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L_1 -norm minimization for quaternion signals

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Abstract: An algorithm for recovering the quaternion signals in both noiseless and noise contaminated scenarios by solving an L_1-norm minimization problem is presented. The L_1-norm minimization problem over the quaternion number field is solved by converting it to an equivalent second-order cone programming problem over the real number field, which can be readily solved by convex optimization solvers like SeDuMi. Numerical experiments are provided to illustrate the effectiveness of the proposed algorithm. In a noiseless scenario, the experimental results show that under some practically acceptable conditions, exact signal recovery can be achieved. With additive noise contamination in measurements, the experimental results show that the proposed algorithm is robust to noise. The proposed algorithm can be applied in compressed-sensing-based signal recovery in the quaternion domain.

Key words: quaternion; signal recovery; compressed sensing
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The problem of L_1-norm minimization plays an important role in the recently developed compressed sensing (CS) theory[1-3], which is a new approach for data acquisition and has wide applications in the field of signal and image processing. CS has been conventionally used to real input data. Recently, special attention has also been paid to the complex input data such as blind source separation[4] and terahertz imaging[5].

On the other hand, the theory and application of quaternion or hypercomplex algebra, invented by Hamilton[6], have received much attention in recent years[7-14]. Ell and Sangwine[7] applied the quaternion Fourier transform to color image processing. Jiang and Wei[9] proposed an algorithm for solving the quaternion least squares problem:

$$\min \ \|\boldsymbol{Ax}-\boldsymbol{y}\|_2 \qquad \text{s. t.} \ \ \boldsymbol{Bx}=\boldsymbol{z} \tag{1}$$

where $\boldsymbol{A}\in \mathbf{Q}^{n\times m}$, $\boldsymbol{B}\in \mathbf{Q}^{u\times m}$, $\boldsymbol{x}\in \mathbf{Q}^{m\times 1}$, $\boldsymbol{y}\in \mathbf{Q}^{n\times 1}$, $\mathbf{Q}$ denotes the quaternion number field, and $\|\cdot\|_2$ denotes the L_2-norm of quaternion vector. The algorithm reported in Ref. [9] was further extended by Jiang et al. [10] to a two-dimensional scenario where the inputs are quaternion matrices. Note that the algorithms reported in Refs. [9 – 10] solve the overdetermined system, that is, $n\geqslant m$ for measurement matrix $\boldsymbol{A}$.

In this paper, we consider the recovery problem of quaternion signals for the case where $n\leqslant m$, that is, we deal with the underdetermined linear system with measurement matrix $\boldsymbol{A}$, which is quite different from that of (1). We will do that by solving an L_1-norm minimization problem. To the authors' knowledge, the L_1-norm minimization problem for the quaternion signals has not yet been investigated. Winter et al. [4] converted the L_1-norm minimization of complex signals to the second-order cone programming (SOCP), which was then solved by SeDuMi software[15]. In this paper, we extend the algorithm[4] to the quaternion signals with and without noise contamination, which are, respectively, defined by the following two optimization problems:

$$\min \ \|\boldsymbol{x}\|_1 \qquad \text{s. t.} \ \ \boldsymbol{y}=\boldsymbol{Ax} \tag{2}$$

$$\min \ \|\boldsymbol{x}\|_1 \qquad \text{s. t.} \ \ \|\boldsymbol{Ax}-\boldsymbol{y}\|_2\leqslant\varepsilon \tag{3}$$

where $\|\cdot\|_1$ denotes the L_1-norm of the quaternion vector and ε is the noise penalty level for noise term $\boldsymbol{e}$ in noise measurements $\boldsymbol{y}=\boldsymbol{Ax}+\boldsymbol{e}$.

1 Definitions

A quaternion q is a hypercomplex number which consists of one real part $R(q)$ and three imaginary parts $I(q)$, $J(q)$ and $K(q)$ as follows:

$$q=R(q)+I(q)i+J(q)j+K(q)k \tag{4}$$

where $R(q)$, $I(q)$, $J(q)$, $K(q)\in\mathbf{R}$; and i, j and k are three imaginary units obeying the following rules:

$$i^2=j^2=k^2=ijk=-1 \tag{5}$$

$$ij=-ji=k,\ jk=-kj=i,\ ki=-ik=j \tag{6}$$

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The conjugate and modulus of a quaternion are, respectively, defined as

