Journal of Southeast University (English Edition) Vol. 29, No. 1, pp. 52 - 56 Mar. 2013 ISSN 1003—7985

Reliability assessment considering dependent competing failure process and shifting-threshold

Su Chun Qu Zhongzhou Hao Huibing

(School of Mechanical Engineering, Southeast University, Nanjing 211189, China)

Abstract: The reliability assessment problem for products subject to degradation and random shocks is investigated. Two kinds of probabilistic models are constructed, in which the dependent competing failure process is considered. First, based on the assumption of cumulative shock, the probabilistic models for hard failure and soft failure are built respectively. On this basis, the dependent competing failure model involving degradation and shock processes is established. Furthermore, the situation of the shifting-threshold is also considered, in which the hard failure threshold value decreases to a lower level after the arrival of a certain number of shocks. A case study of fatigue crack growth is given to illustrate the proposed models. Numerical results show that shock has a significant effect on the failure process; meanwhile, the effect will be magnified when the value of the hard threshold shifts to a lower level.

Key words: degradation; hard failure; dependent competing failure process; cumulative shock model; shifting-threshold

doi: 10.3969/j. issn. 1003 - 7985. 2013. 01. 011

For mechanical parts, failures usually result from the competition between soft failure (degradation) and hard failure, and shock processes can speed up both of the failures. In recent years, studies on competing failures have been extensively explored; but most of them assume that the two kinds of failure processes are independent of each other[1-4].

In real circumstances, however, there exist correlations between the degradation process and random shocks. For example, the degradation process makes the system more vulnerable to random shocks, and random shocks can accelerate the degradation process. Up to now, different categories of random shock assumptions have been constructed, including the cumulative shock model[5], the extreme shock model[6], the mixed model[7] and the δ-shock model[8]. Moreover, effects of correlations, competing processes between shocks and degradation have also been studied. Su et al. [9] proposed a reliability assessment method considering competing failure based on the Wiener process. Peng et al. [10] developed reliability models for systems that undergo multiple dependent competing failure processes, in which two kinds of failure processes are dependent upon each other due to the impact from the same shock processes. Investigating both the degradation process and the shock process, Wang et al. [11] constructed a system reliability model on competitive failure processes with fuzzy degradation data, which was evaluated with a multi-state system reliability theory.

In this paper, the reliability analysis is conducted on the basis of the dependent competing failure process. In which, hard failure is caused by the shock process, while soft failure is the result of continuous degradation and abrupt damage from the same process. The cumulative shock model is applied for the sake of establishing two probabilistic models, and the correlation between hard failure and soft failure is considered. Based on that, reliability is consequently estimated including the situation of the shifting-threshold value. The case study with sensitivity analysis implies that the proposed models are in line with the actual situation, which also demonstrates that the proposed models can be applied to the components that endure dependent failure processes.

1 Dependent Competing Failure Process (DCFP)

As shown in Fig. 1, let $X(t)$ be the wear volume of the continuous degradation by time t, and it is monotonically increased with time. Shock loads will cause additional abrupt damage $Y_i(i=1, 2, \ldots)$ and speed up the degradation process. Soft failure will occur when the overall degradation, $X_s(t)$, is beyond the critical strength level H.

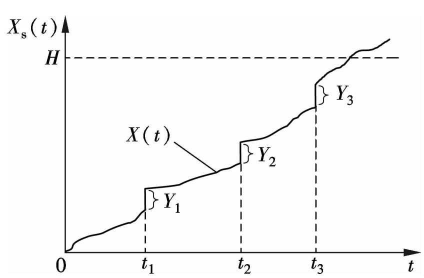

Fig. 1 Soft failure process

Jiang et al. [12] pointed out that when sustaining shocks,

Received 2012-09-16.

Biography: Su Chun (1970—), male, doctor, associate professor, suchun@ seu. edu. cn.

Foundation items: The National Natural Science Foundation of China (No. 50405021), Graduate Training Innovative Projects Foundation of Jiangsu Province (No. CXLX12_0081).

