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The NBEE and NWEE classes of lifetime distributions and their properties

Wang Guanjun

(Department of Mathematics, Southeast University, Nanjing 211189, China)

Abstract: A class of lifetime distributions, new better than equilibrium in expectation (NBEE), and its dual, new worse than equilibrium in expectation (NWEE), are studied based on the comparison of the expectations of lifetime X and its equilibrium X_e. The relationships of the NBEE (NWEE) and other lifetime distribution classes are discussed. It is proved that the NBEE is very large, and increasing failure rate (IFR), new better than used (NBU) and the L class are its subclasses. The closure properties under two kinds of reliability operations, namely, convolution and mixture, are investigated. Furthermore, a Poisson shock model and a special cumulative model are also studied, in which the necessary and sufficient conditions for the NBEE (NWEE) lifetime distribution of the systems are established. In the homogenous Poisson shock model, the system lifetime belongs to NBEE(NWEE) if and only if the corresponding discrete failure distribution belongs to the discrete NBEE(NWEE). While in the cumulative model, the system has an NBEE lifetime if and only if the stochastic threshold of accumulated damage is NBEE.

Key words: lifetime distribution; survival function; closure property; new better than equilibrium in expectation (NBEE); equilibrium distribution; shock model; cumulative damage model

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In performing reliability analysis, many classes of lifetime distribution have been studied, such as increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), new better than used in expectation (NBUE), decreasing mean residual lifetime (DMRL), harmonic new better than used (HNBUE), the L class, and their duals (see Refs. [1 - 2]). Many classes can be described by their residual lifetimes X_t and equilibrium lifetimes X_e under kinds of stochastic ordering. About stochastic ordering, we refer to Refs. [2 - 4]. Some well-known descriptions of these classes are listed as follows:

$$X_t \downarrow \text{st} \Leftrightarrow F \in \text{IFR},\ X_t \leqslant_{\text{st}} X \Leftrightarrow F \in \text{NBU}$$
$$\text{E}X_t \leqslant \text{E}X \Leftrightarrow F \in \text{NBUE},\ X_t \downarrow \text{c} \Leftrightarrow F \in \text{DMRL}$$
$$X_e \leqslant_{\text{st}} X \Leftrightarrow F \in \text{NBUE},\ F_e(t) \in \text{IFR} \Leftrightarrow F \in \text{DMRL}$$
$$X_t \leqslant_{\text{c}} X \Leftrightarrow F \in \text{NBUC},\ X_e \leqslant_{\text{st}} Y \Leftrightarrow F \in \text{HNBUE}$$

where Y yields an exponential distribution with the same expectation as X, and the notations $\downarrow$ st, $\leqslant_{\text{st}}$, $\downarrow$ c, $\leqslant_{\text{c}}$ represent stochastic decreasing, stochastic smaller, stochastic decreasing in concave ordering and stochastic smaller under concave ordering, respectively. Recently, Abouammoh et al. [5] introduced two new classes, namely, new better than renewal used in expectation (NBRUE) and harmonic new better than renewal used in expectation (HNBRUE), based on stochastic comparisons of X_e and X_t, which continues the research along this direction. Some more discussions on this topic can be seen in Refs. [6 - 11].

The $L(\bar{L})$ class defined by Klefsjö[12] consists of lifetime distributions for which

$$\int_0^\infty \mathrm{e}^{-st}\bar{F}(t)\,\mathrm{d}t \geqslant (\leqslant)\ \frac{\mu}{1+\mu s} \tag{1}$$

In this paper, we show that $F \in L \Leftrightarrow E[\mathrm{e}^{-sX}] \leqslant E[\mathrm{e}^{-sY}] \Leftrightarrow E[\mathrm{e}^{-sX}] \leqslant E[\mathrm{e}^{-sX_e}]$, where Y is an exponential variable with expectation μ. We will study a new class of lifetime distributions, namely, new better than equilibrium in expectation (NBEE) and new worse than equilibrium in expectation (NWEE), based on the comparison of the expectation of X and X_e; i. e., $F \in$ NBEE(NWEE) if $E[X_e] \geqslant (\leqslant)$ EX. Furthermore, we will prove that NBEE (NWEE) is larger than $L(\bar{L})$.

1 NBEE and Its Relationships

Let F be the distribution function of lifetime X with mean μ and the second moment $E[X^2]=\mu_2$, and its survival function is $\bar{F}(t)=1-F(t)$. X_e denotes the equilibrium lifetime of X with distribution $F_e(t)=\dfrac{1}{\mu}\displaystyle\int_0^t \bar{F}(u)\,\mathrm{d}u$, and μ_e represents its expectation.

