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Approach to estimation of vehicle-road longitudinal friction coefficient

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Abstract: According to the road adaptive requirements for the vehicle longitudinal safety assistant system, an estimation method of the road longitudinal friction coefficient is proposed. The method can simultaneously be applied to both the high and the low slip ratio conditions. Based on the simplified magic formula tire model, the road longitudinal friction coefficient is preliminarily estimated by the recursive least squares method. The estimated friction coefficient and the tires model parameters are considered as extended states. The extended Kalman filter algorithm is employed to filter out the noise and adaptively adjust the tire model parameters. Then the final road longitudinal friction coefficient is accurately and robustly estimated. The Carsim simulation results show that the proposed method is better than the conventional algorithm. The road longitudinal friction coefficient can be quickly and accurately estimated under both the high and the low slip ratio conditions. The error is less than 0. 1 and the response time is less than 2 s, which meets the requirements of the vehicle longitudinal safety assistant system.

Key words: road friction coefficient; recursive least squares; extended Kalman filter; vehicle longitudinal safety assistant system

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With the implementation of active safety control systems, vehicles have become safer to drive with less involvement in fatal accidents. These active safety control systems can greatly profit from being made road-adaptive; i. e., the control algorithms can be modified to account for the external road conditions if the actual tire-road friction coefficient information is available in real time. The longitudinal tire-road friction coefficient is an essential parameter for the vehicle longitudinal active safety control systems. For example, in an adaptive cruise control (ACC) system, road condition information from the friction coefficient estimation can be used to adjust the longitudinal spacing headway from the preceding vehicle that the ACC vehicle should maintain.

The tire-road friction coefficient must be estimated in real-time to meet the requirements of the vehicle longitudinal active safety control systems under normal driving conditions. So the method of tire-road friction coefficient estimation based on vehicle longitudinal dynamics is most feasible.

The relationship between the normalized longitudinal tire force and the slip ratio is different under different road conditions, which is the basis of utilizing the vehicle longitudinal dynamics to estimate the tire-road friction coefficient[1]. The most well known research in this area is on the use of slip-slope for friction coefficient identification[2−5]. In this method, the normalized longitudinal force is considered proportional to the slip ratio at low slip ratios. The slope of the relationship between the normalized longitudinal force and the slip ratio at low slip ratios is called slip-slope. The basic idea behind the use of slip-slope for friction coefficient estimation is that at low slip ratios, the tire-road friction coefficient is proportional to slip-slope. Thus, by estimating slip-slope, the tire-road friction coefficient can be estimated. But this method is only suitable for the condition of low slip ratios. The parameter estimation method is another commonly used method[6−7]. But only at the large slip ratios, the estimation results will be close to the true value. Domestic researches[8−9] are based on the above two methods, the drawbacks as mentioned above also exist. Shim et al.[10] assumed a tire-road friction coefficient, and then the response of the vehicle is estimated based on the vehicle dynamics model. According to the differences between the estimated response and the actual vehicle response, the tire-road friction coefficient can be calculated. But the method is difficult to apply to complex road conditions since it requires a lot of experience.

As mentioned above, the main problem of the tire-road friction coefficient estimation algorithms is that the algorithms cannot be applied to both high and low slip ratios simultaneously. To solve this problem, the recursive least squares (RLS) method with the forgetting fac-

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tor and the extended Kalman filter (EKF) algorithm are employed to estimate the longitudinal tire-road friction coefficient in this paper. The method utilizes the relationship between the normalized longitudinal tire force and the slip ratio to identify the longitudinal tire-road friction coefficient μ, which can be applicable to for both the high and the low slip ratios, and the effectiveness and feasibility are verified by simulation.

1 Proposed Method

If only the longitudinal motion is considered and the lateral force is ignored, the normalized longitudinal tire force ϕ and the slip ratio s at each wheel can be represented as

$$s=\frac{\omega r-v}{\max(\omega r, v)} \tag{1}$$

$$\phi=\frac{F_x}{F_z} \tag{2}$$

where ω is the angular wheel speed; r is the effective tire radius; v is the vehicle's absolute velocity; F_x is the longitudinal force from ground to wheel; and F_z is the normal force.

