Mechanics analysis of vehicle bumping at approach slabs with superelevation

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Abstract: In order to analyze the pavement stress caused by vehicle bumping at an approach slab, a simplified four-wheeled bi-axle vehicle-moving model is proposed. The effect of damping and vibration reduction in the process of vehicle-moving is not considered. Based on the position change of vehicle wheels at the approach slab, the vehicle dynamic vibration equations are summarized. Meanwhile, the undetermined coefficients of the vibration equations are obtained using the boundary and initial conditions of the vehicle. The analytical motion solutions of rear and front wheels at different stages are concluded. Consequently, a four-wheeled vehicle model is developed and vibration equations are provided, which can be used to analyze the impact of complicated stress on pavement. The results show that the excessive stress and stress concentration will occur at the approach slab, and it needs to be strengthened.

Key words: road engineering; mechanics analysis; approach slab; vehicle model; motion equation
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Bridge approach slabs are normally constructed on the embankment soil connecting the bridge deck with the adjacent paved roadway. The slab function is to provide a smooth transition between the bridge deck and the roadway pavement. However, soil embankment settlement often causes the bridge approach slabs to loosen from the support soil. When this occurs, the slab will bend and deform in a concave manner[1]. Due to the differential settlements of the bridge approach embankment and bridge abutments, the junction profile of road and bridge generates an elevation difference resulting in the vertical movement of vehicles as they pass over. This is called vehicle bumping[2].

In the past decade, there was some research conducted on the bumping problem from the aspects of materials, computations and modeling[3-9]. This research tried to reduce the settlement of the bridge approach embankment and bridge abutments. Green et al. [10] modeled the bridge and vehicle separately and combined mode shapes with an iterative procedure for calculating the dynamic bridge response. The nonlinear vehicle models consist of a leaf-spring, four-axle, articulated vehicle. The research shows that the air-suspended vehicle causes lower dynamic responses than the leaf-spring vehicles. Zhao et al. [11] used the 1/2 vehicle model and found the dynamic model of a moving vehicle, and then they established the vibration equation. Combined with the practical engineering, the variability of force caused by a vehicle approaching and leaving was revealed. Zhang et al. [12] investigated the dynamic response analysis of the man-vehicle-road system using the Laplace transform, in which the vehicle is modeled as a three-degree-of-freedom system. Then, they introduced the maximum transient vibration value (MTVV) of acceleration as the comfort evaluation index and gave the effects of some parameters on the determination of allowable differential settlement. Shi et al. [13] developed an analytical vehicle-bridge coupled model and investigated the dynamic behavior of slab bridges with different span lengths under various vehicle speeds and road surface conditions. Szurgott et al. [14] carried out the experimental tests of three permit vehicles and concluded that poor, loose, and slack cargo tie-downs can trigger a hammering load effect resulting in a magnification of the dynamic response of the vehicle-bridge system. Bump-related problems have been commonly recognized and previous studies just use a 1/4 or a 1/2 vehicle model to consider the force analysis. However, in this paper the dynamic model of the four-wheeled bi-axel vehicle is developed and the vehicle width parameter is introduced. Ignoring the effects of damping and vibration reduction in the vehicle moving process, the vehicle vibration equation and the general solution are obtained. Eventually, the research investigates the boundary conditions when the vehicle front/rear wheels are respectively at different stages (bridge deck, approach slab and pavement) and concludes the dynamic motion equations of various stages when the vehicle bumping occurs at approach slabs.

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1 Modeling and Motion Equation

Generally speaking, the nonintegrated vehicle model is adopted to analyze pavement dynamic load and pressure when a vehicle is moving over an uneven pavement and bumping at the approach slabs[10-13, 15]. A four-wheeled bi-axle vehicle model is adopted. Then, the basic motion equation can be established at the approach slab between the bridge abutment and the pavement. Suppose that R is the vehicle gravity center in the freedom system of multi-degrees as shown in Fig. 1 and Fig. 2, where M is the vehicle mass, and I_y is the moment of inertia when the vehicle revolves around the y axis. The primary motion of the moving vehicle is the vibration from the vertical displacement and the rotating vibration around the centroid. l_1 and l_2 are the distances between the centroid and the vehicle's front/rear axles; b_1 and b_2 are the distances between the centroid and the vehicle's left/right axles; l is the distance between front and rear axles; b is the distance between the left and right axles; E_1, E_2, E_3 and E_4 are the spring stiffnesses of the four wheels; v is the vehicle speed; L_p is the horizontal distance of the approach slab; u_1, u_2, u_3 and u_4 are the vertical displacements of the four wheels; u_5 and u_6 are the centroid vertical displacements of the front and rear axles, respectively; u_m is the vertical displacement of the vehicle centroid; and y_1, y_2, y_3 and y_4 are the landing place heights of the four wheels.