$$q^* = R(q) - I(q)i - J(q)j - K(q)k \tag{7}$$

$$|q| = \sqrt{R^2(q) + I^2(q) + J^2(q) + K^2(q)} \tag{8}$$

We also define

$$\boldsymbol{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_m] \tag{9}$$

$$\boldsymbol{x} = [x_1, \cdots, x_m]^{\mathrm{T}} = R(\boldsymbol{x}) + I(\boldsymbol{x})i + J(\boldsymbol{x})j + K(\boldsymbol{x})k \tag{10}$$

$$\boldsymbol{y} = [y_1, \cdots, y_n]^{\mathrm{T}} = R(\boldsymbol{y}) + I(\boldsymbol{y})i + J(\boldsymbol{y})j + K(\boldsymbol{y})k \tag{11}$$

The L_p-norm of a quaternion vector $\boldsymbol{x}$ is given by

$$\|\boldsymbol{x}\|_p = \left(\sum_{r=1}^{m} |x_r|^p\right)^{1/p} \qquad p = 1,2 \tag{12}$$

2 Method

In this section, we derive an algorithm based on SOCP for L_1-norm minimization problems dealing with quaternion signals.

2.1 Noiseless case

The minimization problem shown in (2) is equivalent to its epigraph form:

$$\min\ t \in \mathbf{R}^+ \qquad \text{s.t. } \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x},\ \|\boldsymbol{x}\|_1 \leqslant t \tag{13}$$

By introducing auxiliary variables $t_r \in \mathbf{R}^+$, where $r = 1, 2, \cdots, m$ and $\mathbf{R}^+$ denotes the positive real number field, the second constraint $\|\boldsymbol{x}\|_1 \leqslant t$ can be decomposed into a set of m constraints

$$\|[R(x_r), I(x_r), J(x_r), K(x_r)]^{\mathrm{T}}\|_2 \leqslant t_r \qquad r = 1, 2, \cdots, m \tag{14}$$

and (13) becomes

$$\begin{gathered}\min_{t} \mathbf{1}^{\mathrm{T}}\boldsymbol{t} \in \mathbf{R} \\ \text{s.t. } \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \\ \|[R(x_r), I(x_r), J(x_r), K(x_r)]^{\mathrm{T}}\|_2 \leqslant t_r \qquad r = 1, 2, \cdots, m\end{gathered} \tag{15}$$

After some manipulation, (15) can be written as

$$\begin{gathered}\min_{\hat{x}} \hat{\boldsymbol{c}}^{\mathrm{T}}\hat{\boldsymbol{x}} \in \mathbf{R} \\ \text{s.t. } \hat{\boldsymbol{y}} = \hat{\boldsymbol{A}}\hat{\boldsymbol{x}} \\ \|[R(x_r), I(x_r), J(x_r), K(x_r)]^{\mathrm{T}}\|_2 \leqslant t_r \qquad r = 1, 2, \cdots, m\end{gathered} \tag{16}$$

where

$$\hat{\boldsymbol{x}} = [t_1, R(x_1), I(x_1), J(x_1), K(x_1), \cdots, t_m, R(x_m), I(x_m), J(x_m), K(x_m)]^{\mathrm{T}} \in \mathbf{R}^{5m} \tag{17}$$

$$\hat{\boldsymbol{c}} = [1, 0, 0, 0, 0, \cdots, 1, 0, 0, 0, 0]^{\mathrm{T}} \in \mathbf{R}^{5m} \tag{18}$$

$$\hat{\boldsymbol{y}} = [R(y), I(y), J(y), K(y)]^{\mathrm{T}} \in \mathbf{R}^{4n} \tag{19}$$

