium of the cross section, we have

$$\begin{aligned}M=&\int_c^d b\sigma(x)x\mathrm{d}x+\int_d^h b\sigma(x)x\mathrm{d}x-\int_0^a b\sigma(x)x\mathrm{d}x-\\&\int_a^c b\sigma(x)x\mathrm{d}x-f_yA_sm\end{aligned}\tag{14}$$

By substituting the stress expressions into Eq. (14), the yielding moment of the cross section is given by

$$\begin{aligned}M_y=&bf_c\left\{\frac{h^2}{2}-\frac{(c_y-m)^2\varepsilon_0^2}{12\varepsilon_y^2}-c_y^2\left[\frac{3c_y\varepsilon_y+(c_y-m)\varepsilon_0}{6c_y\varepsilon_y}\right]\right\}-\\&f_yA_sm-\frac{b\sigma_{tc}h_2^2}{2}+bk_2h_2^2\left[\frac{2\varepsilon_yh_2-3c_y\varepsilon_y}{6(c_y-m)}+\frac{\varepsilon_{tc}}{2}\right]\end{aligned}\tag{15}$$

4.3 Derivation of ultimate failure moment of ECC/concrete composite beam

When the maximum strain of the concrete in the compression zone reaches the ultimate strain of the concrete, the cross section reaches the ultimate load carrying capacity, and the ultimate failure moment M_u can be calculated by considering this stress state. The stress distribution for this moment is shown in Fig. 7.

For the ultimate failure stage, the stress distribution of the cross section can be obtained from Eq. (10). At this moment, the compression zone reaches the ultimate strain of the concrete, and the maximum strain of the concrete is ε_{cu}. The maximum strain in the ECC layer is given by

$$\varepsilon_{t\text{-ecc}} = \frac{c}{h-c}\varepsilon_{cu}$$

According to the force equilibrium of the cross section, we can obtain

$$c^2\left(\frac{f_c\varepsilon_0}{3\varepsilon_{t\text{-ecc}}}+f_c\right)-\frac{k_2\varepsilon_{t\text{-ecc}}h_2^2}{2}+ c\left(\sigma_{tc}h_2+k_2h_2(\varepsilon_{t\text{-ecc}}-\varepsilon_{tc})+\frac{f_yA_s}{b}-f_ch\right)=0 \quad (16)$$

By substituting $\varepsilon_{t\text{-ecc}}$ with $c/(h-c)\varepsilon_{cu}$, the height of the central axis can be calculated by

$$(h-c_u)^2f_c\left(\frac{\varepsilon_0\varepsilon_{cu}}{3}-1\right)+c_uk_2h_2\varepsilon_{cu}-\frac{k_2h_2^2}{2}+ (h-c_u)\left(\sigma_{tc}h_2-k_2h_2\varepsilon_{tc}+\frac{f_yA_s}{b}\right)=0 \quad (17)$$

Again, according to the moment equilibrium of the cross section, the ultimate failure moment can be calculated by

$$M_u = bf_c\left\{\frac{h^2}{2}-c_u^2\left[\frac{(h-c_u)^2\varepsilon_0^2}{12c_u^2\varepsilon_{cu}^2}+\frac{1}{2}+\frac{(h-c_u)\varepsilon_{cu}\varepsilon_0}{3c_u}\right]\right\}- f_yA_sm-\frac{bh_2^2}{2}\sigma_{tc}+bk_2h_2^2\left[\frac{\varepsilon_{cu}h_2c_u}{3(h-c_u)}-\frac{c_u\varepsilon_{cu}}{2(h-c_u)}+\frac{\varepsilon_{tc}}{2}\right] \quad (18)$$

4.4 Simplification of calculation method of ultimate failure moment

Just as mentioned above, the ultimate failure moment is reached when the maximum strain of the concrete in the compression zone reaches the ultimate strain of the concrete. Based on the design method of the conventional RC beam, the stress distribution of the concrete in the compression zone can be replaced with a rectangular block by ensuring the same magnitude and the same height of the resultant force for the two stress distributions. The stress distribution of the ECC layer in the tension zone can also be treated in the same way as that of the concrete in the compression zone. At the ultimate failure moment, the stress state of the cross section is shown in Fig. 8.