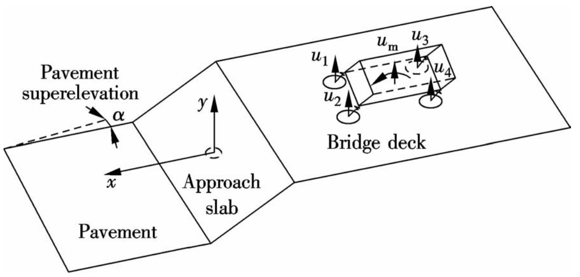

Fig. 1 Schematic diagram of the moving vehicle at pavement and bridge deck

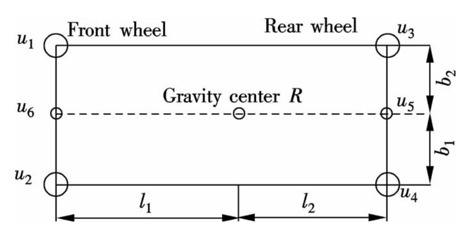

Fig. 2 Position distribution diagram of vehicle wheels and its gravity center

According to $l_1 + l_2 = l$, $b_1 + b_2 = b$, the centroid displacement u_m equation is represented as

$$\sqrt{l^2+b^2}\,u_{m1} = \sqrt{l_2^2+b_1^2}\,u_1 + \sqrt{l_1^2+b_2^2}\,u_3,\quad \sqrt{l^2+b^2}\,u_{m2} = \sqrt{l_2^2+b_2^2}\,u_2 + \sqrt{l_1^2+b_1^2}\,u_4$$

$$u_m = \frac{1}{2}(u_{m1}+u_{m2}) = \frac{1}{2\sqrt{l^2+b^2}}\left(\sqrt{l_2^2+b_1^2}\,u_1 + \sqrt{l_1^2+b_2^2}\,u_3 + \sqrt{l_2^2+b_2^2}\,u_2 + \sqrt{l_1^2+b_1^2}\,u_4\right) \tag{1}$$

The second derivation of Eq. (1) is

$$M\frac{d^2u_m}{dt^2} = \frac{M}{2\sqrt{l^2+b^2}}\left(\sqrt{l_2^2+b_1^2}\frac{d^2u_1}{dt^2} + \sqrt{l_1^2+b_2^2}\frac{d^2u_3}{dt^2} + \sqrt{l_2^2+b_2^2}\frac{d^2u_2}{dt^2} + \sqrt{l_1^2+b_1^2}\frac{d^2u_4}{dt^2}\right) \tag{2}$$

Based on the vertical motion balance conditions, the following equation can be inferred.

$$-M\frac{d^2u_m}{dt^2} = (u_1-y_1)E_1 + (u_2-y_2)E_2 + (u_3-y_3)E_3 + (u_4-y_4)E_4 \tag{3}$$

The following differential equation can be obtained from Eqs. (2) and (3) as

$$\frac{M}{2\sqrt{l^2+b^2}}\left(\sqrt{l_2^2+b_1^2}\frac{d^2u_1}{dt^2} + \sqrt{l_1^2+b_2^2}\frac{d^2u_3}{dt^2} + \sqrt{l_2^2+b_2^2}\frac{d^2u_2}{dt^2} + \sqrt{l_1^2+b_1^2}\frac{d^2u_4}{dt^2}\right) = (u_1-y_1)E_1 + (u_2-y_2)E_2 + (u_3-y_3)E_3 + (u_4-y_4)E_4 \tag{4}$$

When a vehicle revolves around its centroid, the rotation equation is described as

$$I_y\frac{d^2\phi}{dt^2} = (u_1-y_1)E_1l_1 + (u_2-y_2)E_2l_1 - (u_3-y_3)E_3l_2 - (u_4-y_4)E_4l_2 \tag{5}$$

Considering the vehicle rotation angle, we have

$$u_5 = \frac{u_1b_1+u_2b_2}{b},\quad u_6 = \frac{u_4b_1+u_3b_2}{b},\quad \phi = \frac{u_5-u_6}{l} = \frac{u_1b_1+u_2b_2-u_4b_1-u_3b_2}{bl} \tag{6}$$