$$\hat{\boldsymbol{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ R(\boldsymbol{a}_1)^{\mathrm{T}} & I(\boldsymbol{a}_1)^{\mathrm{T}} & J(\boldsymbol{a}_1)^{\mathrm{T}} & K(\boldsymbol{a}_1)^{\mathrm{T}} \\ -I(\boldsymbol{a}_1)^{\mathrm{T}} & R(\boldsymbol{a}_1)^{\mathrm{T}} & K(\boldsymbol{a}_1)^{\mathrm{T}} & -J(\boldsymbol{a}_1)^{\mathrm{T}} \\ -J(\boldsymbol{a}_1)^{\mathrm{T}} & -K(\boldsymbol{a}_1)^{\mathrm{T}} & R(\boldsymbol{a}_1)^{\mathrm{T}} & I(\boldsymbol{a}_1)^{\mathrm{T}} \\ -K(\boldsymbol{a}_1)^{\mathrm{T}} & J(\boldsymbol{a}_1)^{\mathrm{T}} & -I(\boldsymbol{a}_1)^{\mathrm{T}} & R(\boldsymbol{a}_1)^{\mathrm{T}} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ R(\boldsymbol{a}_m)^{\mathrm{T}} & I(\boldsymbol{a}_m)^{\mathrm{T}} & J(\boldsymbol{a}_m)^{\mathrm{T}} & K(\boldsymbol{a}_m)^{\mathrm{T}} \\ -I(\boldsymbol{a}_m)^{\mathrm{T}} & R(\boldsymbol{a}_m)^{\mathrm{T}} & K(\boldsymbol{a}_m)^{\mathrm{T}} & -J(\boldsymbol{a}_m)^{\mathrm{T}} \\ -J(\boldsymbol{a}_m)^{\mathrm{T}} & -K(\boldsymbol{a}_m)^{\mathrm{T}} & R(\boldsymbol{a}_m)^{\mathrm{T}} & I(\boldsymbol{a}_m)^{\mathrm{T}} \\ -K(\boldsymbol{a}_m)^{\mathrm{T}} & J(\boldsymbol{a}_m)^{\mathrm{T}} & -I(\boldsymbol{a}_m)^{\mathrm{T}} & R(\boldsymbol{a}_m)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{4n \times 5m} \tag{20}$$

2.2 Noise case

Similar to the previous case, the minimization problem shown in (3) can be reformulated as

$$\begin{gathered}\min_{\hat{x}} \hat{\boldsymbol{c}}^{\mathrm{T}}\hat{\boldsymbol{x}} \in \mathbf{R} \\ \text{s.t. } \|\hat{\boldsymbol{A}}\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}}\|_2 \leqslant \varepsilon \\ \|[R(x_r), I(x_r), J(x_r), K(x_r)]^{\mathrm{T}}\|_2 \leqslant t_r \qquad r = 1, 2, \cdots, m\end{gathered} \tag{21}$$

The first inequality constraint of (21) can be written as

$$\hat{\boldsymbol{x}}^{\mathrm{T}}\hat{\boldsymbol{A}}^{\mathrm{T}}\hat{\boldsymbol{A}}\hat{\boldsymbol{x}} - 2\hat{\boldsymbol{x}}^{\mathrm{T}}\hat{\boldsymbol{A}}^{\mathrm{T}}\hat{\boldsymbol{y}} + \hat{\boldsymbol{y}}^{\mathrm{T}}\hat{\boldsymbol{y}} \leqslant \varepsilon^2 \tag{22}$$

or equivalently

$$\|\boldsymbol{z}\|_2 \leqslant 2 z_0 z_1 \tag{23}$$

where

$$\boldsymbol{z} = \hat{\boldsymbol{A}}\hat{\boldsymbol{x}}, \quad z_0 = 1, \quad z_1 = \frac{\varepsilon^2}{2} + \hat{\boldsymbol{x}}^{\mathrm{T}}\hat{\boldsymbol{A}}^{\mathrm{T}}\hat{\boldsymbol{y}} - \frac{\hat{\boldsymbol{y}}^{\mathrm{T}}\hat{\boldsymbol{y}}}{2} \tag{24}$$

Here $(\boldsymbol{z}, z_0, z_1)$ belongs to a rotated second-order cone. Taking the linear relationship among $\hat{\boldsymbol{x}}$, $\boldsymbol{z}$, z_0 and z_1 into account, we can obtain a linear constraint:

$$\tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} = \tilde{\boldsymbol{b}}$$

where

$$\tilde{\boldsymbol{b}} = \left[0, 1, \frac{\hat{\boldsymbol{y}}^{\mathrm{T}}\hat{\boldsymbol{y}} - \varepsilon^2}{2}\right] \in \mathbf{R}^{4n+2}$$

$$\tilde{\boldsymbol{x}} = [\hat{\boldsymbol{x}}, \boldsymbol{z}, z_0, z_1] \in \mathbf{R}^{4n+5m+2}$$

$$\tilde{\boldsymbol{A}} = \left[\begin{array}{c:c} \hat{\boldsymbol{y}}^{\mathrm{T}}\hat{\boldsymbol{A}} & \\ 0 & \underbrace{\begin{matrix} -1 & & \\ & \ddots & \\ & & -1 \end{matrix}}_{\text{Negative identity matrix}} \\ \hat{\boldsymbol{A}} & \end{array}\right] \in \mathbf{R}^{(4n+2)\times(4n+5m+2)}$$

