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Ordered successive noise projection cancellation algorithm for dual lattice-reduction-aided MIMO detection

Liu Jinzhu[1, 2] Xing Song[3] Shen Lianfeng[1]

([1]National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China)
([2]School of Electronic & Information Engineering, Nanjing University of Information Science & Technology, Nanjing 210044, China)
([3]Department of Information Systems, California State University, Los Angeles, CA 90032, USA)

Abstract: A novel nonlinear multi-input multi-output (MIMO) detection algorithm is proposed, which is referred to as an ordered successive noise projection cancellation (OSNPC) algorithm. It is capable of improving the computation performance of the MIMO detector with the conventional ordered successive interference cancellation (OSIC) algorithm. In contrast to the OSIC in which the known interferences in the input signal vector are successively cancelled, the OSNPC successively cancels the known noise projections from the decision statistic vector. Analysis indicates that the OSNPC is equivalent to the OSIC in error performance, but it has significantly less complexity in computation. Furthermore, when the OSNPC is applied to the MIMO detection with the preprocessing of dual lattice reduction (DLR), the computational complexity of the proposed OSNPC-based DLR-aided detector is further reduced due to the avoidance of the inverse of the reduced basis of the dual lattice in computation, compared to that of the OSIC-based one. Simulation results validate the theoretical conclusions with regard to both the performance and complexity of the proposed MIMO detection scheme.

Key words: ordered successive noise projection cancellation (OSNPC); dual lattice reduction (DLR); multi-input multi-output (MIMO) detection; ordered successive interference cancellation (OSIC)

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Among various detection schemes for the multi-input multi-output (MIMO) wireless communication systems, the maximum likelihood (ML) MIMO detector provides optimal detection performance, but its computational complexity increases exponentially when numbers of transmit antennas increase[1]. The reduced-complexity detection algorithms, which are not at all optimal, can be classified as linear and nonlinear detectors. The conventional linear zero forcing (ZF)/minimum-mean-squared-error (MMSE) detectors, for example, typically exhibit a low complexity. The nonlinear ordered successive interference cancellation (OSIC) algorithm which detects each symbol sequentially via classic remodulation and subtraction based canceling operations demonstrates its excellent trade-off between computational complexity and error performance. However, all these suboptimal schemes perform significantly worse than the optimal ML detector[2-4].

Recently, the lattice reduction (LR) has been introduced as a promising technique which can improve the performance of many suboptimal MIMO detectors[5]. Particularly, the OSIC-based LR-aided MIMO detection scheme can achieve full diversity and present near optimal detection performance with acceptable complexity[6]. In MIMO detection applications, LR can aim at the primal lattice which is generated by the channel gain matrix, and the corresponding detection scheme is called the primal LR (PLR)-aided MIMO detection. Correspondingly, the dual LR (DLR)-aided MIMO detection is also feasible, which features that the dual lattice basis, i. e., the Moore-Penrose (MP) inverse of the channel gain matrix, is reduced in the detection procedure[6]. Moreover, in particular scenarios the DLR-aided detection is preferable due to its low computational complexity and/or good performance [6-7].

Nevertheless, the complexity of the traditional OSIC-based MIMO detection scheme (also referred to as the VBLAST algorithm) is still considerably high due to its repeated inverse calculations of the successive deflated channel gain matrix[3]. Specifically, when the OSIC algorithm is used in DLR-aided detections, the overall complexity will be further increased. This is due to the fact that the successive interference cancellation is carried out at the primal lattice side. While what is obtained by the DLR is the reduced dual lattice basis, the MP inverse must be still calculated[6].

In this paper, we propose a novel nonlinear MIMO detection algorithm to improve the performance of the MIMO detector with the conventional OSIC algorithm. Un-

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Biographies: Liu Jinzhu (1963—), male, graduate; Shen Lianfeng (corresponding author), male, professor, lfshen@seu.edu.cn.

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like the OSIC algorithm in which the known interferences in the input signal vector are successively cancelled, the proposed algorithm successively cancels the noise projections from the decision statistic vector. Hence we refer to our algorithm as an ordered successive noise projection cancellation (OSNPC) algorithm. Theoretical analysis and simulation results show that the OSNPC algorithm is equivalent to the traditional OSIC algorithm in performance, but it has significantly less complexity in computation. Furthermore, when it is applied to the DLR-aided MIMO detection, the computational complexity is even more reduced due to the avoidance of the inverse of the reduced basis of the dual lattice.

1 OSIC-Based LR-Aided MIMO Detection

Consider a MIMO wireless communication system consisting of N transmitters and M receivers ($M \geqslant N$). The relationship between the N-dimensional transmitted complex symbol vector $\boldsymbol{s}$ and the M-dimensional received vector $\boldsymbol{x}$ is determined by

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{w} \tag{1}$$

where $\boldsymbol{A} \in \mathbf{C}^{M \times N}$ is a complex matrix with full column rank, which presents a flat-fading channel gain matrix; and $\boldsymbol{w}$ is the additive complex noise vector.