Definition 1 The lifetime distribution F is said to be NBEE if $\mu_e \leqslant \mu$. If the reversed inequality holds, then we say that F belongs to NWEE.

Lemma 1 If X is a lifetime with a finite second moment μ_2, then

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Biography: Wang Guanjun (1974—), male, doctor, associate professor, wgjmath@gmail.com.

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$$\mu_e = E[X_e] = \frac{\mu_2}{2\mu} \tag{2}$$

Proof It is easy to be obtained by using the definition of equilibrium distribution.

Lemma 2 $F \in$ NBEE iff $\mu_2 \leqslant 2\mu^2$ or $\sigma^2 \leqslant \mu^2$, where σ^2 is the variance of X; iff means only and if only.

Proof It follows from Lemma 1 and Definition 1.

Definition 2[1] F is a lifetime distribution, if for all $x \geqslant 0, y \geqslant 0$,

$$\bar{F}(x+y) \leqslant (\geqslant) \bar{F}(x)\bar{F}(y) \tag{3}$$

then we say that F belongs to the NBU(NWU) class.

Theorem 1 If $F \in$ NBU(NWU), then $F \in$ NBEE (NWEE).

Proof By integrating both sides of inequality (3), we have

$$\frac{1}{2}\mu_2 \leqslant (\geqslant) \mu^2$$

which means $F \in$ NBEE(NWEE).

Definition 3[1] F is a lifetime distribution with finite mean μ. If for all $t > 0$,

$$\frac{1}{\mu}\int_t^\infty \bar{F}(u)\,\mathrm{d}u \leqslant \bar{F}(t) \tag{4}$$

then we say that F belongs to the NBUE(NWUE) class.

Remark 1 From Definition 3, we can see that $F \in$ NBUE iff $X_e \leqslant_{st} X$. Therefore, $F \in \text{NBUE} \Rightarrow F \in$ NBEE.

Definition 4[5] X is said to be NBRUE, iff

$$\int_x^\infty \left(\int_u^\infty \bar{F}(\omega)\,\mathrm{d}\omega\right)\mathrm{d}u \leqslant \mu\int_x^\infty \bar{F}(u)\,\mathrm{d}u \tag{5}$$

Remark 2 From Definition 4, it is easy to check that $F \in$ NBRUE iff $E[(X_e)_t] \leqslant EX$, for all $t > 0$. Let $t = 0$, and we can see that $F \in$ NBEE.

Definition 5[13] X is a lifetime with distribution F and finite mean μ. If

$$\int_0^\infty \mathrm{e}^{-st}\bar{F}(t)\,\mathrm{d}t \geqslant (\leqslant) \frac{\mu}{1+\mu s} \tag{6}$$

then we say that $F \in L (F \in \bar{L})$.

Remark 3 If Y is an exponential random variable with expectation μ, then $F \in L$ iff

$$E[\mathrm{e}^{-sX}] \leqslant E[\mathrm{e}^{-sY}] = \frac{1}{1+\mu s} \qquad s \geqslant 0$$

which means that X is smaller than Y under the Laplace transform.

Theorem 2 Let $L(s) = \int_0^\infty \mathrm{e}^{-st}\bar{F}(t)\,\mathrm{d}t$, $L_e(s) = \int_0^\infty \mathrm{e}^{-st}\bar{F}_e(t)\,\mathrm{d}t$ be the Laplace transform of $\bar{F}(t)$ and $\bar{F}_e(t)$, respectively; then $F \in L$ iff

$$L(s) \geqslant L_e(s) \qquad s > 0 \tag{7}$$

Proof By using the definition of equilibrium distribution, we have

$$L_e(s) = \frac{1}{\mu s}\int_0^\infty (1-\mathrm{e}^{-su})\bar{F}(u)\,\mathrm{d}u = \frac{1}{\mu s}(\mu - L(s)) \tag{8}$$

If $L(s) \geqslant L_e(s)$, then we obtain $L(s) \geqslant \frac{\mu}{1+\mu s}$, and this proves $F \in L$.

On the other hand, if $F \in L$, by Definition 5, $L(s) \geqslant \frac{\mu}{1+\mu s}$ holds. Then from Eq. (8), we obtain that $L_e(s) \leqslant \frac{\mu}{1+\mu s}$. This completes the proof.