According to Eq. (6), Eq. (5) can be transformed into

$$b_1\frac{\mathrm{d}^2u_1}{\mathrm{d}t^2}+b_2\frac{\mathrm{d}^2u_2}{\mathrm{d}t^2}-b_2\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}-b_1\frac{\mathrm{d}^2u_4}{\mathrm{d}t^2}=\frac{bl}{I_y}[(u_1-y_1)E_1l_1+(u_2-y_2)E_2l_1-(u_3-y_3)E_3l_2-(u_4-y_4)E_4l_2] \tag{7}$$

Because the vehicle front and rear axles share the same suspension system and the superelevation slope is α[15], we have

$$u_1-u_2=b\tan\alpha,\ \frac{\mathrm{d}^2u_1}{\mathrm{d}t^2}=\frac{\mathrm{d}^2u_2}{\mathrm{d}t^2} \tag{8}$$

$$u_4-u_3=b\tan\alpha,\ \frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}=\frac{\mathrm{d}^2u_4}{\mathrm{d}t^2} \tag{9}$$

From the solution of simultaneous Eqs. (4), (7), (8) and (9), we obtain the motion equations as follows:

$$\begin{aligned}\frac{\mathrm{d}^2u_1}{\mathrm{d}t^2}=&\left[\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_1l_1+2\sqrt{l^2+b^2}E_1I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_1-y_1)+\\&\left[\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_2l_1+2\sqrt{l^2+b^2}E_2I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_1-b\tan\alpha-y_2)+\\&\left[\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_3l_2+2\sqrt{l^2+b^2}E_3I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_3-y_3)+\\&\left[\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_4l_2+2\sqrt{l^2+b^2}E_4I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_3+b\tan\alpha-y_4)\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}=&\left[\frac{2\sqrt{l^2+b^2}E_1I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_1l_1}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_1-y_1)+\\&\left[\frac{2\sqrt{l^2+b^2}E_2I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_2l_1}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_1-b\tan\alpha-y_2)+\\&\left[\frac{2\sqrt{l^2+b^2}E_3I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_3l_2}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_3-y_3)+\\&\left[\frac{2\sqrt{l^2+b^2}E_4I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_4l_2}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}\right](u_3+b\tan\alpha-y_4)\end{aligned} \tag{11}$$

2 General Solution of Motion Equation

Based on the second-order linear vehicle motion equations, we introduce coefficients a_1 to a_8, and Eqs. (10) and (11) can be simplified as

$$\left.\begin{aligned}\frac{\mathrm{d}^2u_1}{\mathrm{d}t^2}&=a_1(u_1-y_1)+a_2(u_1-b\tan\alpha-y_2)+a_3(u_3-y_3)+a_4(u_3+b\tan\alpha-y_4)\\\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}&=a_5(u_1-y_1)+a_6(u_1-b\tan\alpha-y_2)+a_7(u_3-y_3)+a_8(u_3+b\tan\alpha-y_4)\end{aligned}\right\} \tag{12}$$

where

$$a_1=\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_1l_1+2\sqrt{l^2+b^2}E_1I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})},\quad a_2=\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_2l_1+2\sqrt{l^2+b^2}E_2I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}$$

$$a_3=\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_3l_2+2\sqrt{l^2+b^2}E_3I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})},\quad a_4=\frac{(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2})MlE_4l_2+2\sqrt{l^2+b^2}E_4I_y}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}$$

$$a_5=\frac{2\sqrt{l^2+b^2}E_1I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_1l_1}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})},\quad a_6=\frac{2\sqrt{l^2+b^2}E_2I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_2l_1}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}$$

$$a_7=\frac{2\sqrt{l^2+b^2}E_3I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_3l_2}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})},\quad a_8=\frac{2\sqrt{l^2+b^2}E_4I_y-(\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})MlE_4l_2}{MI_y(\sqrt{l_1^2+b_2^2}+\sqrt{l_1^2+b_1^2}+\sqrt{l_2^2+b_1^2}+\sqrt{l_2^2+b_2^2})}$$

Assuming that a differential operator $D=\mathrm{d}/\mathrm{d}t$, Eq. (12) can be simplified as

$$\left.\begin{array}{l}(D^2-a_1-a_2)u_1-(a_3+a_4)u_3+(a_1y_1+a_2b\tan\alpha+a_2y_2+a_3y_3+a_4y_4-a_4b\tan\alpha)=0\\(D^2-a_7-a_8)u_3-(a_5+a_6)u_1+(a_5y_1+a_6b\tan\alpha+a_6y_2+a_7y_3+a_8y_4-a_8b\tan\alpha)=0\end{array}\right\}\tag{13}$$

