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Global stabilization for a class of nonlinear time-delay systems using linear output feedback

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Abstract: The stabilization problem via the linear output feedback controller is addressed for a class of nonlinear systems subject to time-delay. The uncertainty of the system satisfies the lower-triangular growth condition and it is affected by time-delay. A linear output feedback controller with a tunable scaling gain is constructed. By selecting an appropriate Lyapunov-Krasovskii functional, the scaling gain can be adjusted to render the closed-loop system globally asymptotically stable. The results can also be extended to the non-triangular nonlinear time-delay systems. The proposed control law together with the observer is linear and memoryless in nature, and, therefore, it is easy to implement in practice. Two computer simulations are conducted to illustrate the effectiveness of the proposed theoretical results.

Key words: global stabilization; nonlinear system; time-delay system; Lyapunov-Krasovskii functional; linear observer; memoryless controller

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In this paper, we consider the problem of global output feedback stabilization for a class of uncertain time-delay systems described by

$$\left.\begin{aligned}
&\dot{x}_1(t)=x_2(t)+\varphi_1(x_1(t),x_1(t-\tau_1))\\
&\quad\vdots\\
&\dot{x}_{n-1}(t)=x_n(t)+\varphi_{n-1}(x_1(t),\ldots,x_{n-1}(t),\\
&\qquad\qquad x_1(t-\tau_1),\ldots,x_{n-1}(t-\tau_{n-1}))\\
&\dot{x}_n(t)=u(t)+\varphi_n(x_1(t),\ldots,x_n(t),\\
&\qquad\qquad x_1(t-\tau_1),\ldots,x_n(t-\tau_n))\\
&y(t)=x_1(t)\\
&\boldsymbol{x}(t)=\boldsymbol{\Psi}(t)\qquad -\tau\leqslant t\leqslant 0
\end{aligned}\right\}\qquad(1)$$

where $\boldsymbol{x}(t)=\{x_1(t),x_2(t),\ldots,x_n(t)\}^{\mathrm{T}}\in\mathbf{R}^n$ is the system state; $u(t)\in\mathbf{R}$ is the control input; $y(t)\in\mathbf{R}$ is the system output; $\tau_i>0$, $i=1,2,\ldots,n$ are given time-delays; while $\tau\geqslant\max\limits_{i=1,2,\ldots,n}\{\tau_i\}$, $\boldsymbol{\Psi}(t)$ is the initial function of the system state vector, and $\varphi_i(\cdot)$ $(i=1,2,\cdots,n-1)$ represent nonlinear perturbations that are not guaranteed to be precisely known. Our objective is to develop an output feedback controller to globally stabilize the uncertain nonlinear system (1).

Owing to the practical importance, the problem of global output feedback stabilization for uncertain nonlinear systems has attracted more attention from the nonlinear control community compared with the state feedback case. Recently, fruitful results of output feedback have been achieved. For a class of lower-triangular nonlinear systems, with the help of feedback domination design[1], some interesting results have been established under the linear growth condition[1] and the higher-order growth condition[2]. Based on the homogeneous domination approach, the homogeneous output controller was designed in Refs. [3 - 5], where the system is under the homogeneous growth condition.

In practice, time-delay is very common in system state, input and output due to the time consumed in sensing, information transmitting and controller computing. However, the aforementioned results did not consider the time-delay effect. Over the past decades, in the case when the nonlinearities contain time-delay, some interesting results were achieved. In Refs. [6 - 7], the backstepping approach was adapted. In Ref. [8], an adaptive approach was employed to design a state feedback controller to globally stabilize a class of upper-triangular systems with time-delay. The work[9] relaxed the growth condition imposed in Ref. [8] by employing a dynamic gain. Some results on state feedback stabilization for some different classes of time-delay nonlinear systems can also be seen in Refs. [10 - 11]. In the case when system state variables are not totally measurable, the problem of output feedback stabilization is more challenging and fewer results have been achieved for nonlinear systems with time-delay. For a linear system with time-delay in the input, the problem of output feedback stabilization was solved in Refs. [12 - 13], where the method of the linear matrix inequality (LMI) was used. For nonlinear system (1) subject to time-delay in uncertainties, the problem of