1.1 OSIC-based PLR-aided MIMO detection

A complex-valued lattice generated by $\boldsymbol{A}$ is defined as

$$L(\boldsymbol{A}) = \{\boldsymbol{A}z,\ z \in \mathbf{Z}^N + i\mathbf{Z}^N\} \tag{2}$$

The columns of $\boldsymbol{A}$ form a basis of the lattice $L(\boldsymbol{A})$. The reduced basis of $L(\boldsymbol{A})$, $\boldsymbol{A}' = \boldsymbol{A}\boldsymbol{U}$, can be obtained by using an LR algorithm such as LLL[8] or SA[9], where $\boldsymbol{U}$ is a unimodular matrix.

To apply LR to MIMO detection, the transformed basis $\boldsymbol{A}'$ is used in the expression of the received signal vector $\boldsymbol{x}$ such that $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{U}\boldsymbol{U}^{-1}\boldsymbol{s} + \boldsymbol{w} = \boldsymbol{A}'\boldsymbol{d} + \boldsymbol{w}$, where $\boldsymbol{d} = \boldsymbol{U}^{-1}\boldsymbol{s}$. Then, the OSIC-based MIMO detection algorithm can be applied to detect the transformed symbol vector $\boldsymbol{d}$ to yield its estimate $\hat{\boldsymbol{d}}$. The detection result of $\boldsymbol{s}$ is then obtained by the linear transformation $\hat{\boldsymbol{s}} = \boldsymbol{U}\hat{\boldsymbol{d}}$. As $L(\boldsymbol{A})$ is generated by the channel gain matrix $\boldsymbol{A}$, it is usually called the primal lattice and the MIMO detection method mentioned above is called the OSIC-based PLR-aided MIMO detection, which is illustrated in Fig. 1.

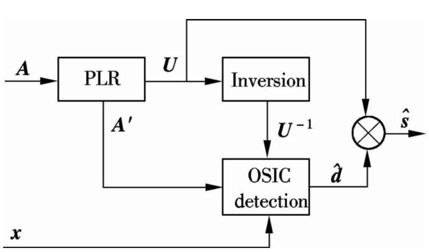

Fig. 1 OSIC-based PLR-aided MIMO detection scheme

1.2 OSIC-based DLR-aided MIMO detection

Let $\boldsymbol{B} \in \mathbf{C}^{N \times M}$ be the Moore-Penrose (MP) inverse of $\boldsymbol{A}$, i. e.,

$$\boldsymbol{B} = \boldsymbol{A}^{\dagger} = (\boldsymbol{A}^{H}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{H}} \tag{3}$$

where $\boldsymbol{A}^{\mathrm{H}}$ denotes the Hermitian of $\boldsymbol{A}$, and the notation $(\cdot)^{\dagger}$ indicates the MP inverse of a matrix. The dual lattice of the primal lattice $L(\boldsymbol{A})$ can be defined as

$$L(\boldsymbol{B}) = \{z^{\mathrm{T}}\boldsymbol{B},\ z \in \mathbf{Z}^N + i\mathbf{Z}^N\} \tag{4}$$

Accordingly, the rows of $\boldsymbol{B}$ form a basis of the lattice $L(\boldsymbol{B})$. The reduced basis of $L(\boldsymbol{B})$, $\boldsymbol{B}' = \boldsymbol{V}\boldsymbol{B}$, can be obtained by using the LR algorithm such as dual LLL or SA[6-7], where $\boldsymbol{V}$ is a unimodular matrix.

The OSIC-based DLR-aided MIMO detection scheme is shown in Fig. 2. Note that the calculation of the MP inverse of the reduced basis $\boldsymbol{B}'$, $\boldsymbol{B}'^{\dagger} = \boldsymbol{B}'^{\mathrm{H}}(\boldsymbol{B}'\boldsymbol{B}'^{\mathrm{H}})^{-1}$, is needed for the subsequent OSIC-based detection. Clearly, $\boldsymbol{A} = \boldsymbol{B}^{\dagger} = \boldsymbol{B}'^{\dagger}\boldsymbol{V}$. Thus, the received signal vector $\boldsymbol{x}$ can be expressed as $\boldsymbol{x} = \boldsymbol{B}'^{\dagger}\boldsymbol{V}\boldsymbol{s} + \boldsymbol{w} = \boldsymbol{B}'^{\dagger}\boldsymbol{d} + \boldsymbol{w}$, where $\boldsymbol{d} = \boldsymbol{V}\boldsymbol{s}$. Then $\boldsymbol{d}$ can be detected from $\boldsymbol{x}$ by using the OSIC algorithm to produce $\hat{\boldsymbol{d}}$. Finally, $\hat{\boldsymbol{s}}$ is derived from $\hat{\boldsymbol{d}}$ via $\hat{\boldsymbol{s}} = \boldsymbol{V}^{-1}\hat{\boldsymbol{d}}$.

