

Reliability analysis of anti-pull piles under excavation

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Abstract: The impact of excavation on the reliability of anti-pull piles is studied, and three cases of reliability analysis, named reliability of ultimate limit state (ULS), reliability of serviceability limit state (SLS) and reliability of system (SYS) are studied. The reduction factor of the pile capacity is used to calculate the reliability indices for the three cases. The ratio ξ of the pile capacity of SLS to the pile capacity of ULS has a significant influence on the reliability indices of SLS and SYS. The mean value μ_ξ of the ratio ξ is considered as a random variable to study the reliability indices of SLS and SYS. The numerical example demonstrates that the excavation depth and the excavation diameter are proved to have significant influences on the reduction factor of the pile capacity and the reliability indices. The reliability indices decrease with the increase in the excavation depth, and the excavation diameter has a considerable influence on the reliability index when the excavation is relatively deep. In addition, μ_ξ has a significant influence on the reliability indices of SLS and SYS. For a more accurate estimation of μ_ξ , further research should be conducted to study μ_ξ .

Key words: excavation; reliability; anti-pull piles; reduction factor; excavation depth; excavation diameter

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Anti-pull piles are increasingly applied in underground engineering, and some researchers have studied the ultimate bearing capacity of anti-pull piles^[1-3]. However, the impact of soil deformation induced by excavation on pile foundations is a complicated problem involving the knowledge about deep excavation and pile foundation engineering. Leung et al.^[4] performed a centrifuge model to study the effects of an uncorbelled deep excavation in dry dense sand on an adjacent existing single pile foundation. Wang et al.^[5-6] proposed a method to analyze and design anti-pull piles under excavation according to the project of the Shanghai World Exposition 500 kV underground substation. Poulos^[7] developed theoretical methods to evalu-

ate the behavior of piles under soil movement induced by excavation. The excavation depths and excavation dimensions are the major factors influencing pile capacities. However, how the reliability index of anti-pull piles can be impacted by excavation has not been investigated. In addition, the reliability study for SLS of pile foundation under excavation has never been reported. Nevertheless, the ability to quantitatively estimate the reliability of piles under excavation enables us to judge the safety of foundations in a more rational way.

The objective of this paper is to evaluate the impact of excavation on the reliability of anti-pull piles under excavation. The pile capacity reduction factor is used to study the influence of excavation on pile capacity, and the calculation formulae of the reliability indices are deduced according to the reduction factor. Finally, a numerical example is presented to illustrate the proposed method.

1 Reliability Analysis of ULS

When analyzing the reliability of anti-pull piles for ULS, the ultimate state equation should be first established. If the pile capacity and the load effect are considered, the ultimate state equation can be written as

$$g = R - Q = 0 \quad (1)$$

where R and Q are the pile capacity and the load effect, respectively. Obviously, Eq. (1) is a linear performance function, and the two factors in it should be random variances due to uncertainties arising from soil parameters, geotechnical materials, construction and so on. Assuming that pile capacity and load effect both follow lognormal distributions, the reliability index β_{ULS} of ULS can be written as^[8]

$$\beta_{\text{ULS}} = \frac{\ln\left(\frac{\mu_{R_{\text{ULS}}}}{\mu_Q} \sqrt{\frac{1 + \text{COV}_Q^2}{1 + \text{COV}_{R_{\text{ULS}}}^2}}\right)}{\sqrt{\ln(1 + \text{COV}_{R_{\text{ULS}}}^2)(1 + \text{COV}_Q^2)}} \quad (2)$$

where $\mu_{R_{\text{ULS}}}$ and μ_Q are the means of pile capacity and load effect, respectively; $\text{COV}_{R_{\text{ULS}}}$ and COV_Q are the coefficients of variation for pile capacity and load effect, respectively. Zhang^[9] considered the load effect to be dead load Q_D and live load Q_L , and gave that

$$\frac{\mu_{R_{\text{ULS}}}}{\mu_Q} = \frac{\lambda_R \text{FOS}(Q_D/Q_L + 1)}{\lambda_{Q_D}(Q_D/Q_L) + \lambda_{Q_L}} \quad (3)$$

