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Linear arboricity of Cartesian products of graphs

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Abstract: A linear forest is a forest whose components are paths. The linear arboricity $\mathrm{la}(G)$ of a graph G is the minimum number of linear forests which partition the edge set $E(G)$ of G. The Cartesian product $G\square H$ of two graphs G and H is defined as the graph with vertex set $V(G\square H)=\{(u,v)\mid u\in V(G),\ v\in V(H)\}$ and edge set $E(G\square H)=\{(u,x)(v,y)\mid u=v$ and $xy\in E(H)$, or $uv\in E(G)$ and $x=y\}$. Let P_m and C_m, respectively, denote the path and cycle on m vertices and K_n denote the complete graph on n vertices. It is proved that $\mathrm{la}(K_n\square P_m)=\left\lceil\frac{n+1}{2}\right\rceil$ for $m\geqslant 2$, $\mathrm{la}(K_n\square C_m)=\left\lceil\frac{n+2}{2}\right\rceil$, and $\mathrm{la}(K_n\square K_m)=\left\lceil\frac{n+m-1}{2}\right\rceil$. The methods to decompose these graphs into linear forests are given in the proofs. Furthermore, the linear arboricity conjecture is true for these classes of graphs.

Key words: linear forest; linear arboricity; Cartesian product

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In this paper, all the graphs are simple, finite and undirected. For a real number x, $\lceil x\rceil$ is the least integer not less than x and $\lfloor x\rfloor$ is the largest integer not larger than x. Let G be a graph. We use $V(G)$, $E(G)$ and $\Delta(G)$ to denote the vertex set, the edge set and the maximum degree of G, respectively.

A linear forest is a forest whose components are paths. The linear arboricity $\mathrm{la}(G)$ of G defined by Harary[1] is the minimum number of linear forests needed to partition the edge set $E(G)$ of G.

Akiyama et al.[2] conjectured that $\mathrm{la}(G)=\lceil(\Delta(G)+1)/2\rceil$ for any regular graph G. They proved that the conjecture is true for complete graphs and graphs with $\Delta=3, 4$[2-3]. Enomoto and Péroche[4] proved that the conjecture is true for graphs with $\Delta=5, 6, 8$. Guldan[5] proved that the conjecture is true for graphs with $\Delta=10$. It is obvious that $\mathrm{la}(G)\geqslant\lceil\Delta(G)/2\rceil$ for every graph G and $\mathrm{la}(G)\geqslant\lceil(\Delta(G)+1)/2\rceil$ for every regular graph G. So the conjecture is equivalent to the following linear arboricity conjecture (LAC)[2]. For any graph G, $\lceil\Delta(G)/2\rceil\leqslant\mathrm{la}(G)\leqslant\lceil(\Delta(G)+1)/2\rceil$.

Akiyama et al.[2] determined the linear arboricity of complete bipartite graphs and trees. Martinova[6] determined the linear arboricity of the maximal outerplanar graphs. Wu et al.[7-8] proved that the LAC is true for all the planar graphs. Wu[9] also determined the linear arboricity of the series-parallel graphs. Some other researches on linear arboricity can be found in Refs. [10 - 12].

The Cartesian product of two graphs G and H (or simply product), denoted by $G\square H$, is defined as the graph with vertex set $V(G\square H)=\{(u,v)\mid u\in V(G),\ v\in V(H)\}$ and edge set $E(G\square H)=\{(u,x)(v,y)\mid u=v$ and $xy\in E(H)$, or $uv\in E(G)$ and $x=y\}$. Let P_m and C_m respectively, denote the path and cycle on m vertices and K_n denote the complete graph on n vertices. In this paper, we determine the linear arboricity of $K_n\square P_m$, $K_n\square C_m$ and $K_n\square K_m$.

The following lemmas are useful in our proofs.

Lemma 1 If H is a subgraph of G, then $\mathrm{la}(H)\leqslant\mathrm{la}(G)$.

Lemma 2 $\mathrm{la}(G\square H)\leqslant\mathrm{la}(G)+\mathrm{la}(H)$.

Lemma 2 holds by the definition of the linear arboricity and the Cartesian product of graphs.