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output feedback stabilization has not been widely investigated. In this paper, we focus on solving the problem by using the output feedback domination approach. First, inspired by the result of Ref. [1], we propose the design procedure for a linear output controller with a scaling gain for system (1) under the linear growth condition. Then, we construct a Lyapunov-Krasovskii functional and use it to choose an appropriate scaling gain in the output feedback controller to guarantee the closed-loop system globally asymptotic stability. The proposed observer and control law are linear and memoryless in nature, and, therefore, are easy to implement in practice. After that, the output feedback controller is verified feasible when it is extended to the non-triangular nonlinear time-delay systems. Two computer simulations are conducted to illustrate the effectiveness of the theoretical results.

1 Linear Output Controller for Lower-Triangular Time-Delay Systems

In this section, we are devoted to the problem of global stabilization of nonlinear systems under the lower-triangular linear growth condition, and the nonlinear time-delay system (1) can be globally stabilized by a linear output feedback controller. Specifically, $\varphi_i(x_1(t),\cdots,x_n(t),x_1(t-\tau_1),\cdots,x_i(t-\tau_i))$, for $i=1,2,\cdots,n$, satisfy the following growth condition.

Assumption 1 For $i=1,2,\cdots,n$, there are constants $c_1\geqslant 0$ and $c_2\geqslant 0$ such that

$$\begin{aligned}|\varphi_i(x_1(t),\cdots,x_n(t),x(t),x(t-\tau_1),\cdots,x_i(t-\tau_i))|\leqslant\\ c_1(|x_1(t)|+\cdots+|x_i(t)|)+\\ c_2(|x_1(t-\tau_1)|+\cdots+|x_i(t-\tau_i)|)\end{aligned} \quad (2)$$

Remark 1 Assumption 1 requires that the nonlinear function $\varphi_i(x_1(t),\cdots,x_n(t),x_1(t-\tau_1),\cdots,x_i(t-\tau_i))$, for $i=1,2,\cdots,n$, should be bounded by linear terms with and without time-delay. Assumption 1 is more general than the linear growth condition imposed in Ref. [1] since Assumption 1 reduces to the nonlinear system in Ref. [1] when $c_2=0$.

With the help of Assumption 1, we are ready to construct a linear output feedback controller for system (1).

Theorem 1 Under Assumption 1, there exists an appropriate gain such that system (1) can be globally stabilized by the following output feedback controller:

$$u(t)=-L^n\left[k_1\hat{x}_1(t)+\frac{1}{L}k_2\hat{x}_2(t)+\cdots+\frac{1}{L^{n-1}}k_n\hat{x}_n(t)\right] \quad (3)$$

$$\left.\begin{aligned}\dot{\hat{x}}_1(t)&=\hat{x}_2(t)+La_1(x_1(t)-\hat{x}_1(t))\\ \dot{\hat{x}}_2(t)&=\hat{x}_3(t)+L^2a_2(x_1(t)-\hat{x}_1(t))\\ &\vdots\\ \dot{\hat{x}}_{n-1}(t)&=\hat{x}_n(t)+L^{n-1}a_{n-1}(x_1(t)-\hat{x}_1(t))\\ \dot{\hat{x}}_n(t)&=u(t)+L^na_n(x_1(t)-\hat{x}_1(t))\end{aligned}\right\} \quad (4)$$

where constants $a_j>0$, $k_j>0$, $j=1,2,\cdots,n$ are the coefficients of the Hurwitz polynomials $p_1(\omega)=\omega^n+a_1\omega^{n-1}+a_2\omega^{n-2}\cdots+a_{n-1}\omega+a_n$ and $p_2(\omega)=\omega^n+k_n\omega^{n-1}+\cdots+k_2\omega+k_1$.