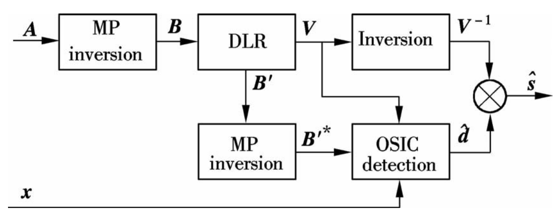

Fig. 2 OSIC-based DLR-aided MIMO detection scheme

2 OSNPC Algorithm and OSNPC-Based DLR-Aided MIMO Detection

2.1 OSIC algorithm review

First, we briefly review the OSIC algorithm[3] in the LR-aided MIMO detection. As mentioned above, the transformed symbol $\boldsymbol{d}$ can be detected from $\boldsymbol{x} = \boldsymbol{A}'\boldsymbol{d} + \boldsymbol{w}$ in PLR-aided detection (or from $\boldsymbol{x} = \boldsymbol{B}'^{\dagger}\boldsymbol{d} + \boldsymbol{w}$ in DLR-aided detection) by using the OSIC algorithm, which is described as follows.

1) Compute $\boldsymbol{A}'^{\dagger}$, the MP inverse of $\boldsymbol{A}'$. Let $\boldsymbol{B}' = \boldsymbol{A}'^{\dagger}$, then multiply $\boldsymbol{B}'$ to the received vector $\boldsymbol{x}$ to obtain the decision statistic vector

$$\boldsymbol{y} = \boldsymbol{B}'\boldsymbol{x} = \boldsymbol{d} + \boldsymbol{B}'\boldsymbol{w} \tag{5}$$

2) Find y_k in $\boldsymbol{y}$ with the highest signal-to-noise ratio (SNR) and detect the corresponding symbol d_k,

$$k = \arg\min_{j}(\boldsymbol{b}_j'\boldsymbol{b}_j'^{\mathrm{H}}) \tag{6}$$

where $\boldsymbol{b}_j'$ denotes the j-th row of $\boldsymbol{B}'$. The detection result of d_k is

$$\hat{d}_k = \boldsymbol{Q}(y_k) \tag{7}$$

where $y_k = d_k + \boldsymbol{b}'_k \boldsymbol{w}$ and $\boldsymbol{Q}(\cdot)$ indicates the slicing or quantization procedure according to the signal constellation in use.

3) Cancel the corresponding interference term of d_k from received signal $\boldsymbol{x}$: $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{d}_k \boldsymbol{a}'_k = \boldsymbol{As} - \hat{d}_k \boldsymbol{a}'_k + \boldsymbol{w}$, where $\boldsymbol{a}'_k$ denotes the k-th column of $\boldsymbol{A}'$. Suppose that d_k is detected correctly, i. e., $\hat{d}_k = d_k$. Then $\tilde{\boldsymbol{x}} = \boldsymbol{A}'\boldsymbol{d} - \hat{d}_k \boldsymbol{a}'_k + \boldsymbol{w} = \widetilde{\boldsymbol{A}}'\tilde{\boldsymbol{d}} + \boldsymbol{w}$, where $\widetilde{\boldsymbol{A}}'$ is a deflated matrix of $\boldsymbol{A}'$ obtained by deleting the k-th column of $\boldsymbol{A}'$, and $\tilde{\boldsymbol{d}}$ is the result of deleting the k-th column of $\boldsymbol{d}$.

4) Let $\boldsymbol{A}' = \widetilde{\boldsymbol{A}}'$, $\boldsymbol{x} = \tilde{\boldsymbol{x}}$ and $\boldsymbol{d} = \tilde{\boldsymbol{d}}$, repeat steps 1) to 3) till all the N symbols in $\boldsymbol{d}$ are detected.

2.2 Proposition of MP inverse of deflated matrix

In the aforementioned OSIC iteration procedure, the repeated computations of the MP inverse of the deflated channel gain matrices $\widetilde{\boldsymbol{A}}'$ are required, which leads to an extraordinary computational cost. Nevertheless, if we observe an important property of the MP inverse of the deflated matrix, which is proposed as the following proposition, the repeated inverse computation in the OSIC can be avoided.