Citation: Su Chun, Qu Zhongzhou, Hao Huibing. Reliability assessment considering dependent competing failure process and shifting-threshold[J]. Journal of Southeast University (English Edition), 2013, 29(1): 52 - 56. [doi: 10.3969/j. issn. 1003 - 7985. 2013. 01. 011]

components become more susceptible to hard failures. Thus, the same random shock process can also result in hard failure. As seen in Fig. 2(a), let W_i denote the magnitude of the i-th shock ($i = 1, 2, \ldots$), hard failure occurs when the cumulative shock load magnitude exceeds the threshold value D_1. The system will fail when either of the two failures occurs. For most materials, the strength will gradually decrease with time. Fig. 2(b) shows that the critical strength value decreases from D_1 to D_2 after the arrival of a run of m shock loads.

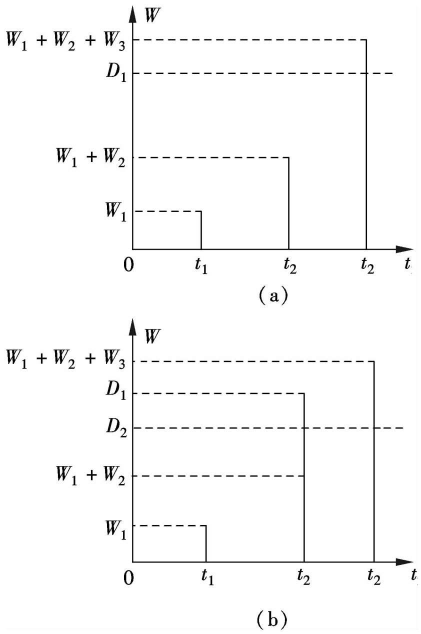

Fig. 2 Hard failure process. (a) Fixed threshold; (b) Shifting threshold

2 Reliability Modeling for DCFP

2.1 Shock process analysis

It is assumed that random shocks arrive according to a homogeneous Poisson process with rate λ. Let $N(t)$ denote the number of shocks until time t, then

$$P\{N(t)=i\} = \frac{(\lambda t)^i}{i!}\mathrm{e}^{-\lambda t} \qquad i = 0, 1, 2\ldots \tag{1}$$

As shown in Fig. 2(a), in the cumulative shock model, let T be the time that the system incurs hard failure, and the system will not fail until the cumulative shock damage exceeds the threshold value D_1. Thus, the reliability function of the hard failure process can take the form as

$$R_{\mathrm{h}} = P(t \leqslant T) = P\left(\sum_{i=1}^{N(t)} W_i < D_1\right) \tag{2}$$

Specifically, when the magnitudes of the shocks are independent identically distributed (i. i. d) random variables following a normal distribution as $W_i \sim N(\mu_W, \sigma_W^2)$, the reliability function can be obtained as

$$R_{\mathrm{h}} = P(t < T) = \Phi\left(\frac{D_1 - i\mu_W}{\sigma_W}\right) \tag{3}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normally distributed variable.

As shown in Fig. 2(b), the hard failure threshold value decreases from D_1 to D_2 right after the arrival of a run of m shocks. In such a shifting-threshold situation, the equivalent reliability function is

$$R_{\mathrm{h}} = P(t < T) = P\left(\sum_{i=1}^{m} W_i < D_1, \sum_{i=m+1}^{N(t)} W_i < D_2\right) \tag{4}$$

2.2 Soft failure model due to degradation and shocks

Descriptions for gradual degradation, $X(t)$, make a great difference in various studies[4, 7-8]. Some researchers apply the linear degradation path model[6, 12], and a stochastic process can also be employed [13].

Each random shock can cause abrupt damage. The abrupt damage in the overall degradation are measured by the shock damage sizes as $\{Y_1, Y_2, \cdots\}$. The cumulative damage size due to random shocks until time t is given as[10]

$$S(t) = \begin{cases} \sum_{j=1}^{N(t)} Y_j & \text{if } N(t) > 0 \\ 0 & \text{if } N(t) = 0 \end{cases} \tag{5}$$

where $N(t)$ is the total number of shocks to the system until time t.

