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Uplink capacity analysis of single-user SA-MIMO system

Dai Jianxin[1,2] Chen Ming[1] Chung Pei-Jung[3]

([1]National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China)

([2]School of Science, Nanjing University of Posts and Telecommunications, Nanjing 210023, China)

([3]Institute for Digital Communications, School of Engineering, the University of Edinburgh, Edinburgh EH9 3JL, UK)

Abstract: A novel framework of multiple antenna systems, which combines smart antennas (SA) with multiple-input multiple-output (MIMO) at the receiver, is proposed. The uplink SA-MIMO system is investigated. The joint optimization problem corresponding to the uplink capacity of the single-user SA-MIMO system is deduced. Then the closed-form expression of the capacity is obtained in the case of equal power allocation and the same direction-of-arrivals (DOAs) from different transmit antennas at the same antenna array, and an upper bound of the capacity is also given in the case of different DOAs at the same antenna array. After that, for the general case, a suboptimal method for the capacity optimization problem is presented. Some numerical results are also given to compare the capacities of conventional MIMO and SA-MIMO systems and show that the proposed method is viable.

Key words: smart antennas; multiple-input multiple-output (MIMO); uplink capacity; beam-forming

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As well known, the multiple-input multiple-output (MIMO) system with multiple antennas at both the transmitter and the receiver in a richly scattering environment can significantly increase the capacity of wireless channels without requiring additional power or bandwidth [1-2]. But the advantages of MIMO systems tend to be lost around the cell edge areas for two principal reasons. The first is the low signal-to-noise ratio (SNR) and inter-cell interference with the low frequency reuse factor. The other is the increase in the signal correlation for different antenna elements. To overcome the problem found in traditional MIMO systems and provide higher data rates, wider coverage and better quality of service (QoS) in the cell edge areas, many new frameworks of multiple antenna systems have been suggested to extend conventional MIMO systems [3-8]. The distributed antenna system (DAS) was investigated in Refs. [3 - 5] whose basic idea is that all antennas are geographically separated from each other and connected by optical fibers to a central processor where all signals are jointly processed. The downlink performance of single cell and multi-cell MIMO relay networks was analyzed which took into account MIMO technology in fixed relay networks in Ref. [6]. Ref. [7] provided a propagation measurement campaign of a MIMO two-hop relay network in a 5 GHz band in an L-shaped corridor environment with various relay locations and a relay placement estimation scheme to identify the optimum relay location. A brief survey of cooperative MIMO, the basic idea of which is to group multiple devices into virtual antenna arrays to emulate MIMO communications, was provided in Ref. [8].

However, the improvement of performance in the above-mentioned multiple antenna systems comes at the price of increased cost, space and computational complexity due to the individual power constraints of each antenna, the cooperation algorithm and the hardware limitations. In this paper, a new smart antennas-MIMO (SA-MIMO) system with the total power constraint of all antennas is studied. The basic idea of the SA-MIMO is to replace each antenna of traditional MIMO systems with a smart antenna array. Smart antennas, which can suppress the interference coming from different directions by beam-forming and hence can make the cell have a wider coverage and greater user capacity [9-10], are considered as a transmission technology of the single channel current in the 3rd generation of mobile communication systems (3G) standard. And the MIMO technology provides multiple independent transmission channels that can increase system throughput in a long term evolution (LTE) standard. However, a key fact to note is that a richly scattering environment also suffers from high loss. Smart antennas technique improves power gains and increases the SNR which lowers the path loss. Thus, it is of great interest to investigate the combination of the MIMO system and smart antennas in order to ensure the future system a smooth evolution, make the most of existing system resources, avoid extensive redesign of antennas and the

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Biographies: Dai Jianxin (1971—), male, graduate, associate professor; Chen Ming (corresponding autor), male, doctor, professor, chenming@seu.edu.cn.

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feeder system as much as possible, and reduce difficulties of network arrangement and cell-site selection. However, the investigation of the SA-MIMO is rare in the current literature.

In this paper, we propose an SA-MIMO system, which is different from the traditional MIMO systems and other multiple antenna systems, i. e., the DAS and the cooperative MIMO system. For the single-user uplink SA-MIMO system, we first propose an optimization model. Then we investigate the capacity of the single-user uplink SA-MIMO system in some cases. For the general case, it is difficult to derive closed-form solutions of the optimal beam-forming vectors to the capacity optimization problem. This paper presents a suboptimal method for the capacity optimization problem, in which the beam-forming vectors are obtained by maximizing the squared Frobenius norm of the channel matrix.