Proposition 1 Assume that matrix $\boldsymbol{A} \in \mathbf{C}^{M\times N}$, and rank $\boldsymbol{A} = N$. Let $\boldsymbol{B}$ be the Moore-Penrose (MP) inverse of $\boldsymbol{A}$, and $\widetilde{\boldsymbol{A}}$ denote the deflated matrix of $\boldsymbol{A}$. $\widetilde{\boldsymbol{A}}$ is obtained by deleting the k-th column of $\boldsymbol{A}$, $k \in \{1,2,\cdots,N\}$, and expressed as $\widetilde{\boldsymbol{A}} = \boldsymbol{A}_{|k}$. Furthermore, let $\widetilde{\boldsymbol{B}}$ be the MP inverse of $\widetilde{\boldsymbol{A}}$, then $\widetilde{\boldsymbol{B}}$ can be simply derived from $\boldsymbol{B}$ as follows:

Delete the k-th row of $\boldsymbol{B}$ to obtain $\overline{\boldsymbol{B}}$ which is expressed as $\overline{\boldsymbol{B}} = \boldsymbol{B}_{\bar{k}}$. Then

$$\tilde{\boldsymbol{b}}_i = \bar{\boldsymbol{b}}_i - \frac{\langle \bar{\boldsymbol{b}}_i, \boldsymbol{b}_k \rangle}{\langle \boldsymbol{b}_k, \boldsymbol{b}_k \rangle} \boldsymbol{b}_k \qquad i = 1,2,\cdots,N-1 \tag{8}$$

where $\tilde{\boldsymbol{b}}_i$ and $\bar{\boldsymbol{b}}_i$ denote the i-th row of $\widetilde{\boldsymbol{B}}$ and $\overline{\boldsymbol{B}}$, respectively; $\boldsymbol{b}_k$ denotes the k-th row of $\boldsymbol{B}$; $\langle \bar{\boldsymbol{b}}_i, \boldsymbol{b}_k \rangle = \bar{\boldsymbol{b}}_i \boldsymbol{b}_k^{\mathrm{H}}$ is the inner product of $\bar{\boldsymbol{b}}_i$ and $\boldsymbol{b}_k$, and $\langle \boldsymbol{b}_k, \boldsymbol{b}_k \rangle = \boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}$.

Proof It is known that $\boldsymbol{A}$ and its unique MP inverse $\boldsymbol{B}$ satisfy four Penrose equations[10]: 1) $\boldsymbol{ABA} = \boldsymbol{A}$; 2) $\boldsymbol{BAB} = \boldsymbol{B}$; 3) $(\boldsymbol{BA})^{\mathrm{H}} = \boldsymbol{BA}$; and 4) $(\boldsymbol{AB})^{\mathrm{H}} = \boldsymbol{AB}$. Thus, we only need to prove that $\widetilde{\boldsymbol{A}}$ and $\widetilde{\boldsymbol{B}}$ which are derived from Eq. (8) satisfy the Penrose equations as well. Considering that the matrix form of Eq. (8) is presented by

$$\widetilde{\boldsymbol{B}} = \overline{\boldsymbol{B}} - \frac{\overline{\boldsymbol{B}}\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} \tag{9}$$

Then we have

$$\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{A}} = \overline{\boldsymbol{B}}\widetilde{\boldsymbol{A}} - \frac{\overline{\boldsymbol{B}}\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k \widetilde{\boldsymbol{A}}}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \boldsymbol{I}_{N-1} - \frac{\overline{\boldsymbol{B}}\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{O}_{1\times(N-1)}}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \boldsymbol{I}_{N-1} \tag{10}$$

where $\boldsymbol{I}_{N-1}$ denotes the identity matrix of order $N-1$, and $\boldsymbol{O}_{1\times(N-1)}$ is the row zero vector with $N-1$ elements. $\boldsymbol{b}_k \widetilde{\boldsymbol{A}} = \boldsymbol{O}_{1\times(N-1)}$ due to $\boldsymbol{BA} = \boldsymbol{I}_N$. Consequently, the following equations hold: 1) $\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{A}} = \widetilde{\boldsymbol{A}}\boldsymbol{I}_{N-1} = \widetilde{\boldsymbol{A}}$; 2) $\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}} = \boldsymbol{I}_{N-1}\widetilde{\boldsymbol{B}} = \widetilde{\boldsymbol{B}}$; 3) $(\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{A}})^{\mathrm{H}} = (\boldsymbol{I}_{N-1})^{\mathrm{H}} = \widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{A}}$. In addition,

$$\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}} = \widetilde{\boldsymbol{A}}\left(\overline{\boldsymbol{B}} - \frac{\overline{\boldsymbol{B}}\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}}\right) = \widetilde{\boldsymbol{A}}\overline{\boldsymbol{B}}\left(\boldsymbol{I}_{\mathrm{M}} - \frac{\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}}\right) \tag{11}$$

By applying $\widetilde{\boldsymbol{A}}\overline{\boldsymbol{B}} = (\boldsymbol{AB} - \boldsymbol{a}_k \boldsymbol{b}_k)$ to Eq. (11), where $\boldsymbol{a}_k$ denotes the k-th column of $\boldsymbol{A}$, we obtain

$$\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}} = \boldsymbol{AB} - \boldsymbol{a}_k \boldsymbol{b}_k - \frac{\boldsymbol{ABb}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} + \frac{\boldsymbol{a}_k \boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \boldsymbol{AB} - \frac{\boldsymbol{ABb}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} \tag{12}$$