Remark 4 $L(s) \geqslant L_e(s)$, $s > 0$ is equivalent to $E[\mathrm{e}^{-sX}] \leqslant E[\mathrm{e}^{-sX_e}]$, which implies that $F \in L$ iff X is smaller than X_e under the Laplace transform.

Theorem 3 If $F \in L$, then $F \in$ NBEE.

Proof By Theorem 2, if $F \in L$ then

$$\int_0^\infty \mathrm{e}^{-st}\bar{F}(t)\,\mathrm{d}t \geqslant \int_0^\infty \mathrm{e}^{-st}\bar{F}_e(t)\,\mathrm{d}t \qquad s \geqslant 0$$

Let $s = 0$, we can obtain $\mu \geqslant \mu_e$.

From Theorem 3, it can be seen that the NBEE (NWEE) class contains the $L(\bar{L})$ class. Furthermore, the following examples indicate that the NBEE(NWEE) class is strictly larger than the $L(\bar{L})$ class.

Example 1 Let $\bar{F}(t)$ be the survival function of a lifetime X,

$$\bar{F}(t) = \begin{cases} \frac{2}{3} & 0 \leqslant t < 1 \\ \frac{1}{3} & 1 \leqslant t < 2 \\ 0 & t \geqslant 2 \end{cases}$$

By a straightforward calculation, one can see that F is in NBEE, but it does not belong to $\bar{L}$.

Example 2 Let $\bar{F}(t)$ be another survival function,

$$\bar{F}(t) = \begin{cases} 1 & 0 \leqslant t < 1 \\ \frac{2}{3} & 1 \leqslant t < 2 \\ \frac{1}{3} & 2 \leqslant t < 30 \\ 0 & t \geqslant 30 \end{cases}$$

It is not difficult to check that F is in NWEE but is not in $\bar{L}$.

2 Preservation of NBEE and NWEE Classes Under Reliability Operations

Many well-known classes such as IFR, IFRA, NBU, NBUE, DMRL, the L class and their duals are known to be closed or not under three reliability operations: formation of coherent structure, convolution, and mixture. In this section, we investigate the preservation property of NBEE and NWEE under these reliability operations. We will prove that NBEE is closed under convolution and NWEE is closed for mixture operations.

Let lifetimes X_1, X_2 be independent with distribution functions F_1 and F_2, respectively. Assume that the first and second moments of X_1, X_2 are finite. F denotes the convolution of F_1 and F_2, and then F is the distribution function of $X = X_1 + X_2$. We have the following theorem.

Theorem 4 If $F_1, F_2 \in$ NBEE, then $F \in$ NBEE. That is, NBEE is closed under convolution.

Proof For $F_1, F_2 \in$ NBEE, we have

$$D(X) = D(X_1) + D(X_2) \leqslant [E(X_1)]^2 + [E(X_2)]^2 \leqslant [E(X)]^2$$

By employing Lemma 2, $F \in$ NBEE holds.

This result can also be derived from Theorem 3.1 of Ref. [6].

Theorem 5 NWEE is closed under mixture.

Proof Let $\bar{F}(t) = \int_{\Omega} \bar{F}_{\alpha}(t) \mathrm{d}G(\alpha)$ be the reliability function of the mixture of a class of distributions $\{F_{\alpha}(t)\}$, where G is the distribution function of α, and $\alpha \in \Omega$. The first and second moments of $\bar{F}(t)$ can be obtained as

$$\mu = \int_0^{\infty} \bar{F}(t)\,\mathrm{d}t = \int_{\Omega} \mu_{\alpha} \mathrm{d}G(\alpha) \tag{9}$$

$$\mu_2 = \int_0^{\infty} 2t\bar{F}(t)\,\mathrm{d}t = \int_{\Omega}\int_0^{\infty} 2t\bar{F}_{\alpha}(t)\,\mathrm{d}t\mathrm{d}G(\alpha) \tag{10}$$

If $\bar{F}_{\alpha}(t) \in$ NWEE, then we have

$$\int_{\Omega}\int_0^{\infty} 2t\bar{F}_{\alpha}(t)\,\mathrm{d}t\mathrm{d}G(\alpha) \geqslant 2\int_{\Omega} \mu_{\alpha}^2\,\mathrm{d}G(\alpha) \geqslant 2\Big[\int_{\Omega} \mu_{\alpha}\,\mathrm{d}G(\alpha)\Big]^2$$

where the last inequality is obtained by using Jensen's inequality. From Lemma 2, the preservation property of NWEE under the mixture operation is proved.