1 Statement of the Problem

1.1 System model

Consider an uplink SA-MIMO system which is shown in Fig. 1. The user transmitter has N antenna transmitting uplink signals, whose mutual distances are greater than the half wavelength of the carrier wave such that all the transmit channels are independent of each other. At the receiver base station, there are M antenna arrays, each of which has L elements whose mutual distance is less than half the wavelength. In addition, all the M antenna arrays are mutually so far that their receive channels are independent of each other.

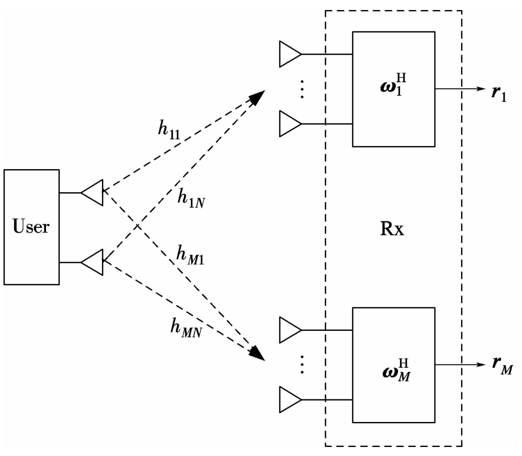

Fig. 1 Single-user uplink SA-MIMO system model

Let h_{mn} be the microscope fading coefficient between the n-th transmit antenna and the m-th receive antenna array for $1 \leqslant m \leqslant M$ and $1 \leqslant n \leqslant N$. The channels are flat Rayleigh fading, i. e., $h_{mn} \sim \mathrm{CN}(0, 1)$, where $x \sim \mathrm{CN}(\mu, \sigma^2)$ means that x is complex Gaussian distributed with mean μ and variance σ^2.

Denote the beam-forming vector of the m-th array at the receiver as $\boldsymbol{\omega}_m = \{\omega_{m,1}, \omega_{m,2}, \ldots, \omega_{m,L}\}^{\mathrm{T}}$ with $\|\boldsymbol{\omega}_m\| = 1$, where $\|\cdot\|$ is the Euclidean norm of a vector. Let $\boldsymbol{a}_{mn} = \boldsymbol{a}(\theta_{mn})$ be the steering vector of the m-th antenna array with respect to the n-th transmit antenna, where θ_{mn} is the angle of departure from the n-th transmit antenna to the m-th receive antenna array. $s_1, s_2, \cdots, s_N$ and $p_1, p_2, \cdots, p_N$ represent the transmitted signals and powers at the N antennas, respectively. Then the received signal at the m-th antenna array after beam-forming can be written as

$$r_m = \sum_{n=1}^{N} s_n \sqrt{p_n} h_{mn} \langle \boldsymbol{\omega}_m, a_{mn} \rangle + \langle \omega_m, z_m \rangle = \sum_{n=1}^{N} s_n \sqrt{p_n} \tilde{h}_{mn} + \tilde{z}_m \tag{1}$$

where $z_m = \{z_{m,1}, z_{m,2}, \cdots, z_{m,L}\}^{\mathrm{T}} \sim \mathrm{CN}(0, \sigma^2 \boldsymbol{I}_L)$ is the noise vector at the m-th antenna array; $\langle \cdot, \cdot \rangle$ denotes the Euclid inner product of two vectors; $\tilde{h}_{mn} = h_{mn} \langle \boldsymbol{\omega}_m, a_{mn} \rangle$, $\tilde{z}_m = \langle \boldsymbol{\omega}_m, z_m \rangle$.

By stacking the received signals of all the antenna arrays into $\boldsymbol{r} = \{r_1, r_2, \cdots, r_M\}^{\mathrm{T}}$, we have

$$\boldsymbol{r} = \boldsymbol{HPs} + z \tag{2}$$

where $z = \{\tilde{z}_1, \tilde{z}_2, \cdots, \tilde{z}_M\}^{\mathrm{T}}$, $\boldsymbol{s} = \{s_1, s_2, \cdots, s_N\}^{\mathrm{T}}$, $\boldsymbol{H} = (\tilde{h}_{mn})_{M \times N}$, $\boldsymbol{P} = \mathrm{diag}[\sqrt{p_1}, \sqrt{p_2}, \cdots, \sqrt{p_N}]$.