Then the overall degradation of the system including gradual degradation and shock damage can be expressed as $X_s(t) = X(t) + S(t)$. Soft failure will occur when the overall degradation is beyond the threshold value H. Thus, the probability that the component survives is

$$P(X_s(t) < H) = \sum_{i=0}^{\infty} P(X(t) + S(t) \mid N(t) = i) \cdot P(N(t) = i) \tag{6}$$

In this paper, it is assumed that the magnitude of gradual degradation, $X(t)$, at time t follows a normal distribution as $N(\mu(t), \sigma^2(t))$. And the shock damage sizes are also i. i. d variables taking the form as $Y_i \sim N(\mu_Y, \sigma_Y^2)$. Considering the independence of the two random variables, $X_s(t)$ takes the distribution as $N(\mu(t) + N(t)\mu_Y, \sigma^2(t) + N(t)\sigma_Y^2)$. Reliability is equal to the probability that the total damage to the system has not exceeded the failure threshold. Using Eqs. (1) and (6), the reliability function for the soft failure process can be obtained as

$$R_s(t) = \Phi\left(\frac{H - \mu(t)}{\sigma(t)}\right)\mathrm{e}^{-\lambda t} + \sum_{i=1}^{\infty} \Phi\left(\frac{H - (i\mu_Y + \mu(t))}{\sqrt{\sigma(t)^2 + i\sigma_Y^2}}\right)\frac{\mathrm{e}^{-\lambda t}}{i!}(\lambda t)^i \tag{7}$$

2.3 System reliability analysis

2.3.1 Reliability analysis under a cumulative shock model

Fig. 2(a) shows a cumulative shock model; that is to say, a system is considered to be failed only when the cumulative magnitudes exceed the threshold value D_1. Although the two failure processes are dependent for being affected by the same shock process, it is still reasonable to assume that they are physically independent of each other[10]. Thus, the probability that the component survives from failure can be expressed as

$$\begin{aligned} &P\Big(X_s(t) < H, \sum_{k=1}^{N(t)} W_k < D_1\Big) = \\ &\quad P\Big(X(t) + \sum_{j=1}^{N(t)} Y_j < H, \sum_{k=1}^{N(t)} W_k < D_1\Big) = \\ &\quad P(X(t) < H)P(N(t) = 0) + \\ &\quad \sum_{i=1}^{\infty} P\Big(\sum_{k=1}^{i} W_k < D_1\Big) P\Big(X(t) + \sum_{j=1}^{i} Y_j < H\Big) \cdot \\ &\quad P(N(t) = i) \end{aligned} \tag{8}$$

It is also assumed that the shock process follows the homogeneous Poisson process described in Section 2.1. Based on the specific assumptions of $X(t)$, W_i, $Y_i (i = 1, 2, \cdots)$, and using Eqs. (3) and (7), the reliability function of the dependent competing failure process can be obtained as

$$\begin{aligned} R(t) &= \Phi\left(\frac{H - \mu(t)}{\sigma(t)}\right) e^{-\lambda t} + \\ &\sum_{i=1}^{\infty} \Phi\Big(\frac{D_1 - i\mu_W}{\sqrt{i}\sigma_W}\Big) \Phi\left(\frac{H - (\mu(t) + i\mu_Y)}{\sqrt{\sigma(t)^2 + i\sigma_Y^2}}\right) \frac{e^{-\lambda t}}{i!}(\lambda t)^i \end{aligned} \tag{9}$$

2.3.2 Reliability analysis due to a shifting-threshold

In Fig. 2(b), the hard failure threshold decreases from D_1 to D_2 right after the arrival of m shocks. This kind of problem was considered by Jiang et al[12]. In their research, a generalized run shock model was given. In this paper, we propose a different approach for reliability analysis based on the cumulative shock model. Similarly, by using Eqs. (4) and (7), we can obtain the reliability that the component survives from failure as

$$\begin{aligned} R(t) &= P(X(t) < H)P(N(t) = 0) + \\ &\sum_{i=1}^{m} P\Big(\sum_{k=1}^{i} W_k < D_1\Big) P\Big(X(t) + \sum_{j=1}^{i} Y_j < H\Big) P(N(t) = i) + \\ &\sum_{i=m+1}^{\infty} P\Big(\sum_{k=1}^{i} W_k < D_2\Big) P\Big(X(t) + \sum_{j=1}^{i} Y_j < H\Big) P(N(t) = i) \end{aligned} \tag{10}$$

When the assumptions made in the previous section are taken into consideration, the specific reliability function take the form as

$$\begin{aligned} R(t) &= \Phi\left(\frac{H - u(t)}{\sigma(t)}\right) e^{-\lambda t} + \\ &\sum_{i=1}^{m} \Phi\Big(\frac{D_1 - i\mu_W}{\sqrt{i}\sigma_W}\Big) \Phi\left(\frac{H - (\mu(t) + i\mu_Y)}{\sqrt{\sigma(t)^2 + i\sigma_Y^2}}\right) \frac{e^{-\lambda t}}{i!}(\lambda t)^i + \\ &\sum_{i=m+1}^{\infty} \Phi\Big(\frac{D_2 - i\mu_W}{\sqrt{i}\sigma_W}\Big) \Phi\left(\frac{H - (\mu(t) + i\mu_Y)}{\sqrt{\sigma(t)^2 + i\sigma_Y^2}}\right) \frac{e^{-\lambda t}}{i!}(\lambda t)^i \end{aligned} \tag{11}$$