It is observed that $\boldsymbol{AB} = (\boldsymbol{AB})^{\mathrm{H}}$, $\boldsymbol{b}_k \boldsymbol{A} = \boldsymbol{i}_k$, and $\boldsymbol{B}^{\mathrm{H}} \boldsymbol{i}_k^{\mathrm{H}} = \boldsymbol{b}_k^{\mathrm{H}}$, where $\boldsymbol{i}_k$ is the k-th row of $\boldsymbol{I}_N$. From Eq. (12), we have

$$\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}} = \boldsymbol{AB} - \frac{(\boldsymbol{AB})^{\mathrm{H}} \boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \boldsymbol{AB} - \frac{\boldsymbol{B}^{\mathrm{H}} (\boldsymbol{b}_k \boldsymbol{A})^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \boldsymbol{AB} - \frac{\boldsymbol{B}^{\mathrm{H}} \boldsymbol{i}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \boldsymbol{AB} - \frac{\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} \tag{13}$$

Thus,

$$(\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}})^{\mathrm{H}} = \left(\boldsymbol{AB} - \frac{\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}}\right)^{\mathrm{H}} = (\boldsymbol{AB})^{\mathrm{H}} - \left(\frac{\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}}\right)^{\mathrm{H}} = \boldsymbol{AB} - \frac{\boldsymbol{b}_k^{\mathrm{H}} \boldsymbol{b}_k}{\boldsymbol{b}_k \boldsymbol{b}_k^{\mathrm{H}}} = \widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}} \tag{14}$$

That is to say, $\widetilde{\boldsymbol{A}}$ and $\widetilde{\boldsymbol{B}}$ satisfy the 4th Penrose equation.

2.3 OSNPC algorithm

By applying the proposition of the MP inverse of the deflated matrix in the LR-aided MIMO detection, we can construct the OSNPC algorithm as follows.

Take the PLR-aided MIMO detection for example. Suppose that the received signal vector $\boldsymbol{x} = \boldsymbol{A}'\boldsymbol{d} + \boldsymbol{w}$ and channel gain matrix $\boldsymbol{A}'$ are known inputs, then the transformed symbol vector $\boldsymbol{d}$ can be detected by using the following steps. Note that steps 1) and 2) are the same as those of the OSIC algorithm.

1) Compute $\boldsymbol{A}'^{\dagger}$, the MP inverse of $\boldsymbol{A}'$. Let $\boldsymbol{B}' = \boldsymbol{A}'^{\dagger}$, then multiply $\boldsymbol{B}'$ to the received vector $\boldsymbol{x}$ to obtain the decision statistic vector $\boldsymbol{y} = \boldsymbol{d} + \boldsymbol{B}'\boldsymbol{w}$.

2) Find y_k in $\boldsymbol{y}$ with the highest signal-to-noise ratio (SNR) and detect the corresponding symbol d_k, where $k = \arg\min_j (\boldsymbol{b}'_j \boldsymbol{b}'^{\mathrm{H}}_j)$. The detection result of d_k is $\hat{d}_k = Q(y_k)$, where $y_k = d_k + \boldsymbol{b}'_k \boldsymbol{w} = d_k + \eta_k$, and $\eta_k (= \boldsymbol{b}'_k \boldsymbol{w})$ indicates the noise term in y_k.

3) Compute the estimate of the noise term in y_k by

$$\hat{\eta}_k = y_k - \hat{d}_k \tag{15}$$

4) Delete the k-th rows of $\boldsymbol{B}'$, $\boldsymbol{y}$ and $\boldsymbol{d}$ to obtain a deflated matrix $\bar{\boldsymbol{B}}'$, $\bar{\boldsymbol{y}}$ and $\tilde{\boldsymbol{d}}$, respectively. Then we have $\bar{\boldsymbol{y}} = \tilde{\boldsymbol{d}} + \bar{\boldsymbol{B}}'\boldsymbol{w}$. Computing

$$\mu_{ik} = \frac{\bar{\boldsymbol{b}}_i'\boldsymbol{b}_k'^{\mathrm{H}}}{\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}}} \qquad i = 1, 2, \cdots N-1 \tag{16}$$

and canceling the known noise projection terms from $\bar{\boldsymbol{y}}$ yield

$$\tilde{y}_i = \bar{y}_i - \mu_{ik}\hat{\eta}_k = \tilde{d}_i + \bar{\boldsymbol{b}}_i'\boldsymbol{w} - \mu_{ik}\hat{\eta}_k \qquad i = 1, 2, \cdots, N-1 \tag{17}$$