Based on the assumption of the plane remaining plane, the strain in the ECC layer can be given by

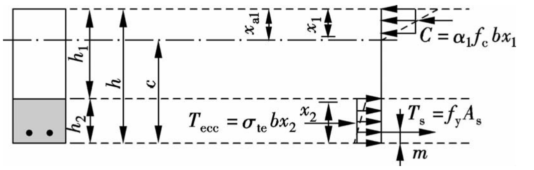

Fig. 8 Simplified stress distribution of cross section at the ultimate failure moment

$$\varepsilon(x)=\frac{c-h_2+x}{h-c}\varepsilon_{cu} \qquad 0\leqslant x\leqslant h_2 \quad (19)$$

At the ultimate failure stage, most of the ECC layer enters the strain hardening stage, and the resultant tensile force of the ECC layer can be given by

$$T_{ecc}=\int_0^{h_2}\sigma(x)b\mathrm{d}x=\left[(\sigma_{tc}-k_2\varepsilon_{tc})+k_2\varepsilon_{cu}\frac{2c-h_2}{2(h-c)}\right]bh_2 \quad (20)$$

Assuming that x_2 is the height of the equivalent block stress distribution of the ECC layer, the equivalent design strength of the ECC layer can be given by

$$\sigma_{te}=\frac{T_{ecc}}{bx_2}=\left[(\sigma_{tc}-k_2\varepsilon_{tc})+k_2\varepsilon_{cu}\frac{2c-h_2}{2(h-c)}\right]\frac{h_2}{x_2} \quad (21)$$

For the ultimate failure moment, the equivalent design strength of the concrete in the compression zone is $\alpha_1 f_c$, and the height of the resultant compression force is $x_1=\beta_1 x_{a1}$. Based on the design code of reinforced concrete structures, the parameters α_1 and β_1 are 1.0 and 0.8, respectively. According to the force and moment equilibrium, we can obtain

$$\left.\begin{aligned} &-\alpha_1f_cbx_1+f_yA_s+\sigma_{te}bx_2=0\\ &M=\alpha_1f_cbx_1\left(h-\frac{x_1}{2}\right)-f_yA_sm-\sigma_{te}b\frac{x_2^2}{2}\end{aligned}\right\} \quad (22)$$

By substituting the expression of σ_{te} into Eq. (22), the height of the central axis can be calculated as

$$c_u=\frac{2h\left[\frac{\alpha_1f_cbx_1-f_yA_s}{bh_2}-(\sigma_{tc}-k_2\varepsilon_{tc})\right]+k_2h_2\varepsilon_{cu}}{2\left[\frac{\alpha_1f_cbx_1-f_yA_s}{bh_2}-(\sigma_{tc}-k_2\varepsilon_{tc})\right]+2k_2\varepsilon_{cu}} \quad (23)$$

With the value of c_u and σ_{te}, the ultimate failure moment of the cross section M_u can be given by

$$M_u=\alpha_1f_cbx_1\left(h-\frac{x_1}{2}\right)-f_yA_sm- \left[(\sigma_{tc}-k_2\varepsilon_{tc})+k_2\varepsilon_{cu}\frac{2c_u-h_2}{2(h-c_u)}\right]bh_2\frac{x_2}{2} \quad (24)$$

5 Relationship Between Moment and Curvature of ECC/Concrete Composite Beam

For a differential element of the ECC/concrete composite beam in Fig. 9, the average curvature of the element can be given by

$$\varphi = \frac{1}{r} = \frac{\varepsilon_c + \varepsilon_{t\text{-ecc}}}{h} \tag{25}$$

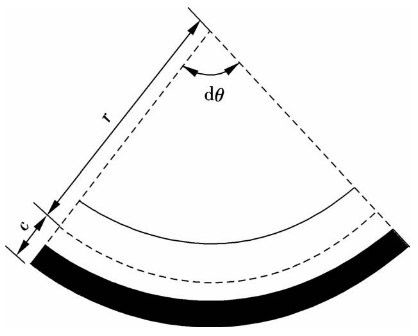

Fig. 9 Curvature of a differential element of ECC/concrete composite beam

According to the assumption of the plane remaining plane, the maximum strain of the concrete in the compression zone can be expressed by the maximum tension strain of the ECC layer, and the relationship between ε_c and $\varepsilon_{t\text{-ecc}}$ is given by $\varepsilon_c = \frac{h-c}{c}\varepsilon_{t\text{-ecc}}$. The curvature of the element can be reconstructed as

$$\varphi = \frac{1}{r} = \frac{\frac{h-c}{c}\varepsilon_{t\text{-ecc}} + \varepsilon_{t\text{-ecc}}}{h} = \frac{\varepsilon_{t\text{-ecc}}}{c} \tag{26}$$

