Journal of Southeast University (English Edition) Vol. 28, No. 3, pp. 287 − 291 Sept. 2012 ISSN 1003—7985

Discrete-time Markov-based dynamic control approach for compressed sampling

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Abstract: To solve the problem that the signal sparsity level is time-varying and not known as a priori in most cases, a signal sparsity level prediction and optimal sampling rate determination scheme is proposed. The discrete-time Markov chain is used to model the signal sparsity level and analyze the transition between different states. According to the current state, the signal sparsity level state in the next sampling period and its probability are predicted. Furthermore, based on the prediction results, a dynamic control approach is proposed to find out the optimal sampling rate with the aim of maximizing the expected reward which considers both the energy consumption and the recovery accuracy. The proposed approach can balance the tradeoff between the energy consumption and the recovery accuracy. Simulation results show that the proposed dynamic control approach can significantly improve the sampling performance compared with the existing approach.

Key words: compressed sampling; signal sparsity level prediction; discrete-time Markov chain

doi: 10.3969/j. issn. 1003 − 7985. 2012. 03. 006

With the rapid development of science and technology, the amount of data that needs to be sampled and processed increases swiftly. This sets higher requirements for data sampling and processing. On the other hand, most samples are thrown away in the process of data compression almost without any loss due to the redundancy in the obtained information. As a result, sampling efficiency can be improved by ignoring the samples that will be thrown away. Motivated by this, Donoho[1] proposed compressed sensing. By taking advantage of data (signal) sparsity in one specific domain, compressed sensing can greatly reduce the sampling rate and recover the original data (signal) with high accuracy.

Compared with the Nyquist sampling theory, compressed sensing allows a lower sampling rate and it has become an emerging subject in signal processing recently. Much work has been done on it recently. In Ref. [2], Tropp et al. designed a new type of data acquisition system to perform compressed sampling, called a random demodulator. In Ref. [3], special sampling and reconstruction techniques were proposed for the SCS-FRI scenario, where the classical finite rate of innovation (FRI) sampling scheme was extended to the sparse common-support scenario (SCS). In Ref. [4], Tropp et al. demonstrated theoretically and empirically that the reliable recovery of a signal in dimension d with m nonzero entries can be achieved by an orthogonal matching pursuit (OMP) algorithm using $O(m\ln d)$ random linear measurements. Candès et al. [5] presented a methodology for how to exactly reconstruct an object (a signal) from highly incomplete frequency information. In Ref. [6], the deterministic construction of binary, bipolar, and ternary compressed sensing matrices was studied. Besides, compressed sensing was applied to a range of applications such as the estimation of sparse multipath channels in many wireless systems[7-8], the random access in energy-efficient underwater sensor networks[9], and so on.

Most of these works focus on either the design of the sampling system or the reconstruction methods based on the assumption that the signal sparsity level is known as a priori. However, in most cases the signal sparsity level is not known before sampling, and it is a variable due to the time-varying property of transmitters. At the same time, both Ref. [1] and Ref. [2] have shown that the number of samples (or the sampling rate) needed by compressed sensing increases with the signal sparsity level and logarithmically with the signal bandwidths. As a result, the prediction of the signal sparsity level for compressed sampling is an essential problem. But to the best of our knowledge, this problem has not been addressed yet. Although the work in Ref. [10] does not need to know the sparsity level of unknown signals, the recovery error should be estimated after acquiring each single sample to determine whether enough samples have been obtained or not. This means that the number of samples is exactly equal to the times that signal recovery and the estimating algorithm has been run. Therefore, the computational complexity is relatively high.

Received 2012-05-20.

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Foundation items: Innovation Funds for Outstanding Graduate Students in School of Information and Communication Engineering in BUPT, the National Natural Science Foundation of China (No. 61001115, 61271182).