Suppose that d_k is detected correctly, i. e., $\hat{d}_k = d_k$. From $y_k = d_k + \boldsymbol{b}_k'\boldsymbol{w}$ and Eq. (15), we have $\hat{\eta}_k = \boldsymbol{b}_k'\boldsymbol{w}$. Hence Eq. (17) becomes

$$\tilde{y}_i = \tilde{d}_i + (\bar{\boldsymbol{b}}_i' - \mu_{ik}\boldsymbol{b}_k')\boldsymbol{w} = \tilde{d}_i + \tilde{\boldsymbol{b}}_i'\boldsymbol{w} \qquad i = 1, 2, \cdots, N-1 \tag{18}$$

where $\tilde{\boldsymbol{b}}_i' = \bar{\boldsymbol{b}}_i' - \mu_{ik}\boldsymbol{b}_k' = \bar{\boldsymbol{b}}_i' - (\bar{\boldsymbol{b}}_i'\boldsymbol{b}'^{\mathrm{H}}_k/(\boldsymbol{b}_k'\boldsymbol{b}'^{\mathrm{H}}_k))\boldsymbol{b}_k'$. We see that $\tilde{\boldsymbol{b}}_i'$ is obtained by subtracting the projection of $\bar{\boldsymbol{b}}_i'$ onto $\boldsymbol{b}_k'$ from $\bar{\boldsymbol{b}}_i'$. Thus, $\tilde{\boldsymbol{b}}_i'$ must be orthogonal to $\boldsymbol{b}_k'$ and also shorter than $\bar{\boldsymbol{b}}_i'$. For convenience, we call $\mu_{ik}\boldsymbol{b}_k'\boldsymbol{w}$ the noise projection of $\bar{\boldsymbol{b}}_i'\boldsymbol{w}$ onto noise term $\boldsymbol{b}_k'\boldsymbol{w}$ and call this proposed detection algorithm as OSNPC.

5) Let $y_i = \tilde{y}_i$, $\boldsymbol{b}_i' = \tilde{\boldsymbol{b}}_i'$, and $d_i = \tilde{d}_i$ for $i = 1, 2, \cdots, N-1$ (or $\boldsymbol{y} = \tilde{\boldsymbol{y}}$, $\boldsymbol{B}' = \tilde{\boldsymbol{B}}'$, and $\boldsymbol{d} = \tilde{\boldsymbol{d}}$ in vector notation). Repeat steps 2) to 4) till all N symbols in $\boldsymbol{d}$ are detected.

2.4 Performance and complexity comparison of OSNPC and OSIC

From the aforementioned proposition of the MP inverse of the deflated matrix we can arrive at the conclusion that the OSNPC algorithm is equivalent to the traditional OSIC in performance, which can be explained as follows.

In the OSIC, after d_k is detected and its corresponding interferences are cancelled from $\boldsymbol{x}$, the updating $\boldsymbol{x}$ is

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{d}_k\boldsymbol{a}_k' = \tilde{\boldsymbol{A}}'\tilde{\boldsymbol{d}} + \boldsymbol{w} + (d_k - \hat{d}_k)\boldsymbol{a}_k' \tag{19}$$

The updating decision statistic vector is $\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{B}}'\tilde{\boldsymbol{x}} = \tilde{\boldsymbol{d}} + \tilde{\boldsymbol{B}}'\boldsymbol{w} + (d_k - \hat{d}_k)\tilde{\boldsymbol{B}}'\boldsymbol{a}_k'$. From Eq. (9) we have $\tilde{\boldsymbol{B}}' = \bar{\boldsymbol{B}}' - \bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}\,\boldsymbol{b}_k'/(\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}})$. Observing that $\bar{\boldsymbol{B}}'\boldsymbol{a}_k' = \boldsymbol{0}$ and $\boldsymbol{b}_k'\boldsymbol{a}_k' = 1$, we obtain

$$\tilde{y} = \tilde{\boldsymbol{d}} + \tilde{\boldsymbol{B}}'\boldsymbol{w} + (d_k - \hat{d}_k)\frac{\bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}}{\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}}} \tag{20}$$

In the OSNPC, after d_k is detected and the corresponding noise projections are cancelled from $\boldsymbol{y}$, the updated $\boldsymbol{y}$ is obtained, and its expression is explained as follows.