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and

$$\text{COV}_Q^2 = \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2 \quad (4)$$

where λ_R is the bias factor of the pile capacity; λ_{Q_D} and λ_{Q_L} are the bias factors for dead and live loads, respectively; Q_D and Q_L are the nominal values of dead and live loads, respectively; COV_{Q_D} and COV_{Q_L} are the coefficients of variation for dead and live loads, respectively; FOS is the factor of safety for traditional allowable stress design. AASHTO^[10] offers the load statistics, which are utilized to analyze the reliability in this paper, and they are $\lambda_{Q_D} = 1.08$, $\lambda_{Q_L} = 1.15$, $\text{COV}_{Q_D} = 0.13$ and $\text{COV}_{Q_L} = 0.18$. Comprehensive investigations conducted by McVay et al.^[11] show that when 3.69 is selected as the value of Q_D/Q_L , the pile foundation is relatively safe. In addition, 2.0 is commonly used as the value of FOS^[12]. Therefore, β_{ULS} is actually the function of λ_R and COV_R associated with the pile capacity.

The pile capacity will inevitably decrease owing to the change of the ground stress field and the normal stress of the pile-soil interface under excavation. How to evaluate the capacity loss is an important issue due to its great contributions to the design and analysis of a pile foundation. Several researchers adopted a loss ratio of the pile capacity to study the impact of excavation on the bearing capacity of uplift piles^[1,6]. To study the reliability index influenced by excavation, the reduction factor for the pile capacity which is defined as the ratio of the pile capacity under excavation to the pile capacity before excavation is proposed as^[13]

$$\eta = R_U/R_B \quad (5)$$

where η is the reduction factor of the pile capacity; R_B is the pile capacity before excavation; R_U is the pile capacity under excavation. Strictly speaking, η should be treated as a random variable. Due to the lack of sufficient statistical data for η , η is treated as a deterministic quantity in this study, but its values depend on the excavation dimension and excavation depth^[5-6]. Once η is obtained, the bias factor and the coefficient of variation of the pile capacity under excavation are given by

$$\lambda_{\eta R} = \lambda_R \eta \quad (6)$$

$$\text{COV}_{\eta R}^2 = \text{COV}_R^2 + \text{COV}_\eta^2 \quad (7)$$

where $\lambda_{\eta R}$ and $\text{COV}_{\eta R}$ are the bias factor and the coefficient of variation for the pile capacity under excavation, respectively; COV_η is the coefficient of variation for the reduction factor of the pile capacity. For the sake of illustration, COV_η is taken as 0.10 to study the impact of excavation on the reliability index. Thus, the reliability index of ULS under excavation can be calculated using Eq. (2), Eq. (3) and Eq. (4) by replacing λ_R and COV_R with $\lambda_{\eta R}$ and $\text{COV}_{\eta R}$, respectively.

2 Reliability Analysis of SLS

For ULS designs, the reliability index of a single pile can be given using Eq. (2). In Eq. (2), the pile capacity is the ULS capacity R_{ULS} . However, for SLS designs, the pile capacity is the SLS capacity R_{SLS} , rather than the ULS capacity R_{ULS} . If the ratio of R_{SLS} to R_{ULS} is defined as ξ , the SLS probability of failure $P_{f, \text{SLS}}$ can be written as^[8]

$$\begin{aligned} P_{f, \text{SLS}} &= \text{Prob}(R_{\text{SLS}} < (Q_D + Q_L)) = \\ &\text{Prob}\{\ln(R_{\text{SLS}}/R_{\text{ULS}})(R_{\text{ULS}}/(Q_D + Q_L)) < \ln(1)\} = \\ &\text{Prob}[(\ln\xi + \ln R_{\text{ULS}} - \ln(Q_D + Q_L)) < 0] \end{aligned} \quad (8)$$

Based on the Monte Carlo simulation method, Eq. (8) can be rewritten as

$$\begin{aligned} P_{f, \text{SLS}} &= \text{Prob}[\xi + R_{\text{ULS}} - Q_D - Q_L < 0] = \\ &\text{Prob}[N_{\text{SLS}} < 0] = \Phi(-\mu_{N_{\text{SLS}}}/\sigma_{N_{\text{SLS}}}) \end{aligned} \quad (9)$$

where $N_{\text{SLS}} = \xi + R_{\text{ULS}} - Q_D - Q_L$ follows a normal distribution on the condition that ξ , R_{ULS} , Q_D and Q_L are the equivalent normal random variables, and the statistical parameters for N_{SLS} (i. e., the bias factor and the coefficient of variation) can be obtained by