3 Properties Under Shock Model

Assume that a device is subjected to shocks governed by a birth process with birth intensities $\lambda_k, k = 0, 1, 2, \ldots$. Let $N(t)$ be the number of shocks in time interval $(0, t]$. The k-th shock arrives at time $T_k, k = 0, 1, 2, \ldots$, where $T_0 \equiv 0$. Let $U_{k+1} = T_{k+1} - T_k$, and then $U_k, k = 0, 1, 2, \ldots$, are mutually independent and U_k yields an exponential distribution with mean $1/\lambda_{k-1}$; i. e.,

$$P(U_k > t) = \mathrm{e}^{-\lambda_{k-1} t} \qquad t > 0 \tag{11}$$

In this section, we always assume that $\sum_{k=0}^{\infty} \lambda_k^{-1} = \infty$. Let $a_k(t) = P(N(t) = k), k = 0, 1, \ldots$. If $\bar{P}_k$ is the probability of the device surviving k shocks, where $1 = \bar{P}_0 \geqslant \bar{P}_1 \geqslant \bar{P}_2 \geqslant \ldots$, and let T be the lifetime of the device under shocks, then the survival probability of the device until time t is

$$\bar{H}(t) = P(T > t) = \sum_{k=0}^{\infty} a_k(t)\bar{P}_k \tag{12}$$

To discuss the property of the lifetime distribution of the system, we first introduce the definition of discrete NBEE (DNBEE) in the following.

Let Z be the number of shocks that the device survives, and then $\bar{P}_k = P(Z > k)$. Denote ν as the expectation of Z, and then $\nu = \sum_{k=0}^{\infty} \bar{P}_k$. Define the equilibrium distribution of Z by $\bar{G}(k) = \frac{1}{\nu}\sum_{j=0}^{\infty} \bar{P}_{k+j}$, and $\nu_e = \sum_{k=0}^{\infty} \bar{G}(k)$ denotes its mean.

Definition 6 If $\nu_e \leqslant \nu$, then $\{\bar{P}_k, k = 0, 1, 2, \ldots\}$ belongs to the discrete NBEE class (DNBEE). If $\nu_e \geqslant \nu$, then $\{\bar{P}_k, k = 0, 1, 2, \ldots\} \in$ DNWEE.

Lemma 3 $\{\bar{P}_k, k = 0, 1, 2, \ldots\} \in$ DNBEE(DNWEE) iff

$$\sum_{k=0}^{\infty} k\bar{P}_k \leqslant (\geqslant)\, \nu(\nu - 1) \tag{13}$$

Proof Computing ν_e, we have

$$\nu_e = \sum_{k=0}^{\infty} \bar{G}(k) = \frac{1}{\nu}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \bar{P}_{k+j} = \frac{1}{\nu}\sum_{i=0}^{\infty}\sum_{j=0}^{i} \bar{P}_i = \frac{1}{\nu}\sum_{i=0}^{\infty} i\bar{P}_i + 1$$

By Definition 6, inequality (13) holds if and only if $\{\bar{P}_k, k = 0, 1, 2, \ldots\} \in$ DNBEE(DNWEE).

Lemma 4 Let $\mu = E[T], \mu_2 = E[T^2]$ be the first and second moments of the device's lifetime T. Then

$$\mu = \sum_{k=0}^{\infty} \frac{\bar{P}_k}{\lambda_k} \tag{14}$$

$$\mu_2 = 2\sum_{i=0}^{\infty} \frac{1}{\lambda_i}\sum_{k=i}^{\infty} \frac{\bar{P}_k}{\lambda_k} \tag{15}$$

Proof Recall that $a_k(t)$ is the probability of exactly k shocks generated in $(0, t]$ by the birth process, U_k is the length between the $(k-1)$-th and the k-th shock, T_k is the arrival time of the k-th shock, i. e. $T_k = U_1 + \ldots + U_k$, we have

$$a_k(t) = P(T_k \leqslant t < T_{k+1}) = \bar{\Psi}_{k+1}(t) - \bar{\Psi}_k(t) \tag{16}$$

where $\Psi_k(t)$ is the distribution function of T_k. Integrating both sides of the above equality, we have

$$\int_0^{\infty} a_k(t)\,\mathrm{d}t = E[T_{k+1}] - E[T_k] = E[U_{k+1}] = \frac{1}{\lambda_k} \tag{17}$$

So, Eq. (14) holds.