The solution of simultaneous equations can be obtained as

$$\begin{aligned}&\frac{\mathrm{d}^4u_3}{\mathrm{d}t^4}-(a_1+a_2+a_7+a_8)\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}+[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]u_3+\\&\quad\left(a_5\frac{\mathrm{d}^2y_1}{\mathrm{d}t^2}+a_6\frac{\mathrm{d}^2y_2}{\mathrm{d}t^2}+a_7\frac{\mathrm{d}^2y_3}{\mathrm{d}t^2}+a_8\frac{\mathrm{d}^2y_4}{\mathrm{d}t^2}+a_6b\tan\alpha-a_8b\tan\alpha\right)-\\&\quad(a_1+a_2)(a_5y_1+a_6b\tan\alpha+a_6y_2+a_7y_3+a_8y_4-a_8b\tan\alpha)+\\&\quad(a_5+a_6)(a_1y_1+a_2b\tan\alpha+a_2y_2+a_3y_3+a_4y_4-a_4b\tan\alpha)=0\end{aligned}\tag{14}$$

Actually, Eq. (14) is a fourth-order, non-homogeneous, linear differential equation, and its solution is a component of the general solution of homogeneous equation (15) and the particular solution of non-homogeneous equation (14).

$$\frac{\mathrm{d}^4u_3}{\mathrm{d}t^4}-(a_1+a_2+a_7+a_8)\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}+[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]u_3=0\tag{15}$$

A characteristic equation of Eq. (15) is obtained as

$$\lambda^4-(a_1+a_2+a_7+a_8)\lambda^2+(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)=0\tag{16}$$

The solution of characteristic equation (16) is

$$\lambda_{1,2}=\pm\sqrt{\frac{(a_1+a_2+a_7+a_8)+\sqrt{(a_1+a_2+a_7+a_8)^2-4[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]}}{2}}$$

$$\lambda_{3,4}=\pm\sqrt{\frac{(a_1+a_2+a_7+a_8)-\sqrt{(a_1+a_2+a_7+a_8)^2-4[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]}}{2}}$$

The general solution of Eq. (14) can be obtained as

$$u_3=C_1\mathrm{e}^{\lambda_1t}+C_2\mathrm{e}^{\lambda_2t}+C_3\mathrm{e}^{\lambda_3t}+C_4\mathrm{e}^{\lambda_4t}+u^*$$

where u^* is the particular solution of Eq. (14).

3 Motion Equation and Solution

There is a slope connection between the bridge and the pavement when the vehicle bumps at approach slabs. In order to satisfy the continuity, it is inferred that the slope is the cosine half-wavelength, which is $y=A\cos(\pi vt/L_\mathrm{p})\ (0\leqslant x\leqslant L_\mathrm{p})$, where $A=h/2$, h is the height difference between the bridge abutment and the pavement, and L_p is the slope length. When we analyze the vehicle bumping, if the bridge abutment elevation is higher than the pavement elevation, and $A>0$, it means that a vehicle moves off the bridge abutment, downhill. Nevertheless, if $A<0$, it indicates that the vehicle heads for the bridge abutment, uphill. The bridge superelevation interior equation is $y=A\ (x\leqslant 0)$ and the pavement superelevation interior equation is $y=-A\ (x\geqslant L_\mathrm{p})$. If the bridge abutment elevation is lower than its adjacent pavement elevation, and $A>0$, it means that the vehicle heads for the bridge abutment, downhill, Nevertheless, if $A<0$, it indicates that the vehicle moves off the bridge abutment, uphill. The bridge superelevation interior equation is $y=-A(x\geqslant L_\mathrm{p})$ and the pavement superelevation interior equation is $y=A(x\leqslant 0)$. Therefore, the stage equations are founded and listed as follows according to the relative positions of the four wheels.

1) The first stage: The vehicle front wheels head for the slope and the rear wheels remain on bridge abutment.

$$y_1=A\cos\frac{\pi vt}{L_\mathrm{p}}+b\tan\alpha,\quad y_2=A\cos\frac{\pi vt}{L_\mathrm{p}}\qquad 0\leqslant t\leqslant\frac{l}{v}\tag{17}$$