$$\mu_{N_{\text{SLS}}} = \mu_\xi + \mu_{R_{\text{ULS}}} - \mu_{Q_D} - \mu_{Q_L} \quad (10)$$

$$\text{COV}_{N_{\text{SLS}}}^2 = \text{COV}_\xi^2 + \text{COV}_{R_{\text{ULS}}}^2 + \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2 \quad (11)$$

where μ_ξ and COV_ξ are the mean and the coefficient of variation of ξ , respectively. The reliability index for SLS can be written as

$$\beta_{\text{SLS}} = \frac{\ln\left[\frac{\mu_{R_{\text{ULS}}}}{\mu_{Q_D} + \mu_{Q_L}} \sqrt{\frac{1 + \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2}{1 + \text{COV}_{R_{\text{ULS}}}^2}}\right] + \ln\left[\frac{\mu_\xi}{\sqrt{1 + \text{COV}_\xi^2}}\right]}{\sqrt{(1 + \text{COV}_{R_{\text{ULS}}}^2)(1 + \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2) \ln(1 + \text{COV}_\xi^2)}} \quad (12)$$

According to Eqs. (3) and (4), Eq. (12) can be expressed as

$$\begin{aligned} \beta_{\text{SLS}} &= \frac{\ln\left[\frac{\lambda_R \text{FOS}(Q_D/Q_L) + 1}{\lambda_{Q_D}(Q_D/Q_L) + \lambda_{Q_L}} \sqrt{\frac{1 + \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2}{1 + \text{COV}_{R_{\text{ULS}}}^2}}\right]}{\sqrt{(1 + \text{COV}_{R_{\text{ULS}}}^2)(1 + \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2) \ln(1 + \text{COV}_\xi^2)}} + \\ &\frac{\ln(\mu_\xi/\sqrt{1 + \text{COV}_\xi^2})}{\sqrt{(1 + \text{COV}_{R_{\text{ULS}}}^2)(1 + \text{COV}_{Q_D}^2 + \text{COV}_{Q_L}^2) \ln(1 + \text{COV}_\xi^2)}} \end{aligned} \quad (13)$$

The reliability index for SLS under excavation can be calculated using Eq. (13) by replacing λ_R and COV_R with $\lambda_{\eta R}$ and $\text{COV}_{\eta R}$ described in Eq. (6) and Eq. (7), respectively. However, a key random variable which should be determined before calculating the reliability index of SLS is $\xi = R_{\text{ULS}}/R_{\text{SLS}}$ and its probabilistic characteristics. Wang et al.^[14] utilized the load-displacement model to study the probabilistic characteristics (i. e., the mean and the coef-

ficient of variation), and the closed-form approximation method and the Monte Carlo simulation method to calculate the mean and the coefficient of variation of ξ for augered cast-in-place pile foundations (ACIP). $\mu_\xi = 1.484$ and $COV_\xi = 0.214$ for the closed-form approximation method are obtained, while $\mu_\xi = 1.505$, $COV_\xi = 0.303$ for the Monte Carlo simulation method are obtained. Accordingly, different probabilistic characteristics of ξ may be obtained using different calculation methods. To date, the precise method has rarely been studied in existing references or notebooks. Therefore, there is a need for further research. For a more meaningful evaluation, several values for ξ are used for parametric study. In addition, the average value of COV_ξ , 0.256, is used, and ξ is assumed to be a lognormal random variance in this paper according to the investigation outcomes in Ref. [14].

3 Reliability Analysis of SYS

The design of a pile foundation should satisfy the requirements of both the ultimate limit state and serviceability limit state. Systemic failure probability means that the applied load is greater than the load carrying capacity or the pile is subjected to a greater settlement than the serviceable limit, and the systemic failure probability can be described as^[8]

$$P_f((Q \geq R_{ULS}) \cup (S \geq S_{SLS})) = \Phi^{-1}(-\beta_{ULS}) + \Phi^{-1}(-\beta_{SLS}) - \Phi^{-1}(-\beta_{ULS/SLS}) \Phi^{-1}(-\beta_{SLS}) \quad (14)$$

where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution; $P_f((Q \geq R_{ULS}) \cup (S \geq S_{SLS}))$ is the failure probability of the foundation when the applied load is greater than or equal to its ultimate capacity or when the settlement is greater than or equal to its serviceable settlement. The conditional reliability index, $\beta_{ULS/SLS}$, is obtained utilizing the ratio of the number for samples exceeding the ultimate capacity of the foundation to the total number of samples^[9]. According to Eq. (14), the system reliability index can be written as

$$\beta_{SYS} = \Phi^{-1}(1 - P_f((Q \geq R_{ULS}) \cup (S \geq S_{SLS}))) \quad (15)$$