Lemma 3[2] $\mathrm{la}(K_n)=\lceil n/2\rceil$.

Lemma 4[13] For $n\geqslant 3$, the complete graph K_n is decomposable into edge disjoint Hamilton cycles if and only if n is odd. For $n\geqslant 2$, the complete graph K_n is decomposable into edge disjoint Hamilton paths if and only if n is even.

Lemma 5[14] Let $V(K_{2n})=\{v_0, v_1, \dots, v_{2n-1}\}$. For $0\leqslant i\leqslant n-1$, put

$$F_i=v_{0+i}v_{1+i}v_{2n-1+i}v_{2+i}v_{2n-2+i}\dots v_{n+1+i}v_{n+i}$$

where the indices of v_j's are taken modulo $2n$. Then F_0, F_1, …, F_{n-1} are disjoint Hamilton paths of K_{2n}; i.e., K_{2n} is decomposed into edge disjoint Hamilton paths F_0, F_1, …, F_{n-1}.

1 la($K_n\square P_m$)

Let $V(K_n)=\{u, v_0, v_1, \dots, v_{n-2}\}$ and $V(P_m)=\{y_0, y_1, \dots, y_{m-1}\}$. For convenience, we denote any vertex $(x, y_j)\in V(K_n\square P_m)$ by $x^{(j)}$. For a fixed j ($j=0, 1, \dots, m-1$), we use $K_n^{(j)}$ to denote the complete graph induced by $\{u^{(j)}, v_0^{(j)}, v_1^{(j)}, \dots, v_{n-2}^{(j)}\}$.

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The following lemma deals with the decomposition of the complete graph K_{2n+1}.

Lemma 6 $E(K_{2n+1}) = nP_{2n+1} \cup M_n$, where M_n is a matching of order n.

Proof Let $V(K_{2n+1}) = \{u, v_0, v_1, \ldots, v_{2n-1}\}$. For $0 \leqslant i \leqslant n-1$, put

$$F_i = v_{0+i}v_{1+i}v_{2n-1+i}v_{2+i}v_{2n-2+i}\cdots v_{n+1+i}v_{n+i}$$

where the indices of v_j's are taken modulo $2n$. Then, by Lemma 5, the complete graph $K_{2n+1}\setminus\{u\}$ is decomposed into n disjoint Hamilton paths: $F_0, F_1, \ldots, F_{n-1}$. For $0 \leqslant i \leqslant n-1$, let e_i be the n-th edge of F_i and $M_n = \{e_0, e_1, \ldots, e_{n-1}\}$. Then $e_i = v_{i+\lceil n/2\rceil}v_{i-\lceil n/2\rceil}$ for $i = 0, 1, \ldots, n-1$ and $M_n = \{v_0v_n, v_1v_{n+1}, \ldots, v_{n-1}v_{2n-1}\}$. Clearly, M_n is a matching of order n. For each $0 \leqslant i \leqslant n-1$, by deleting e_i from F_i and adding two edges uv_i, uv_{n+i} to F_i, we obtain a path on $2n+1$ vertices. The n paths obtained in this way together with M_n form a decomposition of K_{2n+1} as claimed in the lemma.

Theorem 1 $\operatorname{la}(K_n \square P_m) = \left\lceil \frac{n+1}{2} \right\rceil$ for $m \geqslant 2$.

Proof If $m = 2$, then $\operatorname{la}(K_n \square P_m) \geqslant \left\lceil \frac{n+1}{2} \right\rceil$ since $K_n \square P_m$ is n-regular. If $m \geqslant 3$, then $\operatorname{la}(K_n \square P_m) \geqslant \left\lceil \frac{\Delta}{2} \right\rceil = \left\lceil \frac{n+1}{2} \right\rceil$, where $\Delta = \Delta(K_n \square P_m)$. We now prove the reverse inequality. If n is even, then $\operatorname{la}(K_n \square P_m) \leqslant \operatorname{la}(K_n) + \operatorname{la}(P_m) = \left\lceil \frac{n}{2} \right\rceil + 1 = \left\lceil \frac{n+1}{2} \right\rceil$ by Lemmas 2 and 3. Thus Theorem 1 holds for even n.