With the maximum tension strain in the tension zone, the maximum tension strain in steel reinforcement and the maximum strain of the concrete in the compression zone can be calculated. Once the maximum tension strain in the ECC layer is obtained, the stress distribution of the cross section can be determined, resulting in the obtainment of the values of the curvature, the height of the central axis and the moment of the composite beam. The relationship between the curvature and the moment can be obtained. Meanwhile, the ductility index of the composite beam can also be calculated as

$$\Delta = \frac{\varphi_u}{\varphi_y} \tag{27}$$

6 Verification of Calculation Method for ECC/ Concrete Composite Beam

To verify the validity of the calculation methods in the above sections, the calculation results based on the parameters of a tested ECC/concrete composite beam[7] are compared with the test results. The dimensional information of the tested ECC/concrete composite beam is as follows. The clear span of the beam is 914. 4 mm, and the height and the width of the cross section are 152. 4 and 114. 3 mm, respectively (see Fig. 10). The thickness of the ECC layer is 50. 8 mm. The beam is loaded with a three-point loading configuration, and the distance between the loading point and the support is 304. 8 mm. The reinforcement ratio of the beam is 1. 47%. For the ECC layer, the initial cracking strength and strain are tested to be 3. 2 MPa and 0. 018%, respectively, and the ultimate tensile strength and the strain are tested to be 5. 2 MPa and 5. 4%, respectively. According the test results[7], the compression strength of the PE-ECC material is 42. 3 MPa. The compression and tensile strength of the concrete are 41. 7 and 2. 9 MPa, respectively, and the ultimate tensile strain of the concrete is 0. 008%. Although PE fibers are distinct from PVA fibers for manufacturing ECC materials, the PE-ECC shows similar mechanical properties with the PVA-ECC both under the uniaxial tension and the compression. The constitutive models for the PVA-ECC can also be used to model the mechanical behaviors of the PE-ECC. The yielding strength of the tensile steel reinforcement is 414 MPa and the Young modulus is 200 GPa. For the ECC/concrete composite beam, the obtained maximum compression strain of the concrete is 0. 52%, and the maximum strain in the ECC layer is 2. 6%.

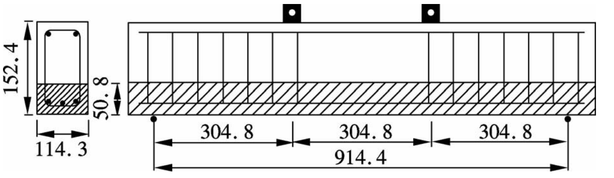

Fig. 10 Dimensional information of the tested beam (unit: mm)

Based on the above geometric and mechanical parameters of the ECC/concrete composite beam, the yielding moment and the ultimate failure moment can be calculated and compared with the test results. As shown in Fig. 11, the calculated moments have good consistency with the test results, but the calculated curvatures are a little smaller than the test results. Cracking of the composite beam leads to degradation of the flexural stiffness and results in greater curvature for a certain external moment. In the calculation, the effect of the flexural stiffness degradation of the composite beam cannot be considered. Furthermore, the calculation method also assumes no sliding between the concrete and the ECC layer and no relative

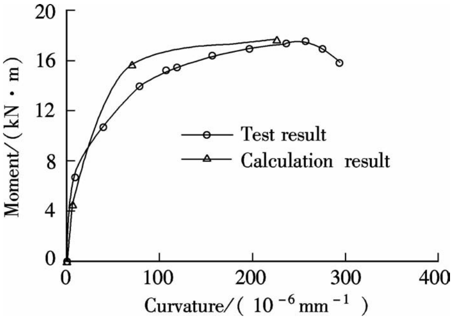

Fig. 11 Comparison of the calculation and test results

slip between the steel reinforcement and the ECC material, which will also underestimate the curvature of the ECC/concrete composite beam. The proposed calculation method is feasible for designing the steel reinforced ECC/concrete composite beam.