If the pile foundation is subjected to be excavated, the reliability index for the system can be calculated using Eq. (14) and Eq. (15) by replacing β_{ULS} and β_{SLS} with corresponding reliability indices under excavation.

4 Numerical Example

Wang et al.^[5] reported the project of the Shanghai World Exposition 500 kV underground substation. According to the finite element method and the Mindlin solution, Wang et al.^[6] focused on the bearing capacity reduction of uplift piles caused by the off-loading of excavation which led to a pressure loss of the soil around the piles and the axial tension force of the piles caused by the foundation rebound after the excavation. Two main fac-

tors, excavation dimension and excavation depth, are considered as major contributions to pile capacity loss, and the pile capacity reduction factor defined in Eq. (5) is summarized in Fig. 1 and Fig. 2, where H , r and L denote excavation depth, excavation dimension and valid pile length below the foundation pit bottom, respectively. As expected, the reduction factor of the pile capacity decreases significantly with the excavation depth from Fig. 1, especially for $r/L = 2.0$. For example, the reduction factor is 1.0 for $H/L = 0$, while the reduction factor is 0.689 for $H/L = 1.0$, and the difference between them is 0.311. From Fig. 2, r/L has a minor effect on the capacity when H/L is relatively large.

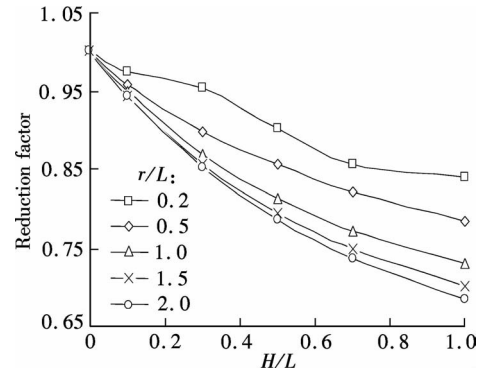


Fig. 1 Relationships between pile capacity reduction factor and H/L

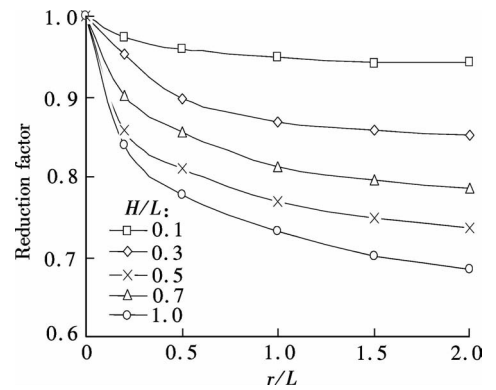


Fig. 2 Relationships between pile capacity reduction factor and r/L

Based on in-situ tests, Liu et al.^[13] studied the bearing capacity of 10 uplift piles, and the bias factor and the coefficient of variation are $\lambda_R = 0.947$ and $COV_R = 0.003$, respectively. In order to illustrate, $\lambda_R = 0.947$ and $COV_R = 0.003$ are utilized to analyze the reliability index for ULS, SLS and SYS in this paper. Tab. 1 gives the calculation results for $\lambda_{\eta R}$. Fig. 3 and Fig. 4 show the curves of the reliability indices vs. the excavation depth and excavation dimension, respectively. The reliability index decreases significantly due to the effect of excavation depth, especially when the excavation is relatively deep (e.g., r/L is greater than 1.0) according to Fig. 3. For instance, for $r/L = 2.0$, the reliability index is 2.61 when

no excavation is obtained, while the reliability index is 0.89 when H/L is 1.0. The reliability index at zero excavation depth is 2.61, and the corresponding probability of failure is 4.53×10^{-3} ; the pile foundation is highly safe^[14]. However, the reliability index for $H/L = 1.0$ is 0.89, and the corresponding failure probability is 0.187; the pile foundation is significantly dangerous^[8]. In addition, as shown in Fig. 4, the reliability index is sensitive to the excavation dimension when the excavation is relatively deep (e. g., H/L is greater than 0.5). The excavation dimension has a minor effect on the reliability index when the excavation is shallow (e. g., $H/L = 0.3$ or 0.1).