Now suppose that n is odd. Let $n = 2k+1$, where $k \geqslant 1$. For $0 \leqslant i \leqslant k-1$ and $0 \leqslant j \leqslant m-1$, put

$$F_i^{(j)} = v_{0+i}^{(j)}\, v_{1+i}^{(j)}\, v_{2k-1+i}^{(j)}\, v_{2+i}^{(j)}\, v_{2k-2+i}^{(j)} \cdots v_{k+1+i}^{(j)}\, v_{k+i}^{(j)}$$

where the indices of $v_i^{(j)}$'s are taken modulo $2k$. Then by Lemmas 4 and 5, for $0 \leqslant j \leqslant m-1$, $K_n^{(j)}$ is decomposed into k edge disjoint Hamilton cycles $C_i^{(j)} = u^{(j)}F_i^{(j)}u^{(j)}$ ($i = 0, 1, \ldots, k-1$).

Let $x_i^{(j)}y_i^{(j)}$ be the k-th edge of $F_i^{(j)}$ and $H_i^{(j)} = C_i^{(j)} \setminus \{x_i^{(j)}y_i^{(j)}\}$ for $0 \leqslant i \leqslant k-1$ and $0 \leqslant j \leqslant m-1$. From the proof of Lemma 6, each complete graph $K_n^{(j)}$ can be decomposed into k edge disjoint Hamilton paths $H_0^{(j)}, H_1^{(j)}, \ldots, H_{k-1}^{(j)}$ and a matching $M_k^{(j)} = \{x_0^{(j)}\ y_0^{(j)}, x_1^{(j)}y_1^{(j)}, \ldots, x_{k-1}^{(j)}y_{k-1}^{(j)}\}$.

Let $N_{x_i} = \{x_i^{(j)}\ x_i^{(j+1)} \mid j = 1, 3, \ldots, s\}$, where $s = m-2$ if m is odd and $s = m-3$ if m is even; and $N_{y_i} = \{y_i^{(j)}y_i^{(j+1)} \mid j = 0, 2, \ldots, t\}$, where $t = m-3$ if m is odd and $t = m-2$ if m is even.

Let $L_i = (\cup_{j=0}^{m-1} H_i^{(j)}) \cup N_{x_i} \cup N_{y_i}$ for $0 \leqslant i \leqslant k-1$. Then $L_0, L_1, \ldots, L_{k-1}$ are k edge disjoint Hamilton paths of $K_n \square P_m$. After we take away these Hamilton paths from $K_n \square P_m$, the remaining edges form a linear forest. Thus, $\operatorname{la}(K_n \square P_m) \leqslant k + 1 = \frac{n-1}{2} + 1 = \left\lceil \frac{n+1}{2} \right\rceil$. This completes the proof.

2 $\operatorname{la}(K_n \square C_m)$

Theorem 2 $\operatorname{la}(K_n \square C_m) = \left\lceil \frac{n+2}{2} \right\rceil$.

Proof Since $K_n \square C_m$ can be decomposed into a $K_n \square P_m$ and a matching of size n, we have $\operatorname{la}(K_n \square C_m) \leqslant \operatorname{la}(K_n \square P_m) + 1 = \left\lceil \frac{n+1}{2} \right\rceil + 1$ by Theorem 1. On the other hand, since $K_n \square C_m$ is $(n+1)$-regular, $\operatorname{la}(K_n \square C_m) \geqslant \left\lceil \frac{n+1+1}{2} \right\rceil = \left\lceil \frac{n+2}{2} \right\rceil$. If n is odd, then $\operatorname{la}(K_n \square C_m) \leqslant \frac{n+3}{2} = \left\lceil \frac{n+2}{2} \right\rceil$. Therefore the theorem holds for odd n.