1.2 Information-theoretic capacity

Let $\boldsymbol{W} = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \cdots, \boldsymbol{\omega}_M\}$. If the input signal $\boldsymbol{s}$ is a circularly symmetric complex Gaussian vector with covariance matrix $\boldsymbol{E}\{\boldsymbol{ss}^{\mathrm{T}}\} = \boldsymbol{I}_N$, the instantaneous capacity of the SA-MIMO system for each channel use can be formulated as

$$\begin{aligned} C = \max_{\boldsymbol{W},\boldsymbol{P}} & \left\{ \log_2 \det\left(\boldsymbol{I}_M + \frac{1}{\sigma^2} \boldsymbol{HPP}^{\mathrm{H}} \boldsymbol{H}^{\mathrm{H}} \right) \right\} \\ \text{s. t. } & \|\boldsymbol{\omega}_m\| = 1, \ \forall \boldsymbol{\omega}_m \in \boldsymbol{W} \\ & \mathrm{Tr}(\boldsymbol{PP}^{\mathrm{H}}) = \sum_{n=1}^{N} p_n = P \\ & \boldsymbol{H} = (h_{mn} \langle \boldsymbol{\omega}_m, a_{mn} \rangle)_{1 \leqslant m \leqslant M,\ 1 \leqslant n \leqslant N} \end{aligned} \tag{3}$$

where $(\ )^{\mathrm{H}}$ denotes the complex-conjugate transpose of a vector or matrix.

1.3 Analysis of the capacity

In this section, we investigate the capacity in the case of equal power allocation.

Denote $\rho = P/\sigma^2$, and let $\lambda_i (i = 1, 2, \cdots, M)$ be the eigenvalues of matrix $\boldsymbol{HH}^{\mathrm{H}}$.

Denote $\widetilde{\boldsymbol{H}}_{M \times N} = \begin{bmatrix} h_{11} & \cdots & h_{1N} \\ \vdots & & \vdots \\ h_{M1} & \cdots & h_{MN} \end{bmatrix}$, and let $\tilde{\lambda}_1, \tilde{\lambda}_2, \cdots, \tilde{\lambda}_M$ be the eigenvalues of the matrix $\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\mathrm{H}}$.

Theorem 1 In the case that the DOAs from all the transmit antennas are the same in a receive antenna array, i. e., $\boldsymbol{a}_{m1} = \boldsymbol{a}_{m2} = \cdots = \boldsymbol{a}_{mN} = \boldsymbol{a}_m$, and the power is allocated to the transmit antennas equally, the capacity of the SA-MIMO system can be formulated as

$$C = \sum_{i=1}^{M} \log_2 \left(1 + \frac{\rho L}{N} \tilde{\lambda}_i \right) \tag{4}$$

if and only if the optimal beam-forming vector can be written as

$$\boldsymbol{\omega}_i^* = \frac{1}{\sqrt{L}}\boldsymbol{a}_i \qquad i = 1, 2, \cdots, M \tag{5}$$

Proof If the DOAs of all the elements of one smart antenna array are the same, the channel matrix $\boldsymbol{H}$ can be written as

$$\boldsymbol{H} = \widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{H}} \tag{6}$$

where

$$\widetilde{\boldsymbol{W}}_{M\times M} = \begin{bmatrix} \boldsymbol{\omega}_1^{\mathrm{H}}\boldsymbol{a}_1 & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{\omega}_M^{\mathrm{H}}\boldsymbol{a}_M \end{bmatrix}$$

Moreover, if the power is allocated to the transmit antennas equally, Eq. (3) can be further written as

$$C(\boldsymbol{\omega}) = \max_{\boldsymbol{\omega}}\left\{\log_2 \det\left(\frac{\rho}{N}\widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\mathrm{H}}(\widetilde{\boldsymbol{W}})^{\mathrm{H}} + \boldsymbol{I}_M\right)\right\}$$
$$\text{s. t. } \|\boldsymbol{\omega}_m\| = 1 \qquad m = 1, 2, \cdots, M \tag{7}$$