Tab. 1 Relationships between pile capacity reduction factor and r/L

H/L	Bias factor $\lambda_{\eta R}$					
	$r/L = 0$	$r/L = 0.2$	$r/L = 0.5$	$r/L = 1.0$	$r/L = 1.5$	$r/L = 2.0$
0	0.947	0.947	0.947	0.947	0.947	0.947
0.1	0.947	0.922	0.907	0.898	0.893	0.892
0.3	0.947	0.902	0.849	0.822	0.812	0.807
0.5	0.947	0.854	0.811	0.770	0.753	0.744
0.7	0.947	0.812	0.778	0.731	0.708	0.697
1.0	0.947	0.795	0.743	0.692	0.665	0.649

Fig. 5 and Fig. 6 show the curves of the reliability index for SLS and SYS vs. the excavation depth for three values of μ_ξ (i. e., $\mu_\xi = 1.2, 1.4$ and 1.6), and it can be drawn that both the excavation depth and the excavation dimension have great contributions to reliability indices of SLS and SYS.

Fig. 7 shows the characteristic curves of the reliability index vs. the ratio of dead load to live load considering three values of μ_ξ (i. e., $\mu_\xi = 1.2, 1.4$ and 1.6) when $r/L = 1.0$ and $H/L = 0.3$. It can be seen that the reliability index of SYS lies between the reliability index of ULS and the reliability index of SLS, and $\beta_{ULS} > \beta_{SYS} > \beta_{SLS}$ can be also obtained from Fig. 7(a). However, β_{SLS} becomes great when μ_ξ increases, and $\beta_{SLS} > \beta_{SYS} > \beta_{ULS}$ is

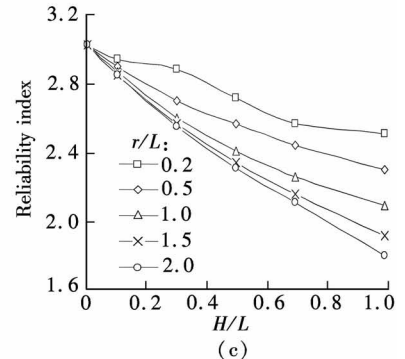
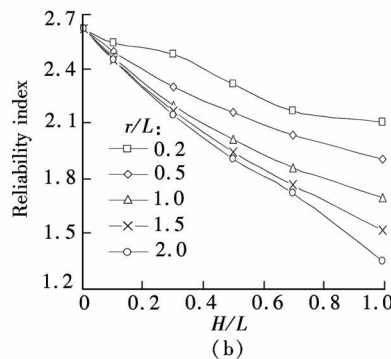
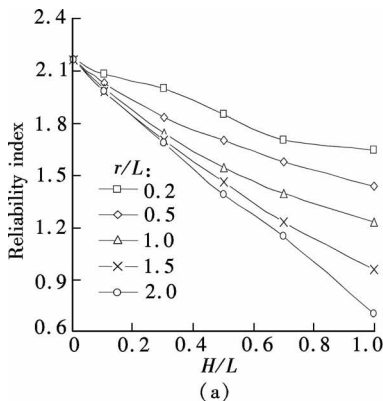


Fig. 5 Reliability indices for SLS with various H/L . (a) $\mu_\xi = 1.2$; (b) $\mu_\xi = 1.4$; (c) $\mu_\xi = 1.6$