The matrix form of Eq. (17) is $\tilde{\boldsymbol{y}} = \bar{\boldsymbol{y}} - \boldsymbol{\mu}_k\hat{\boldsymbol{\eta}}_k = \tilde{\boldsymbol{d}} + \bar{\boldsymbol{B}}'\boldsymbol{w} - \boldsymbol{\mu}_k\hat{\eta}_k$, where $\boldsymbol{\mu}_k = \bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}/(\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}})$ according to Eq. (16). Observing that $\hat{\eta}_k = y_k - \hat{d}_k = d_k - \hat{\boldsymbol{d}}_k + \boldsymbol{b}_k'\boldsymbol{w}$, we have

$$\begin{aligned}\tilde{\boldsymbol{y}} &= \tilde{\boldsymbol{d}} + \bar{\boldsymbol{B}}'\boldsymbol{w} - \frac{\bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}}{\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}}}(d_k - \hat{d}_k + \boldsymbol{b}_k'\boldsymbol{w}) = \\ &\tilde{\boldsymbol{d}} + \bar{\boldsymbol{B}}'\boldsymbol{w} - \frac{\bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}}{\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}}}\boldsymbol{b}_k'\boldsymbol{w} - \frac{\bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}}{\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}}}(d_k - \hat{d}_k) = \\ &\tilde{\boldsymbol{d}} + \tilde{\boldsymbol{B}}'\boldsymbol{w} - (d_k - \hat{d}_k)\frac{\bar{\boldsymbol{B}}'\boldsymbol{b}_k'^{\mathrm{H}}}{\boldsymbol{b}_k'\boldsymbol{b}_k'^{\mathrm{H}}}\end{aligned} \tag{21}$$

It can be clearly seen that Eqs. (20) and (21) are the same, which means that the decision statistics in the OSNPC algorithm are always equal to those in the OSIC algorithm. Thus, we conclude that the OSNPC algorithm is equivalent to the OSIC in performance.

Now we compare the complexity of the OSNPC with that of the OSIC briefly. To compute the MP inverse of a matrix $\boldsymbol{A}$, the most efficient method is based on the Cholesky factorization of the matrix $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}$[11]. If this calculation method is applied, the numbers of multiplications and additions of the OSIC algorithm are $(9/4)N^4 + (4/3)N^3M + (29/6)N^3 + (5/2)N^2M$ and $(9/4)N^4 + (4/3)N^3M + (25/6)N^3 + (5/2)N^2M$, respectively[11]. Here N is the number of transmitters and M the number of receivers in the MIMO system, and the terms below the third order are ignored for brevity. Note that the multiplications and additions herein refer to complex value operations. By contrast, when applying the same MP inverse calculation method to the OSNPC-algorithm, it is easily obtained that the numbers of multiplications and additions of the OSNPC algorithm are $(2/3)N^3 + 3N^2M$ and $(1/2)N^3 + (5/2)N^2M$, respectively. These results reveal that the complexity of the OSNPC algorithm is considerably lower than that of the OSIC algorithm.

2.5 OSNPC-based DLR-aided MIMO detection

Although the aforementioned OSNPC algorithm is described on the basis of the PLR-aided MIMO detection, it can also be applied to the DLR-aided MIMO detection. The OSNPC-based DLR-aided MIMO detection scheme is shown in Fig. 3. In contrast to the OSIC block in Fig. 2 whose inputs are signal vector $\boldsymbol{x}$ and MP inverse $\boldsymbol{B}'^{\dagger}$ of the reduced basis of the dual lattice, the OSNPC block in Fig. 3 needs $\boldsymbol{x}$ and reduced basis $\boldsymbol{B}'$ as its inputs. Obviously, the inverse calculation of the reduced basis $\boldsymbol{B}'$ is avoided in the OSNPC-based DLR-aided MIMO detection scheme, which further reduces the overall computational complexity of the detector.

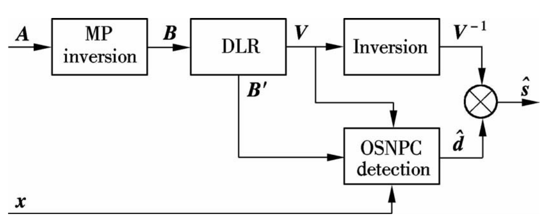

Fig. 3 OSNPC-based DLR-aided MIMO detection scheme

3 Simulation Results

Numerical simulations are performed to evaluate the complexity and performance of the proposed OSNPC-based DLR-aided MIMO detection scheme (OSNPC-DLR). They are also conducted on the OSIC-based DLR-aided MIMO detection scheme (OSIC-DLR) for comparison.

First, we compare the computational complexity of the two schemes. It is well known that for a common floating point implementation of an algorithm, the floating-point operations (flops) dominate the calculation, and the number of flops is a consistent measure of the algorithm computational complexity, independent of what machine it runs on. Therefore, we carry out numerical experiments to count the number of flops in computation used in the detection schemes. In our experiments, the dual LLL[7] is selected as the DLR algorithm in both OSNPC-DLR and OSIC-DLR. Because the computational complexity of the dual LLL algorithm is randomly varying, we calculate the average flops on 10^6 experiments for each case. Fig. 4 shows the average number of flops of OSNPC-DLR and OSIC-DLR with $N=M$. It is observed that the number of flops of OSNPC-DLR increases much more slowly than OSIC-DLR as N increases.