Proof First we introduce the following changes of coordinates with a constant scaling gain $L\geqslant 1$ to be determined later.

$$z_i(t)=\frac{\boldsymbol{x}_i(t)}{L^{i-1}},\ \hat{z}_i(t)=\frac{\hat{\boldsymbol{x}}_i(t)}{L^{i-1}},\ v(t)=\frac{u(t)}{L^n},\ \boldsymbol{\varepsilon}_i=z_i-\hat{z}_i \quad i=1,2,\cdots,n \quad (5)$$

A simple calculation gives

$$\dot{\boldsymbol{\varepsilon}}=L\boldsymbol{A}_1\boldsymbol{\varepsilon}+\boldsymbol{\Phi} \quad (6)$$

with

$$\boldsymbol{A}_1=\begin{bmatrix}-a_1 & 1 & \cdots & 0\\ \vdots & \vdots & & \vdots\\ -a_{n-1} & 0 & \cdots & 1\\ -a_n & 0 & \cdots & 0\end{bmatrix},\ \boldsymbol{\Phi}=\begin{bmatrix}\varphi_1(\cdot)\\ \frac{1}{L}\varphi_2(\cdot)\\ \vdots\\ \frac{1}{L^{n-1}}\varphi_n(\cdot)\end{bmatrix}$$

In addition, with the help of the coordinates change (5), the output feedback control law defined in Eq. (3) can be rewritten as

$$u(t)=L^nv(t),\ v(t)=-k_1\hat{z}_1(t)-\cdots-k_n\hat{z}(t) \quad (7)$$

Moreover, the observer (4) with the control law (3) can be rewritten as

$$\dot{\hat{z}}(t)=L\boldsymbol{A}_2\hat{z}(t)+L\boldsymbol{HC\varepsilon} \quad (8)$$

where $\boldsymbol{A}_2=\begin{bmatrix}0 & 1 & \cdots & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & 1\\ -k_1 & -k_2 & \cdots & -k_n\end{bmatrix}$, $\boldsymbol{H}=\begin{bmatrix}a_1\\ \vdots\\ a_{n-1}\\ a_n\end{bmatrix}$, $\boldsymbol{C}=[1\ \ 0\ \ \cdots\ \ 0]$.

In what follows, we shall prove that the transformed closed-loop systems (6) and (8) can be rendered globally asymptotically stable.

By the definitions of a_i and k_i, it can be verified that both $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are Hurwitz matrices. As a result, there are two positive definite matrices $\boldsymbol{P}_1=\boldsymbol{P}_1^{\mathrm{T}}>\boldsymbol{0}$ and $\boldsymbol{P}_2=\boldsymbol{P}_2^{\mathrm{T}}>\boldsymbol{0}$ such that

$$\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_1+\boldsymbol{P}_1\boldsymbol{A}_1=-\boldsymbol{I},\ \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{P}_2+\boldsymbol{P}_2\boldsymbol{A}_2=-\boldsymbol{I} \quad (9)$$

Construct the Lyapunov function as $V(\boldsymbol{\varepsilon},\hat{z})=(M+1)\cdot\boldsymbol{\varepsilon}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{\varepsilon}(t)+\hat{z}(t)^{\mathrm{T}}\boldsymbol{P}_2\hat{z}(t)$ with $M=2\|\boldsymbol{P}_2\boldsymbol{HC}\|^2$.