Now we consider the case that n is even. Note that $\operatorname{la}(K_n \square C_m) \geqslant \frac{n+2}{2}$, we only need to show that $K_n \square C_m$ can be decomposed into $\frac{n+2}{2}$ linear forests. Let $n = 2k$, where $k \geqslant 1$. Let $V(K_n) = \{v_0, v_1, \ldots, v_{2k-1}\}$ and $V(C_m) = \{y_0, y_1, \ldots, y_{m-1}\}$. For convenience, we denote any vertex $(v_i, y_j) \in V(K_n \square C_m)$ by $v_i^{(j)}$. For a fixed j ($j = 0, 1, \ldots, m-1$), we use $K_n^{(j)}$ to denote the complete graph induced by $\{v_0^{(j)}, v_1^{(j)}, \ldots, v_{2k-1}^{(j)}\}$. By Lemma 5, for $0 \leqslant j \leqslant m-1$, each $K_n^{(j)}$ can be decomposed into k edge disjoint Hamilton paths $F_i^{(j)}$ ($i = 0, 1, \ldots, k-1$), where

$$F_i^{(j)} = v_{0+i}^{(j)}v_{1+i}^{(j)}v_{2k-1+i}^{(j)}v_{2+i}^{(j)}v_{2k-2+i}^{(j)}\cdots v_{k+1+i}^{(j)}v_{k+i}^{(j)}$$

and the subscripts are taken modulo $2k$.

For $i = 0, 1, \ldots, k-1$, let $L_i = (\cup_{j=0}^{m-1} F_i^{(j)}) \cup \{v_i^{(0)}v_i^{(1)}\} \cup \{v_{i+k}^{(m-2)}v_{i+k}^{(m-1)}\}$. It is easy to see that $L_0, L_1, \ldots, L_{k-1}$ are k edge disjoint linear forests and the remaining edges in $K_n \square C_m$ form one linear forest. Thus, $\operatorname{la}(K_n \square C_m) \leqslant k + 1 = \frac{n+2}{2}$, which completes the proof.

3 $\operatorname{la}(K_n \square K_m)$

Theorem 3 $\operatorname{la}(K_n \square K_m) = \frac{n+m}{2}$ if n and m are both even.

Proof By Lemmas 2 and 3, $\operatorname{la}(K_n \square K_m) \leqslant \operatorname{la}(K_n) + \operatorname{la}(K_m) = \frac{n}{2} + \frac{m}{2} = \frac{n+m}{2}$. Since $K_n \square K_m$ is $(n+m-2)$-regular, $\operatorname{la}(K_n \square K_m) \geqslant \left\lceil \frac{n+m-2+1}{2} \right\rceil = \frac{n+m}{2}$.

Now, we consider the case that at least one of n, m is odd.

Theorem 4 $\operatorname{la}(K_n \square K_m) = \frac{n+m-1}{2}$ if n is even and m is odd.

Proof Let $n = 2k$, $k \geqslant 1$. Let $V(K_n) = \{v_0, v_1, \ldots, v_{n-1}\}$ and $V(K_m) = \{y_0, y_1, \ldots, y_{m-1}\}$. For convenience, we denote any vertex $(v_i, y_j) \in V(K_n \square K_m)$ by $v_i^{(j)}$. For a fixed j ($j = 0, 1, \ldots, m-1$), we use $K_n^{(j)}$ to denote the complete graph induced by $\{v_0^{(j)}, v_1^{(j)}, \ldots, v_{n-1}^{(j)}\}$. For a fixed i ($i = 0, 1, \ldots, n-1$), we use $K_m^{(i)}$ to denote the complete graph induced by $\{v_i^{(0)}, v_i^{(1)}, \ldots, v_i^{(m-1)}\}$. By Lemma 5, for $0 \leqslant j \leqslant m-1$, each $K_n^{(j)}$ can be decomposed into k edge disjoint Hamilton paths $F_i^{(j)}$ ($i = 0, 1, \ldots, k-1$), where

$$F_i^{(j)} = v_{0+i}^{(j)} v_{1+i}^{(j)} v_{2k-1+i}^{(j)} v_{2+i}^{(j)} v_{2k-2+i}^{(j)} \ldots v_{k+1+i}^{(j)} v_{k+i}^{(j)}$$

and the subscripts are taken modulo $2k$.