3 Numerical Example

In this section, a case study is provided to illustrate the proposed models in Section 2. This example is based on the fatigue crack growth data of an alloy[14]. The loading cycles are considered to be the loading time t. Information about the shocking process is used to demonstrate the proposed model.

Fig. 3 shows the fatigue crack growth path. We select 13 samples out of a total of 21 samples since the remaining samples are not completed. The least squares method is employed to evaluate the parameters, and the results are

$$\mu(K) = 4.58 \times 10^{-6} K + 0.8745$$
$$\sigma(K) = 1.33 \times 10^{-6} K + 0.0008$$

where K is the number of cycles.

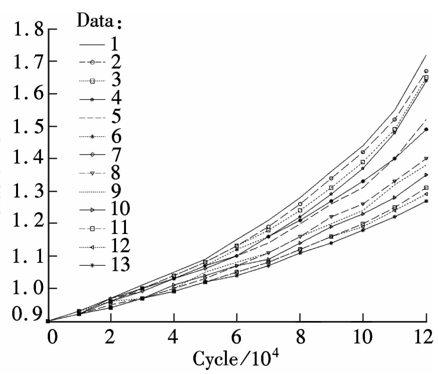

Fig. 3 Fatigue crack growth paths of samples

Sizes of random shock loads, $W_i (i = 1, 2, \cdots)$, which are measured in units of component life[3], are assumed to follow a normal distribution, $W_i \sim N(2, 0.5)$; the shock damage sizes take the form as $Y_i \sim N(0.02, 0.01)$ for $i = 1, 2, \cdots$, and the threshold value of soft failure $H = 2.0$ inches (1 inch = 25.4 mm); the threshold value of hard failure $D_1 = 35$ units; and the arrival rate $\lambda = 0.5 \times 10^{-4}$. In addition, we assume that $m = 3$, and the lower level of hard failure threshold value $D_2 = 25$ units.

According to Eqs. (9) and (11), the reliability curves of the two models constructed in Section 2.3 are plotted in Fig. 4, respectively. For Case 1, reliability almost remains at 1 when $K < 5 \times 10^4$ cycles. This is because the effect of random shocks is not significant and the gradual wear degradation amount is not large enough to cause any failure. In the next time period, with the gradual degradation and random shocks, reliability drops quickly, and the effect of shocks becomes a significant factor of the reliability performance. Eventually, reliability drops to a quite low level when $K > 2.5 \times 10^5$ cycles. Analysis of Case 2 can be obtained similarly.

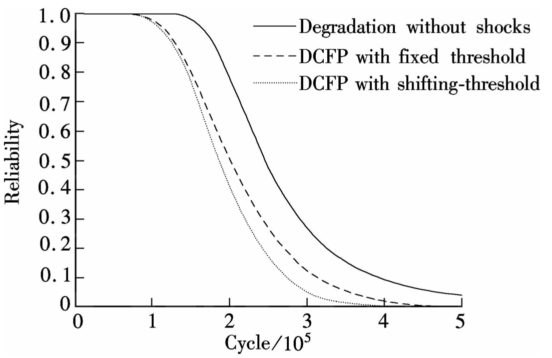

Fig. 4 Reliability function of different models

In addition, the reliability curve based on degradation without shocks is also provided in comparison with the two shock-considering models in Fig. 4. According to the results, it can be concluded that shocks will accelerate the failure process, which validates the effectiveness of models constructed in Section 2. Moreover, when the hard failure threshold value decreases from D_1 to D_2, failure will occur sooner.

Fig. 5 shows the reliability distributions with different rates of random shocks in Case 2. A higher arrival rate will make the reliability drop more quickly. It is reasonable because more intensively frequent random shocks, larger sizes of shock loads and shock damages will resultantly make a component more vulnerable.