In the following, we compute μ_2.

$$\mu_2 = \int_0^{\infty} t^2\mathrm{d}H(t) = \int_0^{\infty} 2t\bar{H}(t)\,\mathrm{d}t \tag{18}$$

Notice that

$$\int_0^\infty 2ta_k(t)\,dt = \int_0^\infty 2t(\overline{\Psi}_{k+1}(t) - \overline{\Psi}_k(t))\,dt = E[T_{k+1}^2] - E[T_k^2] = E[2(U_1 + \ldots + U_k)U_{k+1} + U_{k+1}^2] = \frac{2}{\lambda_k}\sum_{i=0}^{k}\frac{1}{\lambda_i} \tag{19}$$

Substituting Eqs. (12) and (19) into Eq. (18), we obtain that

$$\mu_2 = 2\sum_{k=0}^{\infty}\overline{P}_k\frac{1}{\lambda_k}\sum_{i=0}^{k}\frac{1}{\lambda_i} = 2\sum_{i=0}^{\infty}\frac{1}{\lambda_i}\sum_{k=i}^{\infty}\frac{\overline{P}_k}{\lambda_k} \tag{20}$$

The main results are given as follows.

Theorem 6 Assume that $\lambda_k \geqslant \lambda_0 = 1$, $k = 0, 1, 2, \ldots$. If for all $i = 0, 1, 2, \ldots$,

$$\sum_{k=i}^{\infty}\frac{\overline{P}_k}{\lambda_k} \leqslant (\geqslant)\,\overline{P}_i\sum_{k=0}^{\infty}\frac{\overline{P}_k}{\lambda_k} \tag{21}$$

then $H \in$ NBEE(NWEE).

Proof Multiplying $1/\lambda_i$ on both sides of inequation (21), and summing up i from 0 to ∞, we have

$$\frac{1}{2}\mu_2 = \sum_{i=0}^{\infty}\frac{1}{\lambda_i}\sum_{k=i}^{\infty}\frac{\overline{P}_k}{\lambda_k} \leqslant (\geqslant) \sum_{i=0}^{\infty}\frac{\overline{P}_i}{\lambda_i}\sum_{k=0}^{\infty}\frac{\overline{P}_k}{\lambda_k} = \mu^2 \tag{22}$$

Therefore, $H \in$ NBEE(NWEE) holds.

If the birth intensity $\lambda_k = \lambda$ is a constant, then the birth process becomes a homogeneous Poisson process and the reliability function of T becomes

$$\overline{H}(t) = \sum_{k=0}^{\infty}\frac{(\lambda t)^k}{k!}e^{-\lambda t}\overline{P}_k \tag{23}$$

Theorem 7 For the reliability function (23), if $\{\overline{P}_k, k = 0, 1, 2, \ldots\} \in$ DNBEE(DNWEE), then $H \in$ NBEE(NWEE).

Proof First, we calculate the expectation and second moment of lifetime T as follows:

$$\mu = \int_0^\infty \overline{H}(t)\,dt = \sum_{k=0}^{\infty}\overline{P}_k\int_0^\infty\frac{(\lambda t)^k}{k!}e^{-\lambda t}\,dt = \frac{\nu}{\lambda} \tag{24}$$

$$\mu_2 = 2\int_0^\infty t\overline{H}(t)\,dt = 2\sum_{k=0}^{\infty}\overline{P}_k\int_0^\infty t\frac{(\lambda t)^k}{k!}e^{-\lambda t}\,dt = \frac{2}{\lambda^2}\sum_{k=0}^{\infty}(k+1)\overline{P}_k \tag{25}$$

If $\{\overline{P}_k, k = 0, 1, 2, \ldots\} \in$ DNBEE(DNWEE), then by using Lemma 3, we have $\mu_2 \leqslant (\geqslant)\, 2\nu^2/\lambda^2 = 2\mu^2$. So we conclude that $H \in$ NBEE(NWEE), and the proof is completed.

Remark 5 μ and μ_2 can also be derived from Lemma 4 by letting $\lambda_k = \lambda$, $k = 0, 1, \ldots$.

Remark 6 The corresponding shock models were studied by Klefsjö et al.[13-15] for other lifetime distribution classes such as IFR, NBU, NBUE, and the L class etc. Along this direction, we have extended the shock model theory for the NBEE(NWEE) class here.