Using singular value decomposition (SVD), we have

$$\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\mathrm{H}} = \widetilde{\boldsymbol{U}}^{\mathrm{H}}\widetilde{\boldsymbol{\Lambda}}\widetilde{\boldsymbol{U}} \tag{8}$$

where $\widetilde{\boldsymbol{U}}$ is a unitary matrix, and $\widetilde{\boldsymbol{\Lambda}}$ is a non-negative diagonal matrix whose diagonal elements are the singular values of the matrix $\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\mathrm{H}}$, i. e., $\widetilde{\boldsymbol{\Lambda}} = \mathrm{diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \cdots, \tilde{\lambda}_M)$. Then, using $\det(\boldsymbol{I} + \boldsymbol{AB}) = \det(\boldsymbol{I} + \boldsymbol{BA})$, we obtain

$$\log_2 \det\left(\frac{\rho}{N}\widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\mathrm{H}}\widetilde{\boldsymbol{W}}^{\mathrm{H}} + \boldsymbol{I}_M\right) =$$
$$\log_2 \det\left(\frac{\rho}{N}\widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{U}}^{\mathrm{H}}\widetilde{\boldsymbol{\Lambda}}\widetilde{\boldsymbol{U}}\widetilde{\boldsymbol{W}}^{\mathrm{H}} + \boldsymbol{I}_M\right) =$$
$$\log_2 \det\left(\frac{\rho}{N}\widetilde{\boldsymbol{\Lambda}}\widetilde{\boldsymbol{U}}\widetilde{\boldsymbol{W}}^{\mathrm{H}}\widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{U}}^{\mathrm{H}} + \boldsymbol{I}_M\right) \tag{9}$$

To simplify the rest of the derivations, let

$$\boldsymbol{A} = \widetilde{\boldsymbol{U}}\widetilde{\boldsymbol{W}}^{\mathrm{H}}\widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{U}}^{\mathrm{H}}, \quad \boldsymbol{B} = \frac{\rho}{N}\widetilde{\boldsymbol{\Lambda}}\boldsymbol{A} + \boldsymbol{I}_{\mathrm{M}}$$

It is obvious that $\boldsymbol{B}$ is a positive definite Hermitian matrix. Now, from the Hadamard theorem[11], we obtain

$$\log_2 \det\left(\frac{\rho}{N}\widetilde{\boldsymbol{\Lambda}}\widetilde{\boldsymbol{U}}\widetilde{\boldsymbol{W}}^{\mathrm{H}}\widetilde{\boldsymbol{W}}\widetilde{\boldsymbol{U}}^{\mathrm{H}} + \boldsymbol{I}_M\right) \leqslant$$
$$\log_2 \prod_{i=1}^{M} B_{ii} = \log_2 \prod_{i=1}^{M}\left(1 + \tilde{\lambda}_i \frac{\rho}{N}A_{ii}\right) \tag{10}$$

where A_{ii} and B_{ii} are the diagonal elements of $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. In Eq. (10), there can be equality only if $\boldsymbol{B}$ is a diagonal matrix. If $\boldsymbol{A}$ is a diagonal matrix, $\boldsymbol{B}$ is a diagonal matrix. So Eq. (10) is established only when $\boldsymbol{A}$ is a diagonal matrix. According to the expression of $\boldsymbol{A}$, it can be known that all the diagonal elements are independent. Therefore,

$$A_{ii} = \sum_{j=1}^{M} |\boldsymbol{\omega}_j^{\mathrm{H}}\boldsymbol{a}_j|^2 |u_{ij}|^2 \leqslant$$

$$\sum_{j=1}^{M} \|\boldsymbol{\omega}_j^{\mathrm{H}}\|^2 \|\boldsymbol{a}_j\|^2 |u_{ij}|^2 = L\sum_{j=1}^{M} |u_{ij}|^2 = L \tag{11}$$

In Eq. (11), there can be equality if and only if $\boldsymbol{\omega}_i$ is given by

$$\boldsymbol{\omega}_i^* = \frac{1}{\sqrt{L}}\boldsymbol{a}_i \qquad i = 1, 2, \cdots, M \tag{12}$$

And after substitution, the capacity can be written as

$$C = \sum_{i=1}^{M} \log_2\left(1 + \frac{\rho}{N}\tilde{\lambda}_i L\right)$$

if and only if the beam-forming vector is given by Eq. (12).