For $i = 0, 1, \ldots, k-1$, let $N_i = \{v_i^{(j)} v_i^{(j+1)} \mid j = 0, 2, \ldots, m-3\}$ and $N_{i+k} = \{v_{i+k}^{(j)} v_{i+k}^{(j+1)} \mid j = 1, 3, \ldots, m-2\}$. It is easy to see that each N_i ($i = 0, 1, \ldots, 2k-1$) is a matching of $K_m^{(i)}$ and $|N_i| = \frac{m-1}{2}$. By Lemma 6, the edges in each $E(K_m^{(i)}) \backslash N_i$ ($i = 0, 1, \ldots, 2k-1$) can be partitioned into $\frac{m-1}{2}$ Hamilton paths. So the edges in $E(K_m^{(i)}) \backslash N_i$ ($i = 0, 1, \ldots, 2k-1$) form $\frac{m-1}{2}$ linear forests together. Furthermore, for $0 \leqslant i \leqslant k-1$, each $(\cup_{j=0}^{m-1} F_i^{(j)}) \cup N_i \cup N_{i+k}$ forms a linear forest. So $\mathrm{la}(K_n \square K_m) \leqslant \frac{m-1}{2} + k = \frac{m-1}{2} + \frac{n}{2} = \frac{n+m-1}{2}$. On the other hand, $\mathrm{la}(K_n \square K_m) \geqslant \left\lceil \frac{n+m-2+1}{2} \right\rceil = \frac{n+m-1}{2}$ since $K_n \square K_m$ is $(n+m-2)$-regular. This completes the proof.

Theorem 5 $\mathrm{la}(K_n \square K_m) = \frac{n+m}{2}$ if n and m are both odd.

Proof We use the same notations in Theorem 4; i.e., let $V(K_n \square K_m) = \{v_i^{(j)} \mid i = 0, 1, \ldots, n-1; j = 0, 1, \ldots, m-1\}$. For a fixed j ($j = 0, 1, \ldots, m-1$), we use $K_n^{(j)}$ to denote the complete graph induced by $\{v_0^{(j)}, v_1^{(j)}, \ldots, v_{n-1}^{(j)}\}$. For a fixed i ($i = 0, 1, \ldots, n-1$), we use $K_m^{(i)}$ to denote the complete graph induced by $\{v_i^{(0)}, v_i^{(1)}, \ldots, v_i^{(m-1)}\}$.

Let $H_i = \{v_i^{(j)} v_i^{(j+1)} \mid j = 0, 2, \ldots, m-3\}$ for $i = 0, 1, 3, 5, \ldots n-2$ and $H_i = \{v_i^{(j)} v_i^{(j+1)} \mid j = 1, 3, \ldots, m-2\}$ for $i = 2, 4, 6, \ldots n-1$. Then each H_i is a matching of $K_m^{(i)}$ with $\frac{m-1}{2}$ edges. By Lemma 6, the edges in each $E(K_m^{(i)}) \backslash H_i$ ($i = 0, 1, \ldots, n-1$) can be partitioned into $\frac{m-1}{2}$ Hamilton paths. So the edges in $E(K_m^{(i)}) \backslash H_i$ ($i = 0, 1, \ldots, n-1$) form $\frac{m-1}{2}$ linear forests together.

Let $L_j = \{v_1^{(j)} v_2^{(j)}, v_3^{(j)} v_4^{(j)}, \ldots, v_{n-2}^{(j)} v_{n-1}^{(j)}\}$ for $j = 0, 1, \ldots m-1$. Then each L_j is a matching of $K_n^{(j)}$ with $\frac{n-1}{2}$ edges. Again by Lemma 6, the edges in each $E(K_n^{(i)}) \backslash L_j$ ($j = 0, 1, \ldots, m-1$) can be partitioned into $\frac{n-1}{2}$ Hamilton paths. So the edges in $E(K_n^{(i)}) \backslash L_j$ ($j = 0, 1, \ldots, m-1$) form $\frac{n-1}{2}$ linear forests together.

It is clear that $(\cup_{i=0}^{n-1} H_i) \cup (\cup_{j=0}^{m-1} L_j)$ forms a linear forest. So $\mathrm{la}(K_n \square K_m) \leqslant \frac{n-1}{2} + \frac{m-1}{2} + 1 = \frac{n+m}{2}$. On the other hand, since $K_n \square K_m$ is $(n+m-2)$-regular, $\mathrm{la}(K_n \square K_m) \geqslant \left\lceil \frac{n+m-2+1}{2} \right\rceil = \frac{n+m}{2}$. This completes the proof.