Citation: An Chunyan, Ji Hong, Li Yi, et al. Discrete-time Markov-based dynamic control approach for compressed sampling[J]. Journal of Southeast University (English Edition), 2012, 28(3): 287 − 291. [doi: 10.3969/j. issn. 1003 − 7985. 2012. 03. 006]

In this paper, based on the modelling and analysis of the signal sparsity level with the discrete-time Markov chain, we propose a dynamic control approach for compressed sampling. The proposed approach can effectively find out the optimal sampling rate with the aim of maximizing the expected reward. The contributions of our work are as follows:

- The discrete-time Markov chain is used to model and analyze the states of the signal sparsity level, which is more suitable for periodical prediction. Given the current state, the signal sparsity level and its probability in the next sampling period can be predicted by analyzing transition probabilities.
- According to the prediction results of the signal sparsity level, the expected reward which considers both the recovery accuracy and the energy consumption is maximized by choosing the optimal sampling rate.
- The computational complexity of the proposed approach in this paper is much lower than that in Ref. [10] since the signal recovery and estimation algorithm is run only one time in a sampling period.

1 System Model

In this paper, we consider a scenario where the receiver needs to receive band-limited signals which are sparse or locally sparse in a specific domain. This scenario of signal processing is very common in practical systems. Here are some examples. First, for the acoustic sensors, the received musical signal usually lies in a range of 300 to 34 000 Hz but only contains a dominant sinusoid and several harmonic overtones. In other words, the received signal of the acoustic sensors is band-limited and sparse in the frequency domain. Secondly, for the sensing-based cognitive radio networks, the received signal of detection devices can be seen as the one whose carrier frequency is unknown but can lie anywhere in a wide bandwidth.

For simplicity, the signal model which is sparse in the frequency domain is considered as

$$f(t) = \sum_{k=0}^{K} a_k \sin(2\pi f_k t + \theta_k) + n(t) \tag{1}$$

where K represents the total number of frequencies contained in the signal. For each frequency f_k, a_k is its amplitude and θ_k is its phase; $n(t)$ means the additive white Gaussian noise.

Since the highest frequency in the signal is f_K, the sampling rate is at least $2f_K$ to recover the signal without error according to the Nyquist sampling theory. However, for the scenarios mentioned above, since the signal is sparse, recent advances in compressed sensing can be used to reduce the sampling rate, thus to save energy and decrease the requirements on hardware. As mentioned above, the minimum sampling rate of compressed sensing with tolerable recovery accuracy has a close relationship with the signal sparsity level. However, the signal sparsity level is often time-varying and unknown before sampling. For acoustic sensors, not only the number of sound sources but also the dominant sinusoid and overtones of one sound source may change randomly. For spectrum sensing in cognitive radio networks, detection devices have no knowledge on the signal sparsity level before spectrum sensing. Besides, the signal to be detected is time-varying due to the dynamic properties of primary networks. In conclusion, there should be an effective control approach for compressed sensing to predict the signal sparsity level and determine the optimal sampling rate.

2 Dynamic Control Approach for Compressed Sampling Based on Discrete-Time Markov Chain

In this section, the proposed dynamic control approach for compressed sampling will be presented. First, the signal sparsity level is modeled and analyzed by the discrete-time Markov chain; then the sparsity state and its probability in next sampling period are predicted according to both the current state and state transition probabilities. Secondly, based on the prediction results, the optimal sampling rate is evaluated with the aim of maximizing the expected reward which is defined as the function of both recovery accuracy and energy consumption.

2.1 Analysis

The state space of the signal sparsity level can be expressed as $S = \{S \mid S = 0/K,\ 1/K,\ \ldots,\ K/K\}$, which totally contains $K+1$ states. State k/K, $k = 0, 1, \ldots, K$ represents that k bands of the total K spectrum bands are occupied. In a sampling period with length T, the number of frequencies that are newly released or occupied is random. This means that one state can transit to any other state with different probabilities, as shown in Fig. 1. In the rest of this subsection, we will analyze the transition probabilities between any two states.

For state $S = i$, $0/K < i < K/K$, the transition probabilities can be divided into four types.

Type 1 To state $j < i$, the number of the released frequencies is $(i-j)K$ greater than that of the occupied frequencies in this sampling period. The corresponding

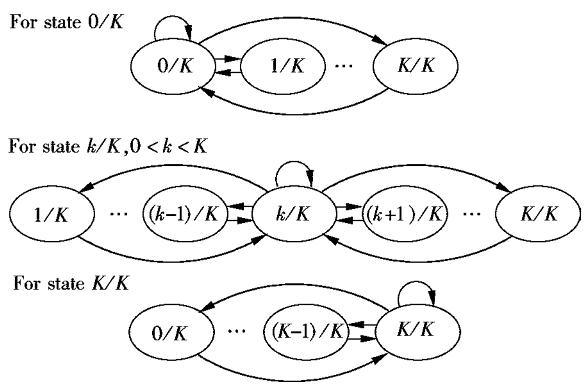

Fig. 1 States transition diagram

transition probability can be described as

$$P(S=j,\ j<i \mid S-1=i) = \sum_{m=(i-j)K}^{iK} P_o(m-(i-j)K)P_r(m) \tag{2}$$

where $P_o(m-(i-j)K)$ means the probability that $m-(i-j)K$ frequencies are newly occupied, while $P_r(m)$ represents the probability that m frequencies are newly released.