The derivative of $V(\varepsilon,\hat{x})$ along the closed-loop systems (6) and (8) is

$$\begin{aligned}\dot{V}(\boldsymbol{\varepsilon},\hat{z})=-L(M+1)\|\boldsymbol{\varepsilon}\|^2+2(M+1)\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\Phi}-\\ L\|\hat{z}(t)\|^2+2L\hat{z}^{\mathrm{T}}(t)\boldsymbol{P}_2\boldsymbol{HC\varepsilon}\end{aligned} \quad (10)$$

By Assumption 1, for any $L\geqslant 1$ the following holds

$$\begin{aligned}2\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\Phi}\leqslant&2\|\boldsymbol{\varepsilon}\|\|\boldsymbol{P}_1\|\left\{\left(1+\frac{1}{L}+\cdots\frac{1}{L^{n-1}}\right)\cdot\right.\\&(c_1|x_1|+c_2|x_1(t-\tau_1)|)+\left(1+\frac{1}{L}+\cdots+\frac{1}{L^{n-2}}\right)\cdot\\&\left(c_1\frac{|x_2|}{L}+c_2\frac{|x_2(t-\tau_2)|}{L}\right)+\cdots+\left(1+\frac{1}{L}\right)\cdot\\&\left(c_1\frac{|x_{n-1}|}{L^{n-2}}+c_2\frac{|x_{n-1}(t-\tau_{n-1})|}{L^{n-2}}\right)+\\&\left.\left(c_1\frac{|x_n|}{L^{n-1}}+c_2\frac{|x_n(t-\tau_n)|}{L^{n-1}}\right)\right\}\leqslant\\&c_3\|\boldsymbol{\varepsilon}\|\left\{|x_1|+|x_1(t-\tau_1)|+\frac{|x_2|}{L}+\right.\\&\left.\frac{|x_2(t-\tau_2)|}{L}+\cdots+\frac{|x_n|}{L^{n-1}}+\frac{|x_n(t-\tau_n)|}{L^{n-1}}\right\}\end{aligned}\tag{11}$$

for a positive constant $c_3=2\max\{c_1,c_2\}\|\boldsymbol{P}_1\|n$. By $\varepsilon_i=z_i-\hat{z}_i=\dfrac{x_i}{L^{i-1}}-\hat{z}_i$, we have

$$\left|\frac{x_i}{L^{i-1}}\right|\leqslant|\hat{z}_i|+|\varepsilon_i|\qquad i=1,2,\cdots,n$$

With this in mind, (11) can be further estimated as

$$\begin{aligned}2\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\Phi}\leqslant&2c_3\|\boldsymbol{\varepsilon}\|\sum_{i=1}^{n}[|\hat{z}_i|+|\varepsilon_i|+|\hat{z}_i(t-\tau_i)|+\\&|\varepsilon_i(t-\tau_i)|]\leqslant2c_3\sqrt{n}\|\boldsymbol{\varepsilon}\|^2+2c_3\sqrt{n}\|\boldsymbol{\varepsilon}\|\cdot\\&\left[\|\hat{\boldsymbol{z}}\|+\sqrt{\sum_{i=1}^{n}\hat{z}_i^2(t-\tau_i)}+\sqrt{\sum_{i=1}^{n}\varepsilon_i^2(t-\tau_i)}\right]\leqslant\\&5c_3\sqrt{n}\|\boldsymbol{\varepsilon}\|^2+c_3\sqrt{n}\left[\|\hat{\boldsymbol{z}}(t)\|^2+\right.\\&\left.\sum_{i=1}^{n}\hat{z}_i^2(t-\tau_i)+\sum_{i=1}^{n}\varepsilon_i^2(t-\tau_i)\right]\end{aligned}\tag{12}$$

In addition, noting that $M=2\|\boldsymbol{P}_2\boldsymbol{HC}\|^2$, the following holds

$$\begin{aligned}2L\hat{\boldsymbol{z}}(t)^{\mathrm{T}}\boldsymbol{P}_2\boldsymbol{HC}\boldsymbol{\varepsilon}\leqslant&2L\left(\frac{1}{\sqrt{2}}\|\hat{\boldsymbol{z}}\|\right)(\sqrt{2}\|\boldsymbol{P}_2\boldsymbol{HC}\|\|\boldsymbol{\varepsilon}\|)\leqslant\\&L\frac{1}{2}\|\hat{\boldsymbol{z}}\|^2+LM\|\boldsymbol{\varepsilon}\|^2\end{aligned}\tag{13}$$