$$y_3=A,\quad y_4=A+b\tan\alpha\qquad 0\leqslant t\leqslant\frac{l}{v}\tag{18}$$

Substituting Eqs. (17) and (18) into Eq. (14), we have

$$\frac{d^4u_3}{dt^4}-(a_1+a_2+a_7+a_8)\frac{d^2u_3}{dt^2}+[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]u_3+$$
$$\left[-a_5A\frac{\pi^2v^2}{L_p^2}\cos\frac{\pi v}{L_p}t-a_6A\frac{\pi^2v^2}{L_p^2}\cos\frac{\pi v}{L_p}t+a_6b\tan\alpha-a_8b\tan\alpha\right]-$$
$$(a_1+a_2)\left[a_5\left(A\cos\frac{\pi v}{L_p}t+b\tan\alpha\right)+a_6b\tan\alpha+a_6A\cos\frac{\pi v}{L_p}t+a_7A+a_8(A+b\tan\alpha)-a_8b\tan\alpha\right]+$$
$$(a_5+a_6)\left[a_1\left(A\cos\frac{\pi v}{L_p}t+b\tan\alpha\right)+a_2b\tan\alpha+a_2A\cos\frac{\pi v}{L_p}t+a_3A+a_4(A+b\tan\alpha)-a_4b\tan\alpha\right]=0 \quad (19)$$

To simplify Eq. (19), we assume that

$$B_1=a_1+a_2+a_7+a_8,\ \frac{\pi v}{L_p}=k,\ B_2=(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)$$
$$B_7=(a_5+a_6)k^2A,\ B_8=(a_3+a_4)(a_5+a_6)A-(a_1+a_2)(a_7+a_8)A+a_6b\tan\alpha-a_8b\tan\alpha$$

Namely, $\frac{d^4u_3}{dt^4}-B_1\frac{d^2u_3}{dt^2}+B_2u_3-B_7\cos kt+B_8=0$, and the particular solution is

$$u^*=\frac{4\sqrt{B_1^2-4B_2}(B_7B_1^2\cos kt-4B_8B_2-4B_8k^4-4B_8B_1k^2)-4B_7\cos kt(B_1^2-4B_2)^{3/2}}{\sqrt{B_1^2-4B_2}(B_1-\sqrt{B_1^2-4B_2})(B_1-\sqrt{B_1^2-4B_2}+2k^2)(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)} \quad (20)$$

2) The second stage: The vehicle rear wheels move onto the approach slab and front wheels remain on the slope.

$$y_1=A\cos\frac{\pi vt}{L_p}+b\tan\alpha,\ y_2=A\cos\frac{\pi vt}{L_p}\qquad \frac{l}{v}\leqslant t\leqslant\frac{L_p}{v} \quad (21)$$

$$y_3=A\cos\frac{\pi(vt-l)}{L_p},\ y_4=A\cos\frac{\pi(vt-l)}{L_p}+b\tan\alpha\qquad \frac{l}{v}\leqslant t\leqslant\frac{L_p}{v} \quad (22)$$

Substituting Eqs. (21) and (22) into Eq. (14), we then use the same method to solve the differential equations.

$$\frac{d^4u_3}{dt^4}-(a_1+a_2+a_7+a_8)\frac{d^2u_3}{dt^2}+[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]u_3-$$
$$\left[(a_5+a_6)A\left(\frac{\pi v}{L_p}\right)^2\cos\frac{\pi vt}{L_p}+(a_7+a_8)A\left(\frac{\pi v}{L_p}\right)^2\cos\frac{\pi(vt-l)}{L_p}-a_6b\tan\alpha+a_8b\tan\alpha\right]+$$
$$[(a_3+a_4)(a_5+a_6)-(a_1+a_2)(a_7+a_8)]A\cos\frac{\pi(vt-l)}{L_p}=0 \quad (23)$$

To simplify Eq. (23), we assume that

$$\frac{\pi v}{L_p}=k,\ B_9=-(a_5+a_6)k^2,\ B_{11}=a_6b\tan\alpha-a_8b\tan\alpha$$
$$B_{10}=-[(a_3+a_4)(a_5+a_6)-(a_1+a_2)(a_7+a_8)-(a_7+a_8)k^2]A$$

Then, $\frac{d^4u_3}{dt^4}-B_1\frac{d^2u_3}{dt^2}+B_2u_3+B_9A\cos kt-B_{10}\cos\left(kt-\frac{\pi l}{L_p}\right)+B_{11}=0$. Applying the Maple software, the particular solution is

$$u^*=\frac{1}{\sqrt{B_1^2-4B_2}(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}\cdot$$
$$2\left[\int-\frac{\exp\left(\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\left[B_9A\cos kt-B_{10}\cos kt\cos\frac{\pi l}{L_p}-B_{10}\sin kt\sin\frac{\pi l}{L_p}+B_{11}\right]}{\sqrt{B_1^2-4B_2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}}dt\cdot\right.$$
$$\exp\left(-\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)(B_1^3+\sqrt{B_1^2-4B_2}B_1^2+\sqrt{B_1^2-4B_2}k^2B_1-4B_2B_1-2B_2\sqrt{B_1^2-4B_2}-$$
$$4B_2k^2+k^2B_1^2)+B_{11}B_1+B_{11}\sqrt{B_1^2-4B_2}+2B_{11}k^2+\left(B_9A\cos kt-B_{10}\cos\frac{-ktL_p+\pi l}{L_p}\right)(B_1+\sqrt{B_1^2-4B_2})+$$