Type 2 To state $j=i$, the number of newly occupied frequencies equals that of the newly released frequencies. The corresponding transition probability can be described as

$$P(S=i \mid S-1=i) = \sum_{m=0}^{iK} P_o(m)P_r(m) \tag{3}$$

Type 3 To state $j>i$, $j\neq K/K$, more frequencies are occupied than those are released. The corresponding transition probability can be described as

$$P(S=j,\ j>i,\ j\neq K/K \mid S-1=i) = \sum_{m=0}^{iK} P_o(m+(j-i)K)P_r(m) \tag{4}$$

Type 4 To state $j=K/K$, all the frequencies are occupied. The corresponding transition probability can be described as

$$P(S=K/K \mid S-1=i) = \sum_{m=0}^{iK} P_r(m) \sum_{n=K-iK+m}^{\infty} P_o(m+n) \tag{5}$$

Blocking may happen in this situation since the frequency resources are not enough.

States $S=0/K$ and $S=K/K$ are the two special cases of state $S=i$, $0/K<i<K/K$. This is because state $S=0/K$ can only transit to state $S=j$, $j\geqslant i$ while state $S=K/K$ can only transit to state $S=j$, $j\leqslant i$. So state $S=0/K$ does not have Type 1 transition probability and state $S=K/K$ does not have Type 3 transition probability.

From Eqs. (2) to (5), we can see that the transition probability $P(S=j \mid S-1=i)$ has a close relationship with the probabilities $P_r(m)$ and $P_o(m)$. As a result, we should analyze $P_r(m)$ and $P_o(m)$ before calculating $P(S=j \mid S-1=i)$. Without loss of generality, only one kind of service is considered for analytical and computational simplicity in this paper. The analysis can be extended to multiple service scenarios easily.

Assume that both the arrival and departure of service are the Poisson process with parameters λ and μ. The probability that m frequencies are newly released can be expressed as

$$P_r(m) = \frac{(\mu T)^m e^{-\mu T}}{m!} \qquad m=0,\ 1,\ 2,\ \dots,\ +\infty \tag{6}$$

Similarly, the probability that n frequencies are newly occupied can be expressed as

$$P_o(n) = \frac{(\lambda T)^n e^{-\lambda T}}{n!} \qquad n=0,\ 1,\ 2,\ \dots,\ +\infty \tag{7}$$

In fact, for a specific scenario, the statistical properties will not change in a long period of time; so do the transition probabilities between any two states. As a result, we can save the transition probabilities in a database.

2.2 Discrete-time Markov chain-based dynamic control approach for compressed sampling

There is a tradeoff between recovery accuracy and energy consumption as the sampling rate R increases. The larger the sampling rate R, the more the energy is consumed, thus worsening the sampling performance. On the other hand, when sampling rate R is too small, recovery accuracy becomes the main factor that may decrease sampling performance. In this paper, we define the expected reward function $E(R)$ by considering both recovery accuracy and energy consumption. Assuming that the current state is $S-1=i$, $E(R)$ can be expressed as

$$E(R) = \sum_{k=0}^{K} P(S=k/K \mid S-1=i)(\alpha A(R,\ k/K) - \beta e_t RT) \tag{8}$$

where $A(R,\ k/K)$ means the recovery accuracy when the sampling rate is R and the signal sparsity level equals k/K; T is the length of the sampling period; RT means the number of samples in a sampling period; e_t represents the required energy to take a single sample; α and β are both the constants, representing the weights of recovery accuracy and energy consumption, respectively.

Due to the fact that the recovery accuracy $A(R,\ k/K)$ cannot be precisely expressed by a formula, we obtain it by simulation, which is similar to the method in Ref. [4].