7 Conclusion

In this paper, the flexural behaviors of the steel reinforced ECC/concrete composite beam are comprehensively investigated. Based on the assumption of the plane remaining plane and the simplified constitutive models of materials, the strain and stress distributions of the cross section of the composite beam are analyzed, and the calculation method of the moment capacities at different stages is proposed. According to the Chinese code for design of concrete structures, a simplified calculation method of the ultimate failure moment is developed. Finally, the proposed calculation methods are employed to calculate the flexural responses of a tested beam. Comparison results show that the calculation results have good consistency with the test results, and the validation of the developed calculation method is proved. The suggested calculation method has certain theoretical significance and reference values.

References

[1] Li V C. On engineering cementitious composites (ECC) [J]. *Journal of Advanced Concrete Technology*, 2003, **1**(3): 215－229.

[2] Li V C. Progress and application of engineered cementitious composites [J]. *Journal of the Chinese Ceramic Society*, 2007, **35**(4): 531－536.

[3] Li V C, Leung C K Y. Steady state and multiple cracking of short random fiber composites [J]. *ASCE Journal of Engineering Mechanics*, 1992, **118**(11): 2246－2264.

[4] Fischer G, Fukuyama H, Li V C. Effect of matrix ductility on the performance of reinforced ECC column members under reversed cyclic loading conditions [C]//*Proceedings of the JCI International Workshop on DFRCC—Application and Evaluation*. Takayama, Japan, 2002: 269－278.

[5] Li V C. Engineered cementitious composites-tailored composites through micromechanical modeling [C]//*Fiber Reinforced Concrete*: *Present and Future*. Montreal: Canadian Society for Civil Engineering, 1998: 64－97.

[6] Cao Qian. Development of pseudo-ductile permanent formwork with glass fiber reinforced plastics reinforcements [D]. Hong Kong: Department of Civil and Environmental Engineering of Hong Kong University of Science and Technology, 2007.

[7] Maalej M, Li V C. Introduction of strain-hardening engineered cementitious composites in design of reinforced concrete flexural members for improved durability [J]. *ACI Structural Journal*, 1995, **92**(2): 167－176.

[8] Cheung Y N. Investigation of concrete components with a pseudo-ductile layer [D]. Hong Kong: Department of Civil and Environmental Engineering of Hong Kong University of Science and Technology, 2004.

[9] Ministry of Housing and Urban-Rural Development of the People's Republic of China. GB 50010—2010 Code for design of concrete structures [S]. Beijing: China Architecture and Building Press, 2010. (in Chinese)

[10] Luo M. Study on the optimal design and mechanical properties of PVA-fiber reinforced cementitious composites [D]. Nanjing: Department of Civil Engineering of Southeast University, 2010. (in Chinese)

[11] Kanda T, Lin Zhong, Li V C. Tensile stress-strain modeling of pseudo strain-hardening cementitious composites [J]. *Journal of Materials in Civil Engineering*, 2000, **12**(2): 147－156.

[12] Zhang J, Li V C. Monotonic and fatigue performance of engineered fiber reinforced cementitious composite in overlay system [J]. *Cement and Concrete Research*, 2002, **32**(3): 415－423.

钢筋增强高延性 ECC/混凝土组合梁的受弯性能

董洛廷[1]　潘金龙[1]　袁　方[1]　梁坚凝[2]

([1] 东南大学混凝土及预应力混凝土教育部重点实验室,南京 210096)

([2] 香港科技大学土木及环境工程学系,香港 999077)

摘要:采用高延性纤维增强水泥基材料部分代替钢筋混凝土梁受拉区的混凝土得到 ECC/RC 组合梁构件,可以有效提高梁的延性及抗裂性能.基于平截面假定和材料本构模型,分析组合梁构件在受力过程中各个阶段的截面应力应变状态,得到各阶段承载力的计算方法.根据混凝土结构设计规范,提出了 ECC/RC 组合梁极限承载力的简化计算方法,并给出了组合梁构件的弯矩-曲率关系,得到组合梁延性的简便计算方法.最后,采用一个组合梁的试验结果对理论公式进行验征.结果表明:计算结果和试验结果吻合得较好,证明所提出的组合梁各阶段受弯承载力计算方法是正确的,具有一定的理论意义和参考价值.

关键词:高延性纤维增强水泥基材料;钢筋混凝土;组合梁;受弯性能;承载力

中图分类号:TU354