Denote $\|\boldsymbol{H}\|_{\mathrm{F}}^2$ as the squared Frobenius norm of $\boldsymbol{H}$. Then $\|\boldsymbol{H}\|_{\mathrm{F}}^2$ in the single-user uplink SA-MIMO system can be written as

$$\|\boldsymbol{H}\|_{\mathrm{F}}^2 = \sum_{m=1}^{M} \boldsymbol{\omega}_m^{\mathrm{H}}\left(\sum_{n=1}^{N} |h_{mn}|^2 \boldsymbol{a}_{mn}\boldsymbol{a}_{mn}^{\mathrm{H}}\right)\boldsymbol{\omega}_m \tag{13}$$

Define $\boldsymbol{A}_m = \sum_{n=1}^{N} |h_{mn}|^2 \boldsymbol{a}_{mn}\boldsymbol{a}_{mn}^{\mathrm{H}}$, $m = 1, 2, \cdots, M$, and let $\lambda_{m,\max}$ be the maximum eigenvalue of $\boldsymbol{A}_m$.

Theorem 2 In the case that the DOAs from different transmit antennas are not the same in a receive antenna array, the upper bound of the capacity of the SA-MIMO system with equal power can be written as

$$C = \log_2 \prod_{i=1}^{R}\left(1 + \frac{\rho}{N}\lambda_i\right) \leqslant R\log_2\left(1 + \frac{\rho}{RN}\sum_{m=1}^{M}\lambda_{m,\max}\right) \tag{14}$$

where R is the number of singular values of matrix $\boldsymbol{HH}^{\mathrm{H}}$.

Proof In the case of equal transmit power allocation, i. e., $\boldsymbol{PP}^{\mathrm{H}} = \frac{P}{N}\boldsymbol{I}_N = \frac{\rho\sigma^2}{N}\boldsymbol{I}_N$, Eq. (3) can be written as

$$C(\omega) = \max_{w}\left\{\log_2 \det\left(\frac{\rho}{N}\boldsymbol{HH}^{\mathrm{H}} + \boldsymbol{I}_M\right)\right\}$$
$$\text{s. t. } \|\omega_m\|_{\mathrm{F}} = 1 \qquad m = 1, 2, \cdots, M \tag{15}$$

Let $\lambda_1, \lambda_2, \cdots, \lambda_R$ be non-zero eigenvalues of matrix $\boldsymbol{HH}^{\mathrm{H}}$. From the theorem of the arithmetic and geometric means, we obtain

$$\log_2 \det\left(\frac{\rho}{N}\boldsymbol{HH}^{\mathrm{H}} + \boldsymbol{I}_M\right) = \log_2 \prod_{i=1}^{R}\left(1 + \frac{\rho}{N}\lambda_i\right) \leqslant$$
$$\log_2\left(\sum_{i=1}^{R}\left(1 + \frac{\rho}{N}\lambda_i\right)/R\right)^R = R\log_2\left(1 + \frac{\rho}{RN}\sum_{i=1}^{R}\lambda_i\right) \tag{16}$$

with equality when

$$\lambda_1 = \lambda_2 = \cdots = \lambda_R \tag{17}$$

From the Rayleigh-Ritz theorem[11], we obtain

$$\sum_{i=1}^{R}\lambda_i = \mathrm{Tr}(\boldsymbol{HH}^{\mathrm{H}}) = \|\boldsymbol{H}\|_{\mathrm{F}}^2 =$$

$$\sum_{m=1}^{M} \boldsymbol{\omega}_m^{\mathrm{H}} \left(\sum_{n=1}^{N} |h_{mn}|^2 \boldsymbol{a}_{mn} \boldsymbol{a}_{mn}^{\mathrm{H}} \right) \boldsymbol{\omega}_m \leqslant \sum_{m=1}^{M} \lambda_{m,\max} \tag{18}$$

In Eq. (18), there can be equality when $\boldsymbol{\omega}_m$ is the main eigenvector of $\boldsymbol{A}_m$, where

$$\boldsymbol{A}_m = \sum_{n=1}^{N} |h_{mn}|^2 \boldsymbol{a}_{mn} \boldsymbol{a}_{mn}^{\mathrm{H}} \tag{19}$$

Namely,

$$\begin{aligned} \boldsymbol{A}_m \boldsymbol{\omega}_m &= \lambda_{m,\max} \boldsymbol{\omega}_m \\ \| \boldsymbol{\omega}_m \| &= 1, \quad \forall m = 1, 2, \cdots, M \end{aligned} \tag{20}$$

So the upper bound of the capacity of the SA-MIMO system with equal power can be written as

$$C = \log_2 \prod_{i=1}^{R} \left(1 + \frac{\rho}{N} \lambda_i \right) \leqslant R \log_2 \left(1 + \frac{\rho}{RN} \sum_{i=1}^{R} \lambda_i \right) \leqslant R \log_2 \left(1 + \frac{\rho}{RN} \sum_{m=1}^{M} \lambda_{m,\max} \right) \tag{21}$$

with equality when Eqs. (17) and (20) are true.