In this paper, our goal is to maximize the expected reward by dynamically adjusting the sampling rate according to the prediction results of the signal sparsity level. In other words, we should solve the following problem:

$$\max_R E(R) = \max_R \sum_{k=0}^{K} P(S=k/K \mid S-1=i) \cdot (\alpha A(R,\ k/K) - \beta e_t RT) \tag{9}$$

The above problem can be easily solved by discrete optimization methods, such as the particle swarm optimization, the ant colony algorithm, and so on.

3 Simulation Results and Analysis

Extensive simulation results are presented in this section to evaluate the performance of our proposed dynamic control approach and the existing approach for compressed sampling. The existing approach here denotes the one in which the sampling rate is determined by considering the maximum signal sparsity level. The following assumptions are adopted in the simulation. The number of frequencies in system K is 8. The highest frequency is 4 kHz. Each time slot lasts 10 ms. As a result, the sampling rate which is at least 8 kHz or 80 samples should be taken in each time slot according to the Nyquist sampling theory. In the simulation process, by using compressed sensing, $N(N<80)$ samples are taken in each time slot and nonlinear optimization is used to recover an original signal. $\alpha=1\,000$ and $\beta=10$ if there is no special explanation.

We first evaluate the recovery accuracy performance of compressed sensing since our expected reward function takes it into consideration. The evaluation process works as follows. For each signal sparsity k/K, an original 80×1 vector $\boldsymbol{x}_{\text{org}}$ is randomly generated. Then as the sampling rate R varies from 1 to 2 f_{m} Hz, the compressed sampling and recovery process is run for 10^4 times. The recovery accuracy $A(R, k/K)$ is defined as the ratio of the times that $\boldsymbol{x}_{\text{org}}$ is successfully recovered to the total times 10^4. Here the vector is successfully recovered when $|\boldsymbol{x}_{\text{org}}-\boldsymbol{x}_{\text{rec}}|<10^{-5}$, where $\boldsymbol{x}_{\text{rec}}$ is the recovered vector.

In Fig. 2, the relationship between the recovery accuracy and the sampling rate is presented. From this figure, we can draw the following conclusions. First, similar to the simulation results in Ref. [2] and Ref. [4], the recovery accuracy increases with the sampling rate for all the sparsity levels. Secondly, the increase rate of recovery accuracy is very slow at first, then becomes rapid, and finally slows down as the sampling rate varies from low to high. Thirdly, the higher the signal sparsity level, the greater the sampling rate, which is in accordance with the theoretical results obtained in Ref. [4].

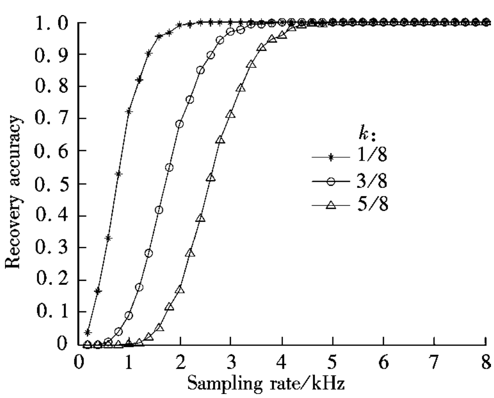

Fig. 2 Recovery accuracy vs. sampling rate for different signal sparsity levels

Fig. 3 shows the performance of our defined expected reward function with different values of the sampling rate. From this figure, we can see that the expected reward first decreases, then increases, but finally decreases when the sampling rate varies from low to high. This is because when the sampling rate is very low, the recovery accuracy increases slowly with the sampling rate, but the energy consumption increases linearly with the sampling rate, so the expected reward function decreases. As the sampling rate increases, the increase rate of recovery accuracy becomes rapid and greater than the increase rate of energy consumption; and the expected reward function increases rapidly. Once the recovery accuracy equals 1, it does not increase. However, energy consumption still increases with the sampling rate. So the expected reward finally decreases linearly. This is also the reason why the three curves combine together when the sampling rate is greater than 5.2 kHz.