Substituting (12) and (13) into (10) yields

$$\begin{aligned}\dot{V}(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})\leqslant&-[L-5c_3\sqrt{n}(M+1)]\|\boldsymbol{\varepsilon}(t)\|^2-\\&\left[\frac{1}{2}L-c_3\sqrt{n}(M+1)\right]\|\hat{\boldsymbol{z}}(t)\|^2+c_3\sqrt{n}(M+1)\cdot\\&\left[\sum_{i=1}^{n}\hat{z}_i^2(t-\tau_i)+\sum_{i=1}^{n}\varepsilon_i^2(t-\tau_i)\right]\end{aligned}\tag{14}$$

Construct the Lyapunov-Krasovskii functional as

$$W(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})=V(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})+\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\hat{z}_i^2(\theta)R\mathrm{d}\theta+\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\varepsilon_i^2(\theta)R\mathrm{d}\theta\tag{15}$$

where $R=c_3\sqrt{n}(M+1)$.

With the help of (14), taking the derivative of (15) yields

$$\begin{aligned}\dot{W}(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})\leqslant&-[L-5c_3\sqrt{n}(M+1)]\|\boldsymbol{\varepsilon}(t)\|^2-\\&\left[\frac{1}{2}L-c_3\sqrt{n}(M+1)\right]\|\hat{\boldsymbol{z}}(t)\|^2+\\&c_3\sqrt{n}(M+1)\left[\sum_{i=1}^{n}\hat{z}_i^2(t)+\sum_{i=1}^{n}\varepsilon_i^2(t)\right]\leqslant\\&-[L-6c_3\sqrt{n}(M+1)]\|\boldsymbol{\varepsilon}\|^2-\\&\left[\frac{1}{2}L-2c_3\sqrt{n}(M+1)\right]\|\hat{\boldsymbol{z}}\|^2\end{aligned}\tag{16}$$

Choosing a large enough gain $L>6c_3\sqrt{n}(M+1)$, we have

$$\dot{W}(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})\leqslant-\rho_1\|\boldsymbol{\varepsilon}\|^2-\rho_2\|\hat{\boldsymbol{z}}(t)\|^2\tag{17}$$

for two positive constants ρ_1 and ρ_2. As a conclusion, the closed-loop system, consisting of systems (6) and (8), is globally asymptotically stable[14]. In other words, system (1) is globally stabilized by the output feedback controller according to Eq. (3) for a large enough L.

Remark 2 Theorem 1 shows that under Assumption 1, the global output feedback stabilization of system (1) can be achieved even if the term $\varphi_i(\cdot)$ is intermixed with disturbances and time-delay. First, we adopt the same format of the observer and the control law introduced in Ref. [11]. Then, with the help of an appropriate functional, the scaling gain is carefully chosen to render the closed-loop system globally asymptotically stable. The proposed observer and control law are linear and memoryless in nature, and, therefore, they are easy to implement in practice.

Remark 3 If all the time-delays of x_i are of the same value, i. e. $\tau_i=\tau$ for $i=1,2,\cdots,n$, then the part of $\sum_{i=1}^{n}\hat{z}_i^2(t-\tau_i)+\sum_{i=1}^{n}\varepsilon_i^2(t-\tau_i)$ in (11) and (14) will be replaced by $\|\hat{\boldsymbol{z}}(t-\tau)\|^2+\|\boldsymbol{\varepsilon}(t-\tau)\|^2$. Additionally, (15) will become the following equation:

$$W(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})=V(\boldsymbol{\varepsilon},\hat{\boldsymbol{z}})+\int_{t-\tau}^{t}\|\hat{\boldsymbol{z}}(\theta)\|^2R\mathrm{d}\theta+\int_{t-\tau}^{t}\|\boldsymbol{\varepsilon}(\theta)\|^2R\mathrm{d}\theta$$

It is easy to verify that (16) and (17) will remain the same. So the proposed observer and controller are still applicable to the system with a unified time-delay.