$$\int \frac{\exp\left(-\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\left(B_9A\cos kt - B_{10}\cos kt\cos\frac{\pi l}{L_{\mathrm{p}}} - B_{10}\sin kt\sin\frac{\pi l}{L_{\mathrm{p}}} + B_{11}\right)}{\sqrt{B_1^2-4B_2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}}\mathrm{d}t\cdot$$
$$\exp\left(\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\left(B_1^3-4B_2B_1-2B_2\sqrt{B_1^2-4B_2}+\sqrt{B_1^2-4B_2}k^2B_1-4B_2k^2+k^2B_1^2\right)\Big] \tag{24}$$

3) The third stage: The vehicle front wheels move off the approach slab and the rear wheels remain on the approach slab.

$$y_1 = -A + b\tan\alpha,\ y_2 = -A \qquad \frac{L_{\mathrm{p}}}{v}\leqslant t\leqslant\frac{L_{\mathrm{p}}+l}{v} \tag{25}$$

$$y_3 = A\cos\frac{\pi(vt-l)}{L_{\mathrm{p}}},\ y_4 = A\cos\frac{\pi(vt-l)}{L_{\mathrm{p}}}+b\tan\alpha \qquad \frac{L_p}{v}\leqslant t\leqslant\frac{L_p+l}{v} \tag{26}$$

Substituting Eqs. (25) and (26) into Eq. (14), we use the same method to solve differential equations.

$$\frac{\mathrm{d}^4u_3}{\mathrm{d}t^4}-(a_1+a_2+a_7+a_8)\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}+[(a_7+a_8)(a_1+a_2)-(a_5+a_6)(a_3+a_4)]u_3+$$
$$[(a_3+a_4)(a_5+a_6)-(a_1+a_2)(a_7+a_8)-(a_7+a_8)k^2]A\cos\frac{\pi(vt-l)}{L_{\mathrm{p}}}+(a_6-a_8)b\tan\alpha=0 \tag{27}$$

Namely, $\frac{\mathrm{d}^4u_3}{\mathrm{d}t^4}-B_1\frac{\mathrm{d}^2u_3}{\mathrm{d}t^2}+B_2u_3-B_{10}\cos\left(kt-\frac{\pi l}{L_p}\right)+B_{11}=0$, and the particular solution is that

$$u^* = \frac{1}{\sqrt{B_1^2-4B_2}\,(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}\cdot$$
$$2\Big[\int -\frac{\exp\left(\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\left(B_{10}\cos kt\cos\frac{\pi l}{L_{\mathrm{p}}}+B_{10}\sin kt\sin\frac{\pi l}{L_{\mathrm{p}}}-B_{11}\right)}{\sqrt{B_1^2-4B_2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}}\mathrm{d}t\cdot$$
$$\exp\left(-\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\Big(\sqrt{B_1^2-4B_2}B_1^2+B_1^3-4B_2B_1-2B_2\sqrt{B_1^2-4B_2}+\sqrt{B_1^2-4B_2}k^2B_1+$$
$$k^2B_1^2-4B_2k^2)-B_{10}\cos\frac{ktL_{\mathrm{p}}-\pi l}{L_{\mathrm{p}}}\left(B_1+\sqrt{B_1^2-4B_2}\right)+B_{11}B_1+B_{11}\sqrt{B_1^2-4B_2}+2B_{11}k^2\Big)+$$
$$\int\frac{\exp\left(-\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\left(B_{10}\cos kt\cos\frac{\pi l}{L_{\mathrm{p}}}+B_{10}\sin kt\sin\frac{\pi l}{L_{\mathrm{p}}}-B_{11}\right)}{\sqrt{B_1^2-4B_2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}}\mathrm{d}t\cdot$$
$$\exp\left(\frac{1}{2}\sqrt{2B_1-2\sqrt{B_1^2-4B_2}}\,t\right)\left(B_1^3-4B_2B_1-2B_2\sqrt{B_1^2-4B_2}+\sqrt{B_1^2-4B_2}k^2B_1+k^2B_1^2-4B_2k^2\right)\Big] \tag{28}$$