2 Method for Determining the Beam-Forming Vectors

For the general case, it is difficult to derive closed-form solutions of the optimal beam-forming vectors to the capacity optimization problem (3). Moreover, although the optimal beam-forming vectors can be found numerically, these methods are complicated, and may not be feasible for practical applications. Here, we present a suboptimal algorithm for determining the beam-forming vectors based on the maximum eigenvalue in this paper.

In Ref. [12], an equivalent scaled AWGN channel induced by the space-time block code for complex constellations was given as

$$\boldsymbol{r} = \| \boldsymbol{H} \|_{\mathrm{F}}^2 \boldsymbol{P} \boldsymbol{s} + \boldsymbol{z} \tag{22}$$

In many other cases, $\| \boldsymbol{H} \|_{\mathrm{F}}$ also has some relationship with the channel quality. Based on the above facts, a suboptimal method for the capacity optimization problem is presented, in which the beam-forming vectors are obtained by maximizing $\| \boldsymbol{H} \|_{\mathrm{F}}^2$.

Lemma 1[11] Let $\boldsymbol{A}$ be a positive definite matrix and $\lambda_{\max}$ be the maximum eigenvalue of $\boldsymbol{A}$. If $\| \boldsymbol{x} \| = 1$, $\max_{\|\boldsymbol{x}\|=1} \boldsymbol{x}^{\mathrm{H}} \boldsymbol{A} \boldsymbol{x} = \lambda_{\max} = \tilde{\boldsymbol{x}}^{\mathrm{H}} \boldsymbol{A} \tilde{\boldsymbol{x}}$, where $\tilde{\boldsymbol{x}}$ is an eigenvector corresponding to the eigenvalue $\lambda_{\max}$.

Recalling the definition of the principal submatrix $\boldsymbol{A}_m$, $m = 1, 2, \cdots, M$, let $\boldsymbol{\omega}_m^*$ be an eigenvector corresponding to the eigenvalue $\lambda_{m,\max}$.

If $\boldsymbol{\omega}_m = \boldsymbol{\omega}_m^*$ for $m = 1, 2, \cdots, M$, we can obtain the maximum value of $\| \boldsymbol{H} \|_{\mathrm{F}}^2$ in Eq. (6), i. e.,

$$\max_{\|\boldsymbol{\omega}_m\|=1} \| \boldsymbol{H} \|_{\mathrm{F}}^2 = \sum_{m=1}^{M} \lambda_{m,\max} = \sum_{m=1}^{M} \boldsymbol{\omega}_m^{*\mathrm{H}} \left(\sum_{n=1}^{N} |h_{mn}|^2 \boldsymbol{a}_{mn} \boldsymbol{a}_{mn}^{\mathrm{H}} \right) \boldsymbol{\omega}_m^* \tag{23}$$

where

$$\begin{aligned} \boldsymbol{A}_m \boldsymbol{\omega}_m^* &= \lambda_{m,\max} \boldsymbol{\omega}_m^* \\ \| \boldsymbol{\omega}_m^* \| &= 1, \quad \forall m = 1, 2, \cdots, M \end{aligned} \tag{24}$$

So we generate the suboptimal beam-forming vectors to solve the capacity optimization problem by the above method.

3 Numerical Results

Monte Carlo simulations are carried out in some cases to compare the capacities of SA-MIMO and conventional MIMO systems. We denote an SA-MIMO system with N transmit antennas and M receive antenna arrays, and each array owns L elements by an $N \times (M, L)$ SA-MIMO system.

Fig. 2 depicts the ergodic channel capacity of the MIMO system and the SA-MIMO system for various values of SNR with $N=2$, $M=2$, $L=4$ and $N=4$, $M=4$, $L=4$, respectively. It is shown that smart antennas can bring significant capacity gain for the MIMO system without additional spatial degrees of freedom. It is also evident that the upper bound is actually very tight for the considered system.