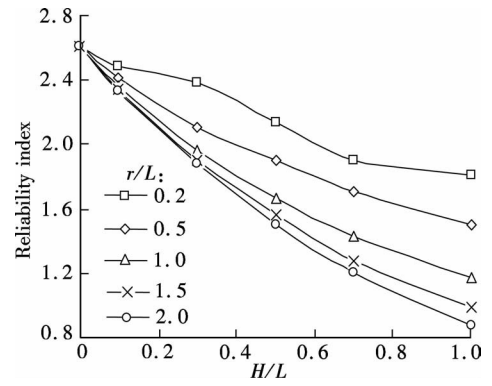


Fig. 3 Reliability indices for ULS with various H/L

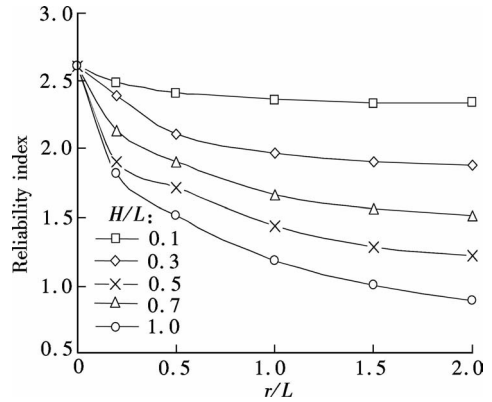


Fig. 4 Reliability indices for ULS with various r/L

shown in Figs. 7(b) and (c). Accordingly, in order to accurately estimate the reliability indices of SLS and SYS, μ_ξ should be further researched. In addition, the reliability index of SYS approaches the reliability index of SLS, and is distant from the reliability index of ULS according to Fig. 7.

5 Conclusions

- 1) The pile capacity decreases significantly with the excavation depth, and the excavation dimension has a significant influence on the pile capacity when the excavation is relatively deep.
- 2) Both the excavation depth and the excavation dimension make great contributions to the reliability index.

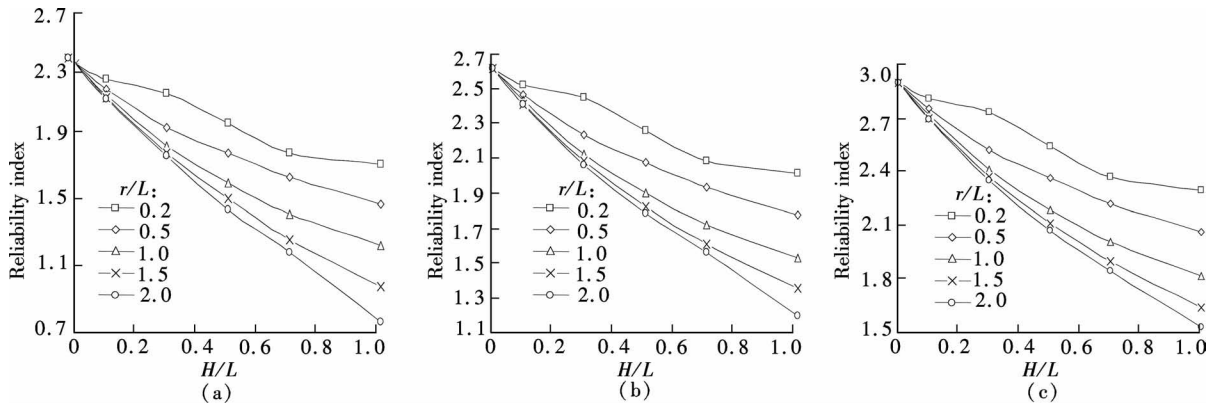


Fig. 6 Reliability indices for SYS with various H/L . (a) $\mu_\xi = 1.2$; (b) $\mu_\xi = 1.4$; (c) $\mu_\xi = 1.6$

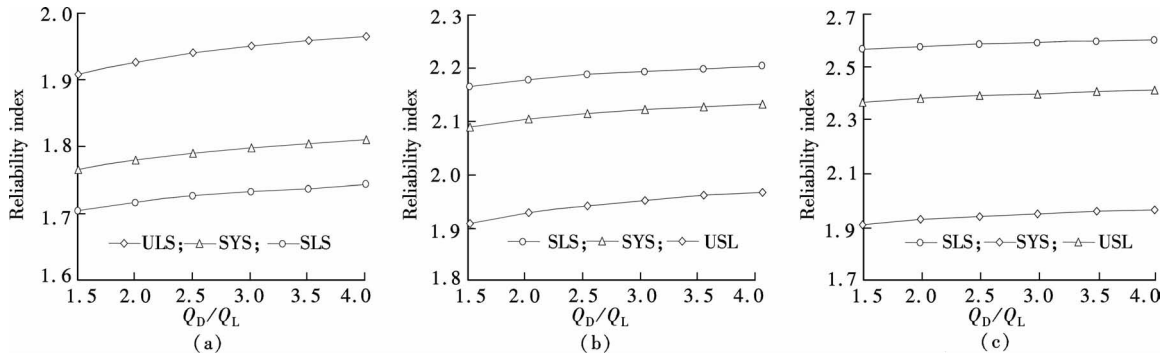


Fig. 7 Relationships between reliability index and Q_D/Q_L for $r/L = 1.0$ and $H/L = 0.3$. (a) $\mu_\xi = 1.2$; (b) $\mu_\xi = 1.4$; (c) $\mu_\xi = 1.6$

In addition, the reliability index of SYS approaches the reliability index of SLS, and is distant from the reliability index of ULS.