In the remainder of this section, we use an example to illustrate the application of Theorem 1.

Example 1 Consider the following time-delay system

$$\left.\begin{aligned}&\dot{x}_1(t)=x_2(t)-0.5x_1(t)+0.5\cos(x_2)x_1(t-1)\\&\dot{x}_2(t)=u(t)+0.5x_1(t)+0.6x_2(t)+\\&\qquad3\sin(x_2(t))\sin(x_1(t-1))+0.5\ln(x_2^2(t-0.9)+1)\\&y=x_1\end{aligned}\right\}\tag{18}$$

By the differential mean-value theorem, there exists a constant $\tilde{c} > 0$ such that $\ln(x_2^2(t-0.9)+1) \leqslant \tilde{c}x_2(t-0.9)$. It is straightforward to verify that Assumption 1 holds for system (18). Hence, by Theorem 1 we now can design an output feedback controller of the form:

$$\left.\begin{aligned} u(t) &= -L^2\left[k_1\hat{x}_1(t) + \frac{1}{L}k_2\hat{x}_2(t)\right] \\ \dot{\hat{x}}_1(t) &= \hat{x}_2(t) + La_1(x_1(t) - x_1(t)) \\ \dot{\hat{x}}_2(t) &= u(t) + L^2a_2(x_1(t) - \hat{x}_1(t)) \end{aligned}\right\} \quad (19)$$

Simulation results are shown in Figs. 1 and 2, where the gains are selected as $k_1 = 0.3$, $k_2 = 1.5$, $a_1 = 2$, $a_2 = 6$, $L = 3.4$ and the initial functions are

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -10\sin(t-0.2) \\ -6\sin(t-0.2) \end{bmatrix}, \quad \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for $-1 \leqslant t \leqslant 0$.

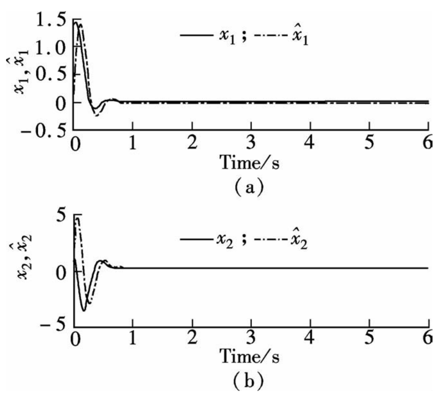

Fig. 1 State trajectories of Eqs. (18) and (19). (a) x_1, $\hat{x}_1$; (b) x_2, $\hat{x}_2$

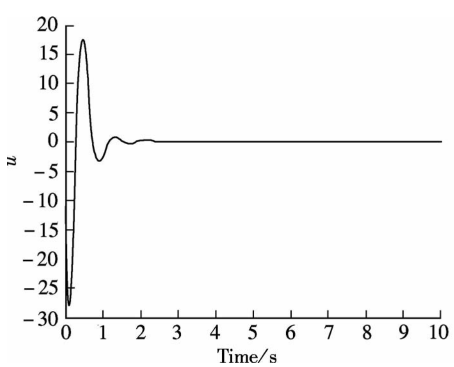

Fig. 2 Time history of the control signal

2 Extensions

Assumption 1 requires nonlinear perturbation in system (1) satisfying a lower-triangular linear growth condition, which is affected by time-delay. In this section, we show that the condition can be further relaxed to encompass some more general nonlinearities, which go beyond triangular growth condition. Specifically, the following general condition will be used in this section.