And the original problem can be turned into a second-order cone problem as

$$\min_{\tilde{x}} \tilde{\boldsymbol{c}}^{\mathrm{T}}\tilde{\boldsymbol{x}} \in \mathbf{R}$$
$$\text{s. t. } \tilde{\boldsymbol{b}} = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}}$$
$$\|[R(x_r), I(x_r), J(x_r), K(x_r)]^{\mathrm{T}}\|_2 \leqslant t_r \qquad r = 1, 2, \cdots, m$$
$$\|z\|_2 \leqslant 2\, z_0 z_1$$
$$\tilde{\boldsymbol{c}} = [\hat{\boldsymbol{c}}, \mathbf{0}] \in \mathbf{R}^{4n+5m+2} \tag{25}$$
$$\tilde{\boldsymbol{x}} = [\hat{\boldsymbol{x}}, z, z_0, z_1] \in \mathbf{R}^{4n+5m+2}$$

(16) and (25) are the standard forms of the SOCP problem and can be solved by using several mature toolboxes, such as SeDuMi [15]. Then, we can easily obtain the recovered quaternion signal $\boldsymbol{xr}$ from $\hat{\boldsymbol{x}}$ or $\tilde{\boldsymbol{x}}$.

3 Numerical Experiments

In a noiseless scenario, just as in Ref. [1], we present numerical experiments that indicate empirical bounds on sparsity s (time domain support of the input signal) relative to n (the number of measurements) for perfectly recovering a quaternion signal $\boldsymbol{x}$. The results can be seen as a set of practical guidelines for situations in which one expects perfect recovery from random Gaussian quaternion measurements using SOCP. Numerical experiments are carried out as follows:

1) An n by m ($m = 512$ is the length of input signal, n is the number of measurements) random Gaussian measurement matrix $\boldsymbol{A} \in \mathbf{Q}^{n \times m}$ ($n \leqslant m$) is produced with random entries sampled from an independent and identically distributed (i. i. d.) Gaussian process with a zero mean and a variance equaling 1 (in quaternion L_2-norm sense).

2) Sparse quaternion input signal $\boldsymbol{x} \in \mathbf{Q}^{m \times 1}$ is produced by selecting a support set T of size $|T| = s$ (sparsity) uniformly at random and sampling a vector $\boldsymbol{x}$ on $\boldsymbol{T}$ with i. i. d. Gaussian entries.

3) Quaternion output signal $\boldsymbol{y} \in \mathbf{Q}^{n \times 1}$ is obtained by multiplying $\boldsymbol{A}$ with input $\boldsymbol{x}$.

4) The vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{c}}$, $\hat{\boldsymbol{y}}$ and matrix $\hat{\boldsymbol{A}}$ are constructed as described in (17) to (20).

5) SeDuMi toolbox[15] is called to solve SOCP problem (16) and the error is computed, which is the L_2-norm of the difference between the recovered signal $\boldsymbol{xr}$ and the input signal $\boldsymbol{x}$, i. e., $\|\boldsymbol{xr} - \boldsymbol{x}\|_2$.

We perform experiments 100 times for each pair of s and n, then we save these errors and count the number of perfect recovered experiments. The criterion for perfect recovery is chosen as $\|\boldsymbol{xr} - \boldsymbol{x}\|_2 \leqslant 10^{-8}$.

Fig. 1 shows the recovery success rate of 512-length signals with different measurement numbers and sparsity level configurations. The image intensity indicates a success ratio with sparsity level s and measurements number n. Fig. 2 depicts its cross section at five different values of n. It can be seen from this figure that for $n \geqslant 32$, the recovery rate is about 80% when $s \leqslant n/5$ and practically 100% when $s \leqslant n/8$. These results are very similar to those reported in Ref. [1] which deals with the real signal with length $m = 512$.