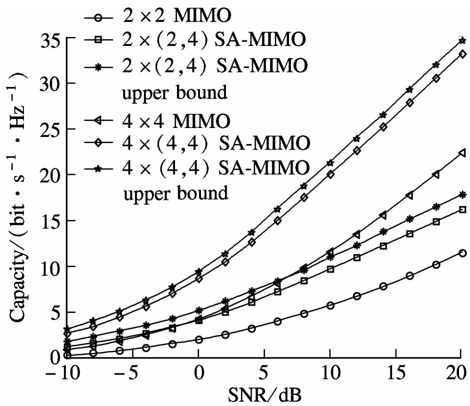

Fig. 2 Ergodic capacity comparison between MIMO system and SA-MIMO system

The following simulations are performed by the proposed method for determining the suboptimal beam-forming vectors. All the simulations are performed under the assumption that the transmit power is equally allocated.

NIA denotes an algorithm that the optimal solution of the capacity problem in the SA-MIMO system is obtained by numerical iteration. MEV denotes the method that the optimal beam-forming vectors are obtained by the main eigenvector method.

Case 1 The same DOAs of one smart antenna

Fig. 3 depicts the capacity of the three systems, with $N = M = 2$ and $L = 8$. It is observed that the capacity obtained by the MEV method is equal to the optimal capacity in case 1. This result also validates theorem 2.

Figs. 4(a) and (b) illustrate the radiation patterns of the two smart antenna arrays at the receiver for the $2 \times (2,8)$ SA-MIMO system for case 1, the beam-forming vectors of which are determined by MEV. It is shown that the pattern gains on the corresponding angles are in

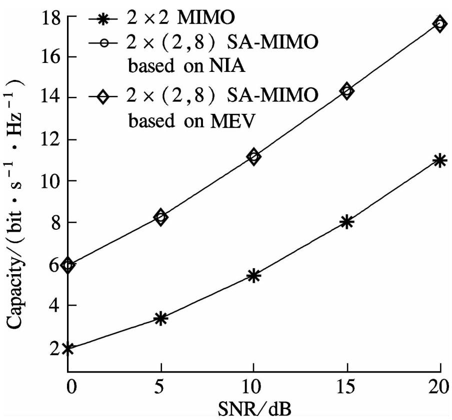

Fig. 3 Ergodic capacity comparison for case 1

the periphery of the peaks, so that the smart antenna will attain a capacity gain.

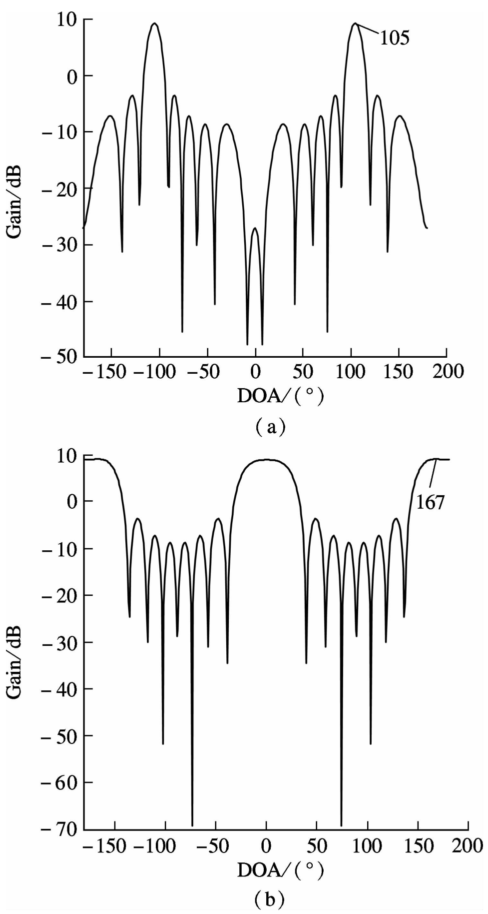

Fig. 4 Two smart antenna patterns for case 1. (a) The first smart antenna pattern (DOAs are 105°); (b) The second smart antenna pattern (DOAs are 167°)

Case 2 The different DOAs of one smart antenna

Fig. 5 depicts the capacity of the four systems mentioned above, where $N = M = 2$ and $L = 8$. It is observed that the gap between the capacity obtained by the MEV method and the optimal capacity is small. It indicates that the performance of beam-forming vectors obtained by MEV is acceptable.