Undetermined coefficients in the general solution of the dynamic motion equations are determined by the boundary conditions. The following discusses how to calculate these coefficients according to various boundary conditions. First, at the first stage, there are three boundary conditions, including $t=0$, $u_1=0$, $u_1'=0$, $u_1''=0$. Thereby, its equations are shown as follows:

$$\frac{4\sqrt{B_1^2-4B_2}\,[B_7B_1^2-4B_8B_2-4B_8k^4-4B_8B_1k^2]-4B_7(B_1^2-4B_2)^{3/2}}{\sqrt{B_1^2-4B_2}\,(B_1-\sqrt{B_1^2-4B_2})(B_1-\sqrt{B_1^2-4B_2}+2k^2)(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}+$$
$$C_1+C_2+C_3+C_4=0$$
$$C_1\lambda_1+C_2\lambda_2+C_3\lambda_3+C_4\lambda_4=0$$
$$\frac{-4\sqrt{B_1^2-4B_2}\,B_7B_1^2k^2+4B_7k^2(B_1^2-4B_2)^{3/2}}{\sqrt{B_1^2-4B_2}\,(B_1-\sqrt{B_1^2-4B_2})(B_1-\sqrt{B_1^2-4B_2}+2k^2)(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}+$$
$$C_1\lambda_1^2+C_2\lambda_2^2+C_3\lambda_3^2+C_4\lambda_4^2=0$$

After analyzing the solution of the characteristic equation and the features of λ_1, λ_2, λ_3 and λ_4, we find that $C_1=C_2$, $C_3=C_4$. Therefore, the undetermined coefficients C_1, C_2, C_3, C_4 in general solution are obtained. Simultaneously, the coefficients in the formula are reduced and the results are shown as follows:

$$D_1 = -\frac{4\sqrt{B_1^2-4B_2}[B_7B_1^2-4B_8B_2-4B_8k^4-4B_8B_1k^2]-4B_7(B_1^2-4B_2)^{3/2}}{\sqrt{B_1^2-4B_2}(B_1-\sqrt{B_1^2-4B_2})(B_1-\sqrt{B_1^2-4B_2}+2k^2)(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}$$

$$D_2 = \frac{-4\sqrt{B_1^2-4B_2}B_7B_1^2k^2+4B_7k^2(B_1^2-4B_2)^{3/2}}{\sqrt{B_1^2-4B_2}(B_1-\sqrt{B_1^2-4B_2})(B_1-\sqrt{B_1^2-4B_2}+2k^2)(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}$$

Then

$$C_1 = C_2 = \frac{(\lambda_3^2+\lambda_4^2)D_1+2D_2}{2(\lambda_3^2+\lambda_4^2-\lambda_1^2-\lambda_2^2)},\quad C_3 = C_4 = \frac{(\lambda_1^2+\lambda_2^2)D_1+2D_2}{2(\lambda_1^2+\lambda_2^2-\lambda_3^2-\lambda_4^2)}$$

The vibration equation of the one-side rear wheel at the first stage is

$$u_3 = C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}+C_3e^{\lambda_3 t}+C_4e^{\lambda_4 t}+ \frac{4\sqrt{B_1^2-4B_2}[B_7B_1^2\cos kt-4B_8B_2-4B_8k^4-4B_8B_1k^2]-4B_7\cos(kt)(B_1^2-4B_2)^{3/2}}{\sqrt{B_1^2-4B_2}(B_1-\sqrt{B_1^2-4B_2})(B_1-\sqrt{B_1^2-4B_2}+2k^2)(B_1+\sqrt{B_1^2-4B_2})(B_1+\sqrt{B_1^2-4B_2}+2k^2)}$$

4 Example Calculation

A Jie-fang CA140 vehicle model is introduced to carry out the example analysis and its parameters are shown as follows: $M=9\,415$ kg; $E_1=E_2=1\,500$ kN/m; $E_3=E_4=2\,060$ kN/m; $I_y=5\,200$ kN · m^2; $l=4$ m; $l_1=2.7$ m; $l_2=1.3$ m; $b=1.8$ m; $b_1=b_2=0.9$ m; $h=0.2$; $\alpha=2\%$; $v=25$ m/s; $L_p=5$ m; $F_3=(u_3-y_3)E_3$.