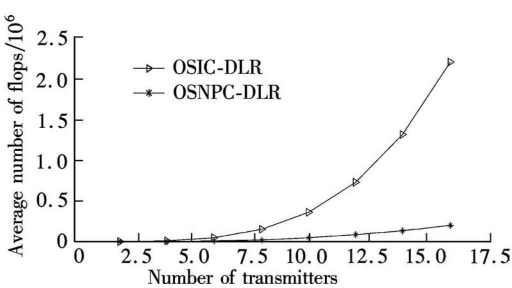

Fig. 4 Average number of flops of OSNPC-DLR and OSIC-DLR vs. number of transmitters with $N=M$

Fig. 5 illustrates the performance comparison of the two schemes in terms of symbol error rate (SER). The corresponding performance of the optimum ML detector is also depicted in Fig. 5 for further comparison. In Fig. 5, the SNR is defined as E_s/N_0 with E_s denoting the average received energy per symbol per receiving antenna and N_0 the power spectrum density of additive white Gaussian noise, respectively. It turns out that OSNPC-DLR performs nearly the same as OSIC-DLR in terms of SER at different values of N and M.

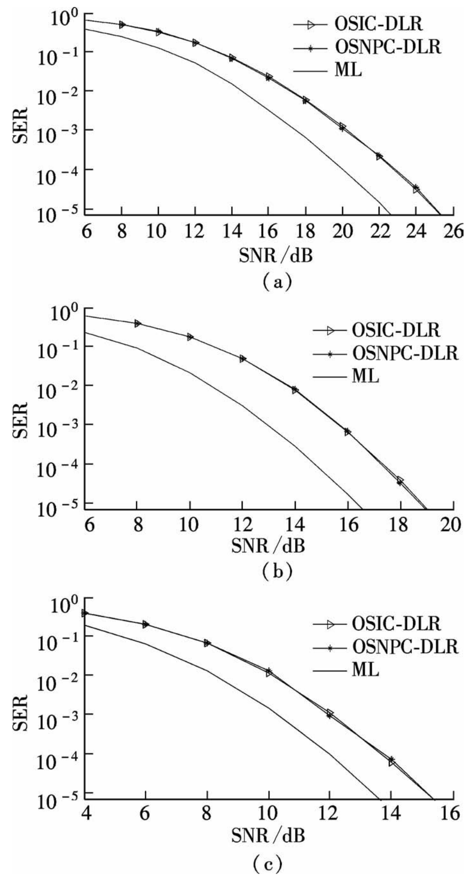

Fig. 5 Symbol error rate of OSNPC-DLR and OSIC-DLR in an $N\times M$ uncoded MIMO system using 16-QAM. (a) $N=M=4$; (b) $N=M=8$; (c) $N=8, M=10$

4 Conclusion

We propose an OSNPC algorithm to improve the performance of the MIMO detector with the conventional OSIC algorithm. The OSNPC-based DLR-aided MIMO detection scheme is further proposed. Analyses and simulation results show the better quality properties of our proposed detector regarding both the error performance and computational complexity. It is noteworthy that the application of the OSNPC algorithm is not restricted to DLR-aided MIMO detection. In fact, it has wider applicability as a general detection algorithm for all the non-orthogonal signaling communication systems (e. g., for non-orthogonal multi-user signal detection). Besides, the proposition of the MP inverse of the deflated matrix proposed in this paper provides some useful insights on the MP inverse problems and needs further investigation of the related works.

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对偶格约减辅助 MIMO 检测的噪声投影按序逐次消去算法

刘金铸[1,2]　邢　松[3]　沈连丰[1]

([1] 东南大学移动通信国家重点实验室, 南京 210096)

([2] 南京信息工程大学电子信息工程学院, 南京 210044)

([3]Department of Information Systems, California State University, Los Angeles, CA 90032, USA)

摘要:提出一种新的多输入多输出(MIMO)非线性检测算法,称为噪声投影按序逐次消去(OSNPC)算法,以改进传统的干扰按序逐次消去(OSIC)算法的计算复杂度方面的性能. OSIC 算法从接收信号向量中逐次消去已知的干扰,而 OSNPC 算法从判决变量向量中逐次消去已知的噪声投影. 理论分析表明,OSNPC 算法在性能上等效于传统的 OSIC 算法,但其计算复杂度却大为降低. 而且,当 OSNPC 算法用于对偶格约减(DLR)辅助 MIMO 检测时,所构成的基于 OSNPC 的 DLR 辅助 MIMO 检测方案,与基于 OSIC 的 DLR 辅助 MIMO 检测方案相比,其整体复杂度得到进一步降低,这是因为在它的检测过程中,省去了对偶格约减基的求逆运算. 仿真结果验证了该检测方案的性能与复杂度的理论分析结论.

关键词:噪声投影按序逐次消去;对偶格约减;多输入多输出检测;干扰按序逐次消去

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