4 Cumulative Damage Model

In this section, we study a cumulative damage model for the survival probabilities. We suppose that a device is subjected to shocks and every shock causes a random damage ξ_k, $k = 1, 2, \ldots$, which yields an exponential distribution with mean $1/\theta$. We also suppose that ξ_k, $k = 1, 2, \ldots$, are independent and accumulate additively. When the accumulated damage exceeds a critical threshold η, the device fails. Assume that η has distribution function F. Then the survival probabilities are given by

$$\overline{P}_k(\theta) = P(\xi_1 + \xi_2 + \ldots + \xi_k < \eta) = \int_0^\infty \frac{\theta^k x^{k-1}}{(k-1)!}e^{-\theta x}\overline{F}(x)\,dx \qquad k = 1, 2, \ldots \tag{26}$$

$$\overline{P}_0(\theta) = 1 \tag{27}$$

We will prove that $\{\overline{P}_k, k = 0, 1, 2, \ldots\} \in$ DNBEE(DNWEE) iff F belongs to NBEE(NWEE). The corresponding results are proved for other lifetime distribution classes, see Refs. [12, 15].

Theorem 8 The survival probability $\{\overline{P}_k(\theta), k = 0, 1, 2, \ldots\}$ belongs to DNBEE iff F belongs to NBEE, for all $\theta > 0$.

Proof For a given θ, let ν and ν_2 be respectively the first and second moments of discrete distribution $\{\overline{P}_k(\theta), k = 0, 1, 2, \ldots\}$. γ and γ_2 denote the first and second moments of F, respectively. From Eqs. (26) and (27), we can compute that

$$\nu = \sum_{k=0}^{\infty}\overline{P}_k(\theta) = \theta\gamma + 1 \tag{28}$$

and

$$\sum_{k=0}^{\infty}k\overline{P}_k(\theta) = \int_0^\infty\sum_{k=0}^{\infty}k\frac{\theta^k x^{k-1}}{(k-1)!}e^{-\theta x}\overline{F}(x)\,dx = \theta\gamma + \frac{1}{2}\theta^2\gamma_2 \tag{29}$$

If F belongs to NBEE, then

$$\theta\gamma + \frac{1}{2}\theta^2\gamma_2 \leqslant \theta\gamma + (\theta\gamma)^2 = \nu(\nu-1) \tag{30}$$

By Lemma 3, we can see that $\{\overline{P}_k(\theta), k = 0, 1, 2, \ldots\} \in$ NBEE.

On the other hand, if $\{\overline{P}_k(\theta), k = 0, 1, 2, \ldots\} \in$ NBEE, then

$$\sum_{k=0}^{\infty}k\overline{P}_k(\theta) \leqslant \nu(\nu-1) = \theta\gamma + (\theta\gamma)^2 \tag{31}$$

Comparing with Eq. (29), we obtain $\gamma_2 \leqslant 2\gamma^2$, which means that F belongs to NBEE. This completes the proof of Theorem 8.

Remark 7 From the proof of Theorem 8, we can also see that $\{\bar{P}_k(\theta), k=0,1,2,\dots\}$ belongs to DNWEE iff F belongs to NWEE, for all $\theta > 0$.

5 Conclusion

In this paper, we introduce a new class of lifetime distribution named by new better than equilibrium in expectation (NBEE). We study some properties of the class under several reliability operations. The relationships between NBEE(NWEE) and other lifetime distribution classes are also investigated. Similar to other well-known lifetime classes, the NBEE(NWEE) lifetime distribution can also be derived from a certain Poisson shock model and a special cumulative model under some suitable conditions.

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NBEE 和 NWEE 寿命分布类及其性质

王冠军

(东南大学数学系,南京 211189)

摘要:在比较寿命 X 与其平衡寿命 X_e 的数学期望的基础上,研究了一类新的寿命分布类 NBEE 和它的对偶类 NWEE. 讨论了 NBEE(NWEE)与其他寿命分布类的关系,证明了 NBEE 分布类是一种很大的分布类,IFR, NBU, L 类等为它的子类. 分析了 NBEE(NWEE)在卷积、混合等可靠性运算下的封闭性质. 针对泊松冲击模型和累积失效模型,分别给出了系统寿命属于 NBEE(NWEE)类的充要条件. 在齐次泊松冲击模型中,系统寿命属于 NBEE(NWEE)类的充要条件是离散冲击失效分布属于 DNBEE(DNWEE)类. 而在累积失效模型中系统中,累积失效寿命属于 DNBEE 类的充要条件是损失量阈值的分布属于 NBEE 类.

关键词:寿命分布;生存函数;封闭性;NBEE;平衡分布;冲击模型;累积失效模型

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