Calculating the parameters respectively, we obtain $a_1=a_2=2.160\,6$; $a_3=a_4=1.541\,1$; $a_5=a_6=-0.954\,8$; $a_7=a_8=-0.518\,9$; $B_1=7.403\,3$; $B_2=1.400\,7$; $B_3=-1.067\times10^3$; $B_7=-47.116\,9$; $B_8=-0.155\,8$; $B_9=471.168\,5$; $B_{10}=-25.468\,3$; $B_{11}=-0.015\,7$; $D_1=0.110\,5$; $D_2=0.185\,4$; $C_1=C_2=-21.986\,7$; $C_3=C_4=22.041\,9$.

Then, the equivalently vertical displacement of the vehicle rear axis spring is analyzed by the vehicle motion equations. The Matlab software is used to calculate the program, and the results are shown in Fig. 3. Furthermore, the equivalent loads on the pavement impacted by the vehicle rear wheels at approach slabs are analyzed (see Fig. 4).

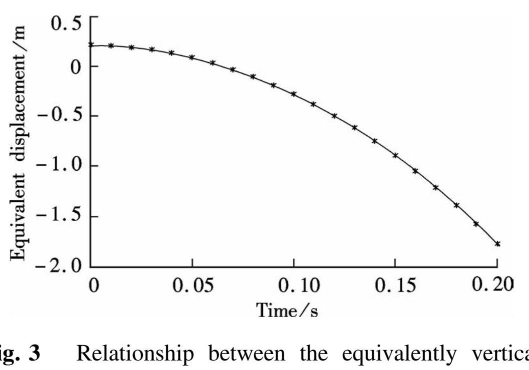

Fig. 3 Relationship between the equivalently vertical displacements of vehicle rear axle spring and time ($h=0.2$ m)

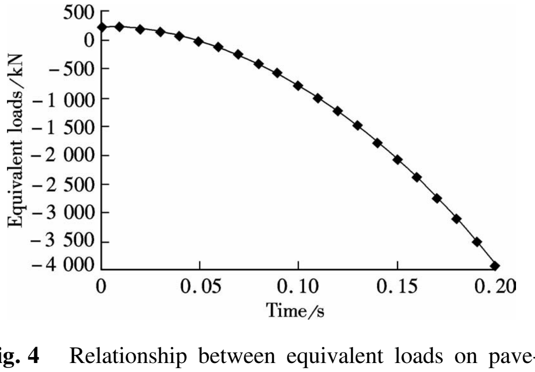

Fig. 4 Relationship between equivalent loads on pavement impacted by vehicle rear wheels at approach slabs and time ($h=0.2$ m)

5 Conclusions

Based on the calculation results, the following conclusions are summarized:

1) When a vehicle is running and bumping at the approach slab, a four wheeled bi-axle vehicle motion model is developed, assuming that the effects of damping and vibration reduction are negligible. Then, according to different positions of the front/rear wheels, the dynamic vibration equations at different stages are summarized.

2) According to the varying elevation and displacement of the front/rear wheels, the vehicle dynamic vibration equation of the rear wheels at the first stage is determined by the initial conditions of the boundary. With regard to the dynamic motion equation of the rear wheels at the second stage, the equation is established by the spring displacement and the vibration speed consecutiveness of the vehicle at the first stage. Meanwhile, using the characteristic equation solutions and features of λ_1, λ_2, λ_3 and λ_4, the undetermined coefficients of the differential equations are established. By the same token, when the height of the bridge deck is higher than the pavement, the dynamic vibration equations of

the second and third stages can be obtained. Furthermore, when the height of the bridge deck is lower than that of the pavement, its equation also can be obtained. Figs. 3 and 4 show that the rear wheels appear to apply negative pressure at the initial stage and the rear wheels do not land on the bridge deck at this short moment. It is dangerous for driving.

3) Considering that the post-construction settlement may appear after the highway is put into operation and driving comfort can be diminished, it is advised that the pavement and bridge deck be reinforced and enhanced in the highway design.

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公路弯道处桥头跳车的路面力学计算分析

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摘要:为了分析汽车行驶在公路弯道桥头过渡段处时发生的桥头跳车现象对路面受力的影响,建立了四轮双轴载重车辆动力运动模型. 忽略汽车行驶过程中的一切阻尼及减振作用的影响,得出公路桥头跳车处的车辆动力运动方程. 根据车轮在桥头位置阶段的变化,充分考虑其边界受力条件,按位置阶段变化总结得出车辆动力运动方程的解析解. 结合工程实例计算,分析和研究得出汽车在高等级公路弯道路桥过渡段处经历桥头跳车过程中后轮对路面产生的荷载压力方程,此方程可推广应用到复杂应力对路面的影响. 结果显示桥头搭板处容易出现应力过大、集中现象,需要进行补强设计.

关键词:道路工程;力学分析;桥头搭板;车辆模型;运动方程

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