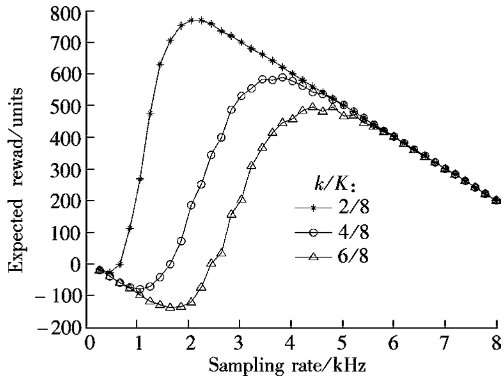

Fig. 3 Expected reward vs. sampling rate for different signal sparsity levels

Fig. 4 shows the relationship between the optimal sampling rate and the signal sparsity level under two conditions. $\alpha:\beta$ is assumed to be 500∶10 for the first condition and 1 500∶10 for the second condition. In Fig. 4, the recovery accuracy and energy consumption have different impacts on the optimal sampling rate. The optimal sampling rate increases as the weight of recovery accuracy increases and decreases as the weight of energy consumption increases.

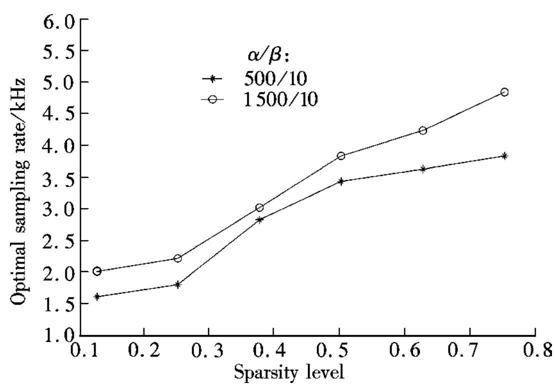

Fig. 4 Optimal sampling rate vs. sparsity level for different ratios of $\alpha:\beta$

In Fig. 5, the performance of our proposed dynamic control approach is compared with the existing one. From this figure, we can draw the conclusion that our proposed approach can greatly improve sampling performance. For example, the reward of our proposed approach is about 1.6

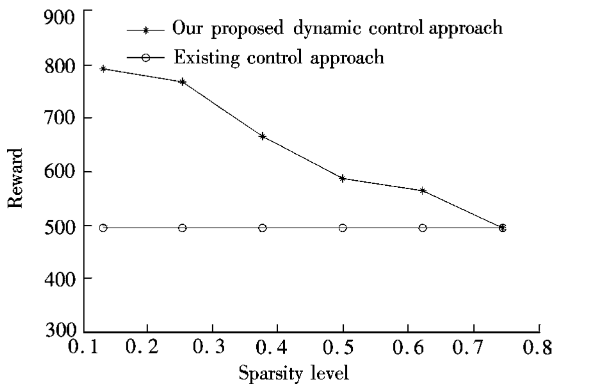

Fig. 5 Reward comparison when using different control approaches for compressed sampling

times as large as that of the existing approach when the signal sparsity level equals 1/8. Additionally, the smaller the sparsity level, the better sampling performance of our proposed approach.

4 Conclusion

In this paper, the sparsity level of the receiving signal is analyzed and modelled by the discrete-time Markov chain. Furthermore, a novel dynamic control approach for compressed sampling is proposed to maximize the expected reward by considering both the recovery accuracy and the energy consumption. The key step of our proposed approach is to analyze the transition probabilities between any two states of signal sparsity level by the discrete-time Markov chain. Finally, the sampling performance of our proposed dynamic control approach is evaluated by extensive simulation results.

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压缩采样中基于离散时间马尔科夫链的动态控制机制

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摘要：针对信号稀疏度在大多数情况下时变且未知的问题，提出了一种实时信号稀疏度预测及最优采样速率确定机制. 利用离散时间马尔科夫链对信号稀疏度进行建模，分析信号稀疏度各状态之间变化的规律，根据当前状态预测下一个采样周期内信号的稀疏度状态及概率. 此外，基于预测结果，综合考虑采样过程中的能量消耗和信号重构的精确度，以最大化预期收益为目的，提出一种控制机制来确定最优采样速率. 该机制能够达到能量消耗和精确度之间的折中. 仿真证明，所提出的基于离散时间马尔科夫链的动态控制机制与现有控制机制相比在采样性能方面具有较大的优势.

关键词：压缩采样；信号稀疏度预测；离散时间马尔科夫链

中图分类号：TN91