Assumption 2 For $i = 1, 2, \cdots, n$, there exist constants $m \geqslant 0$, $v_i > 0$, $c_1 \geqslant 0$ and $c_2 \geqslant 0$ such that for any $L \geqslant 1$ the following holds:

$$\left|\frac{1}{L^{i-1}}\phi_i(x_1(t),\cdots,x_n(t),x_1(t-\tau_1),\cdots,x_n(t-\tau_n))\right| \leqslant L^{1-v_i}\sum_{j=1}^{n}\left(c_1\left|\frac{x_j(t)}{L^{j-1}}\right| + c_2\left|\frac{x_j(t-\tau_j)}{L^{j-1}}\right|\right) \quad (20)$$

By (2), it is apparent that when $L = 1$ and $n = i$, the condition (20) will reduce to condition (1). So Assumption 2 includes Assumption 1 as a special case. As a result, the next theorem is a more general result achieved under Assumption 2. The following lemma is useful in the simulation illustration.

Lemma 1[5] Let c, d be positive real numbers. The following holds for $x \in \mathbf{R}$, $y \in \mathbf{R}$ and any positive real-valued function $\gamma(x,y)$:

$$|x|^c|y|^d \leqslant \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}(x,y)|y|^{c+d}$$

Theorem 2 Under Assumption 2, there exist constants a_i and k_i, $i = 1, 2, \cdots, n$ and $L \geqslant 1$ such that the output feedback controller (3) based on linear observer (4) globally stabilizes system (1).

Proof The proof is similar to that of Theorem 1. We use the exactly same observer (4) and control law (3). Although the nonlinear function is not in the triangular form, Assumption 2 will directly lead to the following equation similar to (12) by using the change of the coordinates of (5).

$$2\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{\Phi} \leqslant 5L^{1-v}c_3\sqrt{n}\|\boldsymbol{\varepsilon}\|^2 + L^{1-v}c_3\sqrt{n}\left[\|\hat{z}(t)\|^2 + \sum_{i=1}^{n}\hat{z}_i^2(t-\tau_i) + \sum_{i=1}^{n}\varepsilon_i^2(t-\tau_i)\right] \quad (21)$$

where $v = \min_{i=1,2,\cdots,n}\{v_i\}$. Then, the global stabilization can be concluded with an appropriate choice of gain $L > [6c_3\sqrt{n}(M+1)]^{1/v}$, which is similar to that in (16). The detailed proof is omitted here for brevity.

We end the section by the following example to illustrate the explicit construction of the global output feedback controller for Theorem 2.

Example 2 Consider the following time-delay system

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) + x_1^{3/5}(t-0.3)\ln(x_2^2(t-0.9)+1) \\ \dot{x}_2(t) &= u(t) \\ y &= x_1 \end{aligned}\right\} \quad (22)$$

Clearly, $\varphi_1 = x_1^{3/5}(t-0.3)\ln(x_2^2(t-0.9)+1)$ is not bounded by a linear growth condition imposed in Assumption 1. However we can show how Assumption 2 holds for (22). Letting $\bar{x} = x_2^{2/5}(t-0.9)$, by the differential mean-value theorem, we have

$$\ln(x_2^2(t-0.9)+1)=\ln(\bar{x}^5+1)\leqslant\bar{c}_1\bar{x} \qquad (23)$$

for a constant $\bar{c}_1>0$. As a matter of fact, by (23) and Lemma 1, we can see that

$$\begin{aligned}&|x_1^{3/5}(t-0.3)\ln(x_2^2(t-0.9)+1)|\leqslant\\&\quad\bar{c}_1|x_1(t-0.3)|^{3/5}|x_2(t-0.9)|^{2/5}\leqslant\\&\quad L^{2/5}\bar{c}_1|x_1^{3/5}(t-0.3)|\left(|x_2(t-0.9)/L|^{3/5}\right)^{2/3}\leqslant\\&\quad L^{1-3/5}\bar{c}_1(|x_1(t-0.3)|+|x_2(t-0.9)/L|)\end{aligned} \qquad (24)$$