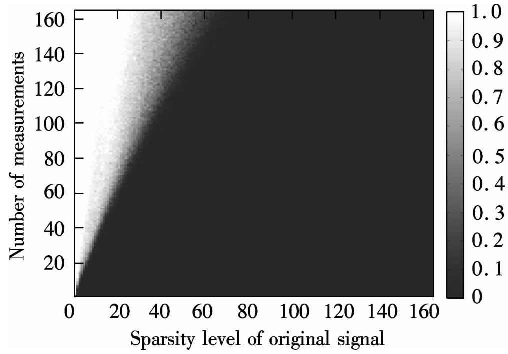

Fig. 1 Recovery experiment for $m = 512$

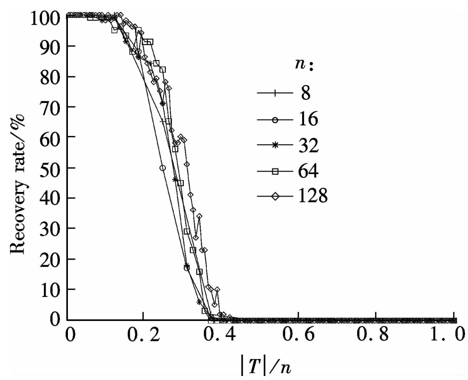

Fig. 2 Cross section of Fig. 1 at $n = 8, 16, 32, 64$ and 128

To further illustrate the recovery results, Fig. 3 and Fig. 4 depict the original quaternion signal $\boldsymbol{x}$ and its corresponding recovered signal $\boldsymbol{xr}$. In Fig. 3, $m = 512$, $n = 160$, and $s = 60$ and the quaternion signal is perfectly recovered. In Fig. 4, $m = 512$, $n = 160$, and $s = 120$ and in this case the recovery process failed.

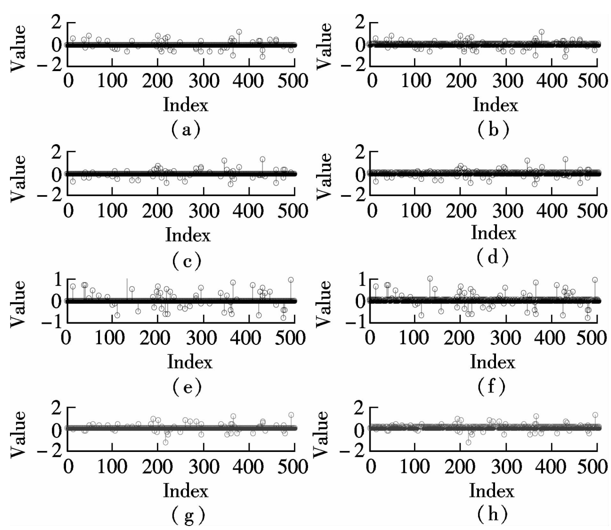

Fig. 3 Illustration of successful recovery of quaternion signal with sparsity level $s = 60$. (a) Original signal, r part; (b) Recovered signal, r part; (c) Original signal, i part; (d) Recovered signal, i part; (e) Original signal, j part; (f) Recovered signal, j part; (g) Original signal, k part; (h) Recovered signal, k part

In the noise case, the original signal $\boldsymbol{x}$ and measurement matrix $\boldsymbol{A}$ are generated as in the noiseless case $n =$

160, $m = 512$, $s = 60$. The measurements are corrupted by the white quaternion Gaussian noise vector comprised of i. i. d. Gaussian variables with mean 0 and variance σ^2, so the squared L_2-norm of noise vector $\|\boldsymbol{e}\|_2^2$ is a chi-square random variable with mean $4n\sigma^2$ and standard deviation $2\sqrt{2n}\sigma^2$. Since the probability that $\|\boldsymbol{e}\|_2^2$ exceeds its mean plus two or three standard deviations is small, here we choose the constraint parameter ε to be $\varepsilon = \sigma\sqrt{4n+4\sqrt{2n}}$.

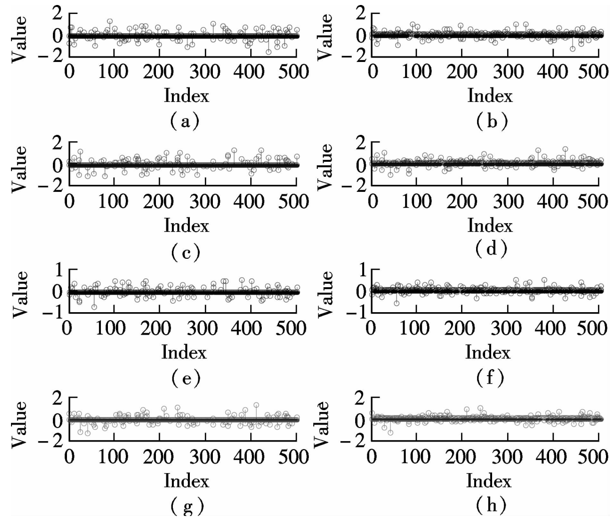

Fig. 4 Illustration of failed recovery of quaternion signal with sparsity level $s = 120$. (a) Original signal, r part; (b) Recovered signal, r part; (c) Original signal, i part; (d) Recovered signal, i part; (e) Original signal, j part; (f) Recovered signal, j part; (g) Original signal, k part; (h) Recovered signal, k part