Fig. 6(a) and Fig. 6(b) illustrate the radiation patterns of the two smart antenna arrays at the receiver for the 2 × (2,8) SA-MIMO system, the beam-forming vectors of which are determined by MEV. It is shown that the pattern gains on the corresponding angles are in the periphery of the peaks, so that SA will attain capacity gain. Such a gain is due to the use of a smart antenna array, rather than diversity.

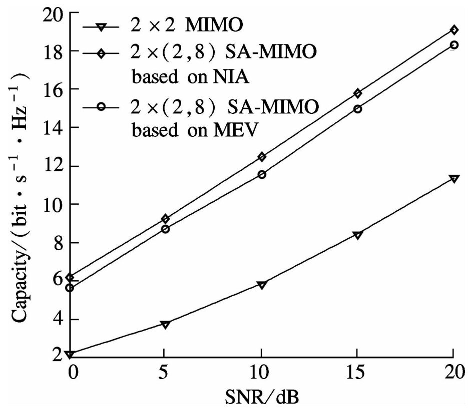

Fig. 5 Ergodic capacity comparison for case 2

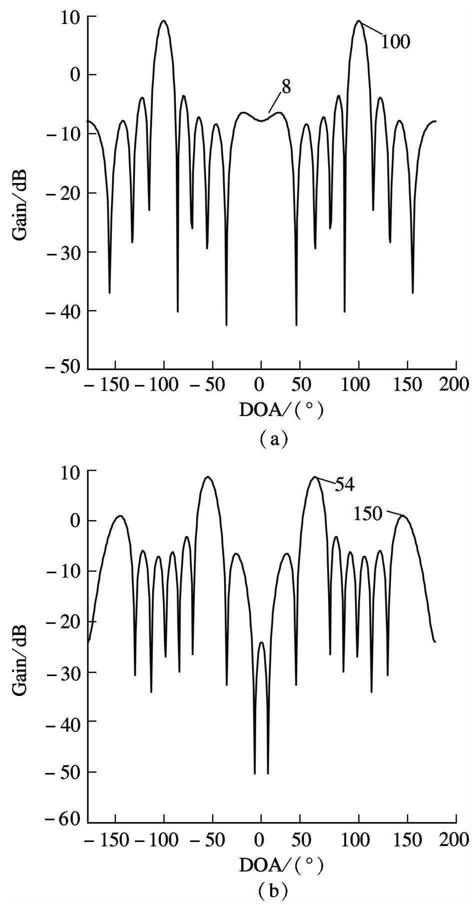

Fig. 6 Two smart antenna patterns for case 2. (a) The first smart antenna pattern (DOAs are 8° and 100°, respectively); (b) The second smart antenna pattern (DOAs are 54° and 150°, respectively)

4 Conclusion

This paper investigates the uplink capacity of single-user SA-MIMO systems. First, in the case of the same

DOAs from different transmit antennas at the same receive antenna array, we deduce the closed-form expression of SA-MIMO systems with equal power allocation. In the case of different DOAs, an upper bound of the capacity with equal power allocation is derived. Then, for the general case, we propose a suboptimal method for the capacity optimization problem in which the beam-forming vectors are obtained by maximizing $\|\boldsymbol{H}\|_F^2$.

Forthcoming work should be focused on analyzing the capacity gain of SA-MIMO systems under the multiple-user case in which the advantage suppressing interference among users can be embodied.

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单用户智能天线-多输入多输出系统的上行容量分析

戴建新[1,2]　陈　明[1]　钟佩蓉[3]

([1] 东南大学移动通信国家重点实验室, 南京 210096)

([2] 南京邮电大学理学院,南京 210023)

([3] Institute for Digital Communications, School of Engineering, the University of Edinburgh, Edinburgh EH9 3JL, UK)

摘要:提出了一种新的多天线系统架构,该系统架构在接收端结合了智能天线技术和多输入多输出技术. 对上行智能天线-多输入多输出系统进行了研究,提出关于单用户智能天线-多输入多输出系统上行容量的联合优化问题. 对于等功率分配情形,当同一天线阵列的来自不同发送天线的到达角相同时,得到了上行容量的闭式解,当同一天线阵列的来自不同发送天线的到达角不同时,得到了上行容量的上限. 接着,对于一般情形,提出了一种求解容量优化问题的次优解方法. 最后, 通过数值仿真比较了传统多输入多输出系统容量和智能天线-多输入多输出系统容量,仿真结果表明所提出的次优解方法是可行的.

关键词:智能天线;多输入多输出;上行容量;波束成型

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