Summarizing Theorems 3 to 5, we have the following theorem.

Theorem 6 $\mathrm{la}(K_n \square K_m) = \left\lceil \frac{n+m-1}{2} \right\rceil$.

References

[1] Harary F. Covering and packing in graphs 1 [J]. *Annals of the New York Academy of Sciences*, 1970, **175**(1): 198 - 205.

[2] Akiyama J, Exoo G, Harary F. Covering and packing in graphs 3: cyclic and acyclic invariants [J]. *Math Slovaca*, 1980, **30**(4): 405 - 417.

[3] Akiyama J, Exoo G, Harary F. Covering and packing in graphs 4: linear arboricity [J]. *Networks*, 1981, **11**(1): 69 - 72.

[4] Enomoto H, Péroche B. The linear arboricity of some regular graphs [J]. *Journal of Graph Theory*, 1984, **8**(2): 309 - 324.

[5] Guldan F. The linear arboricity of 10-regular graphs [J]. *Math Slovaca*, 1986, **36**(3): 225 - 228.

[6] Martinova M K. Linear arboricity of maximal outerplanar graphs [J]. *Godishnik Vissh Uchebn Zaved Prilozhna Math*, 1987, **23**: 147 - 155. (in Bulgarian)

[7] Wu Jianliang. On the linear arboricity of planar graphs [J]. *Journal of Graph Theory*, 1999, **31**(2): 129 - 134.

[8] Wu Jianliang, Wu Yuwen. The linear arboricity of planar graphs of maximum degree seven are four [J]. *Journal of Graph Theory*, 2008, **58**(3): 210 - 220.

[9] Wu Jianliang. The linear arboricity of series-parallel graphs [J]. *Graphs and Combinatorics*, 2000, **16**(3): 367 - 372.

[10] Lu Xiaoxu, Xu Baogang. A note on vertex-arboricity of plane graphs [J]. *Journal of Nanjing University*: *Natural Sciences*, 2007, **43**(1): 13 - 18.

[11] Tan Xiang, Chen Hongyu, Wu Jianliang. The linear arboricity of planar graphs with maximum degree at least five [J]. *Bulletin of the Malaysian Mathematical Sciences Society*, 2011, **34**(3): 541 - 552.

[12] Wu Jianliang, Hou Jianfeng, Liu Guizhen. The linear arboricity of planar graphs with no short cycles [J]. *Theoretical Computer Science*, 2007, **381**(1/2/3): 230 - 233.

[13] Bollobàs B. *Modern graph theory* [M]. New York: Springer-Verlag, 1998.

[14] Chen B L, Huang K C. On the linear k-arboricity of K_n and $K_{n,n}$ [J]. *Discrete Math*, 2002, **254**(1/2/3): 51 - 61.

笛卡尔积图的线性荫度

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摘要:线性森林是指所有分支都是路的森林. 图 G 的线性荫度 $\mathrm{la}(G)$ 是划分 G 的边集 $E(G)$ 所需的线性森林的最小数目. 图 G 和 H 的笛卡尔积图 $G\square H$ 定义为:顶点集 $V(G\square H)=\{(u,v)\mid u\in V(G),\ v\in V(H)\}$. 边集 $E(G\square H)=\{(u,x)(v,y)\mid u=v$ 且 $xy\in E(H)$, 或 $uv\in E(G)$ 且 $x=y\}$. 令 P_m 与 C_m 分别表示 m 个顶点的路和圈, K_n 表示 n 个顶点的完全图. 证明了 $\mathrm{la}(K_n\square P_m)=\left\lceil\frac{n+1}{2}\right\rceil(m\geqslant 2)$, $\mathrm{la}(K_n\square C_m)=\left\lceil\frac{n+2}{2}\right\rceil$以及 $\mathrm{la}(K_n\square K_m)=\left\lceil\frac{n+m-1}{2}\right\rceil$. 证明过程给出了将这些图分解成线性森林的方法. 进一步的线性荫度猜想对这些图类是成立的.

关键词:线性森林;线性荫度;笛卡尔积
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