We carried out ten recovery trials at different noise levels, and the average recovery errors with respect to corresponding noise strengths in those experiments are shown in Tab. 1. The results in Tab. 1 show that the proposed recovery algorithm is robust to noise contamination, and the recovery errors are about two times the noise strengths. An example of a recovery of sparse signals with noisy measurements is shown in Fig. 5 ($\sigma = 0.5$).

Tab. 1 Recovery errors with respect to corresponding noise strengths

σ	0.01	0.02	0.05	0.1	0.2	0.5
ε	0.27	0.53	1.33	2.67	5.34	13.3
$\|\boldsymbol{x}\|_2$	73.0	81.8	76.5	72.3	77.9	76.7
$\|\boldsymbol{e}\|_2$	0.25	0.51	1.30	2.62	5.15	12.7
$\|\boldsymbol{xr}-\boldsymbol{x}\|_2$	1.31	1.64	3.63	5.69	12.3	26.2

4 Conclusions and Future Work

In this paper, we propose an algorithm for solving the L_1-norm minimization problem of quaternion signals, which is converted to SOCP and then solved by SeDuMi software. Numerical examples are provided to illustrate the feasibility of the algorithm. The results can be viewed as a set of practical guidelines for situations where one expects perfect or stable recovery from random Gaussian quaternion measurement matrix information using SOCP. The main advantage of the proposed algorithm is that when converting the quaternion values optimization to that of real values, many mature toolboxes can be applied. However, the converting process decouples the real and imaginary parts of the quaternion signals and no prior phase information is exploited. Further research includes: 1) Considering the prior phase information in quaternion signals optimization[5]; 2) Extending the quaternion signal vector L_1-norm minimization algorithm to that of quaternion matrix nuclear norm minimization, that is, quaternion matrix completion[16]; 3) Studying the problem of robust quaternion principal component analysis[17].

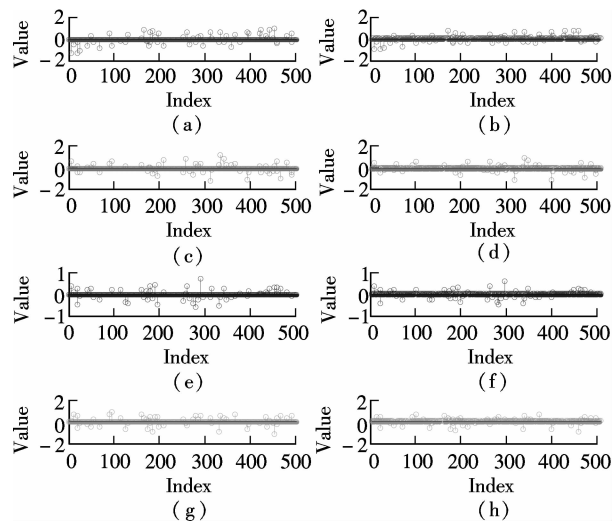

Fig. 5 Example of sparse signal recovery with noisy measurements. (a) Original signal, r part; (b) Recovered signal, r part; (c) Original signal, i part; (d) Recovered signal, i part; (e) Original signal, j part; (f) Recovered signal, j part; (g) Original signal, k part; (h) Recovered signal, k part

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基于 L_1 范数正则化的四元数信号重建

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摘要:提出了一种通过求解L_1范数最小化问题来重建四元数信号的算法,并且同时考虑了有噪声和没有噪声 2 种应用场景. 该算法首先将四元数域的L_1范数最小化问题转化为实数域的二次锥规划问题,然后通过工具包如 SeDuMi 来解决这个二次锥规划问题. 为了验证所提出算法的正确性和有效性,进行了相关的数值试验. 试验结果表明:在没有噪声的情况下,在某些实际可接受的条件下原始信号的精确重建是可以实现的;在有噪声的情况下,所提出的算法对于测量中的加性噪声具有鲁棒性. 该算法可以被应用于四元数域基于压缩感知理论的信号重建中.

关键词:四元数; 信号重建; 压缩感知
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