3) The ratio of the pile capacity for SLS to the pile capacity for ULS has a considerable influence on the reliability index. Further researches should be done to accurately estimate this random variance.

References

- [1] Dash B K, Pise P J. Effect of compressive load on uplift capacity of model piles[J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 2003, **129**(11): 987–992.
- [2] Alawneh A S, Malkawi A I H, Al-deeky H. Tension tests on smooth and rough model piles in dry sand[J]. *Canadian Geotechnical Journal*, 1999, **36**(4): 746–753.
- [3] Liu K F, Xie X Y, Zhang J F, et al. Study of pullout bearing capacity of diving dry-vibrated compacted and anti-pull stone column[J]. *Rock and Soil Mechanics*, 2004, **25**(12): 1937–1941. (in Chinese)
- [4] Leung C F, Chow Y K. Behavior of pile subject to excavation induced soil movement[J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 2000, **129**(11): 947–954.
- [5] Wang W D, Huang M S, Wang J H, et al. Working properties and calculation methods of pile under deep excavation[R]. Shanghai: Shanghai East China Architecture Design & Research Institute Co., Ltd, 2008. (in Chinese)
- [6] Wang W D, Wu J B. Design and analysis of uplift pile under deep excavation[J]. *Chinese Journal of Buildings*, 2010, **31**(5): 202–208. (in Chinese)
- [7] Poulos H G. Pile response due to excavation induced lateral soil movement[J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 1997, **123**(2): 94–99.
- [8] Phoon K K. *Reliability-based design in geotechnical engineering: computations and applications*[M]. New York: Taylor & Francis, 2008.
- [9] Zhang L M. Reliability verification using proof pile load tests[J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 2001, **130**(11): 1203–1213.
- [10] American Association of State Highway and Transportation Officials (AASHTO). Load and resistance factor design (LRFD) bridge design specifications[S]. Washington DC: American Association of State Highway and Transportation Officials, 1997.
- [11] McVay M C, Birgisson B, Zhang L M, et al. Load and resistance factor design (LRFD) for driven piles using dynamic methods — a Florida perspective[J]. *Geotechnical Test Journal*, 2000, **23**(1): 55–66.
- [12] JGJ 94—2008 Technical code for building pile foundations [S]. Beijing: China Architecture & Building Press, 2008. (in Chinese)
- [13] Zhang L M, Li D Q, Tang W H. Impact of routine quality assurance on reliability of bored piles[J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 2006, **132**(5): 622–630.
- [14] Wang Y, Kulhawy F H. Reliability index for serviceability limit state of building foundations [J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 2008, **134**(11): 1587–1594.

开挖条件下抗拔桩的可靠度分析

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摘要:研究了开挖对承载力极限状态(ULS)、正常使用极限状态(SLS)和系统可靠度(SYS)的影响,并基于开挖条件下承载力折减系数分别给出 ULS 可靠度指标、SLS 可靠度指标和 SYS 可靠度指标的计算方法. 研究得出 SLS 承载力和 USL 承载力的比值 ξ 对 SLS 可靠度和 SYS 可靠度有显著影响,并将这一比值的均值 μ_ξ 当作随机变量,研究了 SLS 可靠度和 SYS 可靠度. 算例分析表明:开挖深度和开挖直径对承载力折减系数和可靠度指标有显著影响;随着开挖深度的增大,可靠度指标有很显著的减小;当开挖深度较大时,开挖直径对可靠度指标的影响较大. 此外, μ_ξ 对 SLY 和 SYS 可靠度指标的影响很大,要更精确地估计 μ_ξ ,需对 μ_ξ 进行深入的研究.

关键词:基坑开挖;可靠度;抗拔桩;折减系数;开挖深度;开挖直径

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