By choosing $v_1=3/5$, it can be verified that φ_1 satisfies Assumption 2. As a result, Theorem 2 can be applied to system (22). Hence, by Theorem 2 we now can design an output feedback controller of the form:

$$\left.\begin{aligned}&u(t)=-L^2\left[k_1\hat{x}_1(t)+\frac{1}{L}k_2\hat{x}_2(t)\right]\\&\dot{\hat{x}}_1(t)=\hat{x}_2(t)+La_1(x_1(t)-\hat{x}_1(t))\\&\dot{\hat{x}}_2(t)=u(t)+L^2a_2(x_1(t)-\hat{x}_1(t))\end{aligned}\right\} \qquad (25)$$

Simulation results are shown in Figs. 3 and 4, where the gains are selected as $k_1=0.9$, $k_2=2.2$, $a_1=1.356$, $a_2=3.9$, $L=4.6$, and the initial functions are

$$\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}=\begin{bmatrix}-6\sin(t-0.2)\\-5\sin(t-0.2)\end{bmatrix},\quad\begin{bmatrix}\hat{x}_1(t)\\\hat{x}_2(t)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$
$$-1\leqslant t\leqslant 0$$

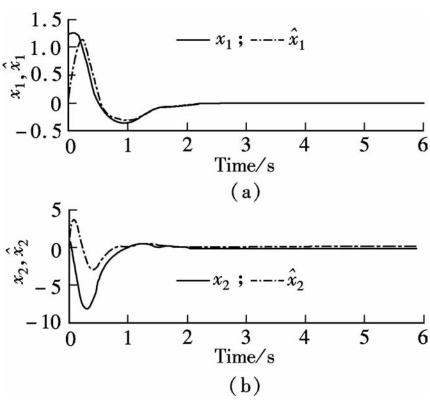

Fig. 3 State trajectories of (22) to (25). (a) x_1, $\hat{x}_1$; (b) x_2, $\hat{x}_2$

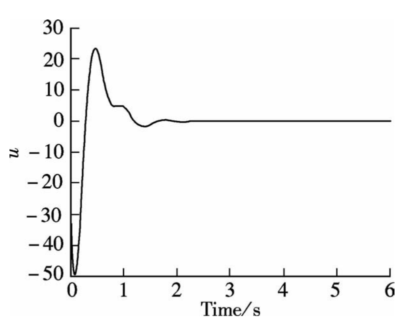

Fig. 4 Time history of the control signal

Remark 4 The uncertain nonlinear time-delay system investigated in this paper is under the linear growth condition. There are still other problems remaining unsolved. For example, an interesting research problem is how to design a homogeneous output feedback controller to globally stabilize the nonlinear time-delay systems under the homogeneous condition.

3 Conclusion

In this paper, we investigate the problem of global output feedback stabilization for a class of time-delay lower-triangular nonlinear systems under the linear growth condition. A linear output feedback controller with a scaling gain is explicitly constructed based on Ref. [1]. Then with the help of the Lyapunov-Krasovskii functional, the scaling gain is carefully adjusted to render the closed-loop system globally asymptotically stable. The output feedback controller can also be extended to the non-triangular nonlinear time-delay system. The linear output feedback controller is memoryless and easy for implementation.

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一类非线性时滞系统的线性输出反馈全局镇定控制

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摘要:针对一类非线性时滞系统,基于线性的输出反馈控制器,研究了该类系统的镇定控制问题. 该系统的不确定部分满足下三角结构的增长条件,且受到时滞参数的影响,通过引入可调增益,构造了线性输出反馈控制器. 通过选择适当形式的 Lyapunov-Krasovskii 泛函,最终证明可以找到使得闭环系统达到全局渐近稳定的可调增益的范围. 该结论也可推广到满足非三角结构的非线性时滞系统系统. 由于所构造的观测器及在其基础上构造的控制器满足线性与无记忆的特性,因此在实践中易于实现. 最后通过 2 个仿真示例验证了所提方法的有效性.

关键词:全局镇定;非线性系统;时滞系统;Lyapunov-Krasovskii 泛函;线性观测器;无记忆控制器
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