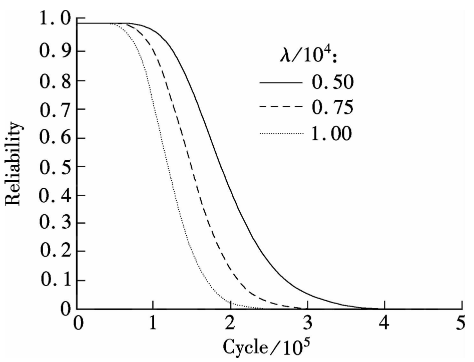

Fig. 5 Sensitivity analysis for λ

4 Conclusion

This paper focuses on the reliability analysis of components with dependent competing failure processes due to hard failure and soft failure. Generalized probabilistic methods based on the cumulative shock model are proposed and two specific models with normal distribution are obtained. Compared with the degradation process without shocks, models established in this paper indicate that random shocks have significant effects on the failure process. When considering the shifting-threshold situation, reliability decreases even faster.

The two proposed models only consider one degradation path. In real situations, components may have multiple degradation measures, and this will be the focus of our future research.

References

[1] Li W J, Pham H. Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks[J]. *IEEE Transactions on Reliability*, 2005, **54**(2): 297 − 303.

[2] Lehmann A. Joint modeling of degradation and failure time data [J]. *Journal of Statistical Planning and Inference*, 2009, **139**(5): 1693 − 1706.

[3] Klutke G A, Yang Y J. The availability of inspected systems subjected to shocks and graceful degradation[J]. *IEEE Transactions on Reliability*, 2002, **51**(3): 371 − 374.

[4] Deloux E, Castanier B, Berenguer C. Predictive maintenance policy for a gradually deteriorating system subject to stress [J]. *Reliability Engineering and System Safety*, 2009, **94**(2): 418 − 431.

[5] Kijima M, Nakagawa T. A cumulative damage shock model with imperfect preventive maintenance[J]. *Naval Research Logistics*, 1991, **38**(2): 145 − 156.

[6] Chen J Y, Li Z H. An extended extreme shock maintenance model for a deteriorating system [J]. *Reliability Engineering and System Safety*, 2008, **93**(8): 1123 − 1129.

[7] Allan Gut. Mixed shock models[J]. *Bernoulli*, 2001, **7**(3): 541 − 555.

[8] Bai J M, Li Z H, Kong X B, Generalized shock models based on a cluster point process[J]. *IEEE Transactions on Reliability*, 2006, **55**(3):542 − 550.

[9] Su Chun, Zhang Ye. System reliability assessment based on Wiener process and competing failure analysis [J]. *Journal of Southeast University: English Edition*, 2010, **26**(4): 554 − 557.

[10] Peng H, Feng Q M, Coit D W. Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes[J]. *IIE Transactions*, 2010, **43**(1):12 − 22.

[11] Wang Z L, Huang H Z. Reliability analysis on competitive failure processes under fuzzy degradation data[J]. *Applied Soft Computing*, 2011, **11**(3):2964 − 2973.

[12] Jiang L, Feng Q M, Coit D W. Reliability analysis for dependent failure processes and dependent failure threshold[C]//2011 *International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering.* Xi'an, China, 2011: 30 − 34.

[13] van Noortwijk J M, van der Weide J A M, Kallen M J, et al. Gamma processes and peaks-over-threshold distributions for time-dependent reliability [J]. *Reliability Engineering and System Safety*, 2007, **92**(12): 1651 − 1658.

[14] Lu C J, Meeker W Q. Using degradation measures to estimate a time-to-failure distribution [J]. *Technometrics*, 1993, **35**(2): 161 − 174.

考虑相关竞争故障过程及变动阈值的可靠性评估

苏　春　瞿众洲　郝会兵

(东南大学机械工程学院,南京 211189)

摘要:研究了受退化和随机冲击共同作用的产品可靠性评估问题. 考虑相关竞争故障过程,建立了2类概率模型. 首先,基于累积冲击假设,分别建立了硬故障概率模型和软故障概率模型. 以此为基础,考虑退化和冲击过程建立了相关竞争故障模型. 此外,研究了因多次冲击载荷作用而导致的硬故障阈值下降的情形,分析故障阈值的变动对产品可靠性的影响. 以一组裂纹增长数据为例,验证模型的有效性. 研究结果表明:冲击对产品故障过程有着显著影响,同时硬故障阈值的下降会进一步加剧上述效应.

关键词:退化;硬故障;相关竞争故障过程;累积冲击模型;变动阈值

中图分类号:TH17