Fig. 1 shows a typical relationship between s and ϕ for various values of the tire-road friction coefficient. μ is the tire-road friction coefficient.

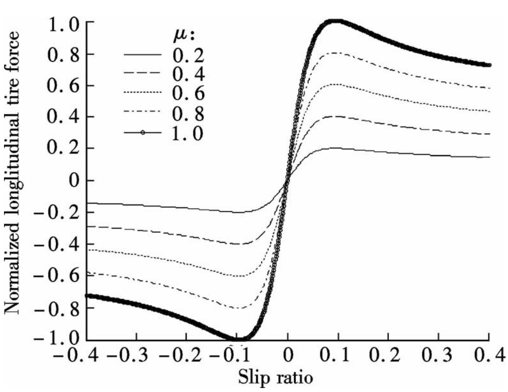

Fig. 1 s-ϕ curves with different friction coefficients

In this paper, the friction coefficient is assumed to be the same at each wheel of the vehicle. By calculating s and ϕ, the longitudinal tire-road friction coefficient μ can be estimated by the RLS method with the forgetting factor, which is based on the simplified magic formula tire model. Then the estimated μ and the tire model parameters are used as extended states. The EKF algorithm is employed to filter out the noise and adaptively adjust the tire model parameters. Then the final road longitudinal friction coefficient is accurately and robustly estimated. The flowchart of the estimation method is shown in Fig. 2.

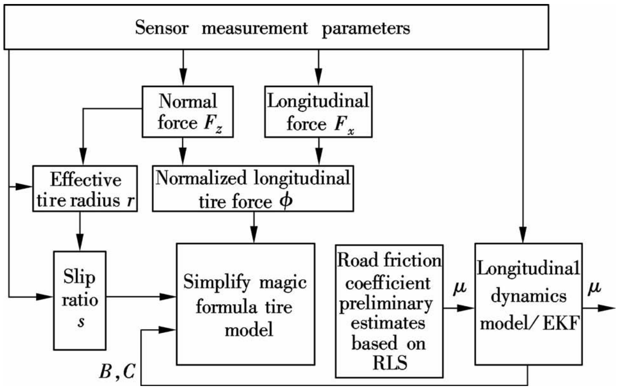

Fig. 2 Flowchart of estimation method

2 Vehicle and Tire Models

The longitudinal vehicle dynamics model can be written as

$$ma_x=F_x-D_a v^2-C_{\mathrm{roll}} mg \tag{3}$$

where m is the mass of the vehicle; a_x is the vehicle longitudinal acceleration; D_a is the air resistance coefficient; C_{roll} is the rolling resistance coefficient; and g is the acceleration of gravity.

A simplified magic formula tire model[11] is adopted in this paper.

$$\phi=\mu \sin[C \arctan(Bs)] \tag{4}$$

where B and C are the model parameters.

3 Road Friction Coefficient Preliminarily Estimated based on RLS

3.1 Longitudinal slip ratio calculation

The effective tire radius r is calculated as

$$r=r_u \frac{\sin[\cos^{-1}(r_s/r_u)]}{\cos^{-1}(r_s/r_u)} \tag{5}$$

where r_u is the undeformed radius of the tire; r_s is the static tire radius and it can be described as $r_s=r_u-F_z/k_t$, k_t is the vertical tire stiffness. The longitudinal slip ratio can be calculated by Eq. (1).

3.2 Normalized longitudinal tire force calculation

Eq. (3) can be rewritten as

$$F_x=F_{xf}+F_{xr}=ma_x+D_a v^2+C_{\mathrm{roll}} mg \tag{6}$$

where F_{xf} and F_{xr} are the traction forces of the front and the rear wheels. The total vehicle longitudinal force F_x can be obtained by Eq. (6).

The normal forces at the front and rear tires can be calculated as follows:

$$F_{zf} = \frac{mgb}{a+b},\ F_{zr} = \frac{mga}{a+b} \tag{7}$$

where F_{zf} and F_{zr} are the normal forces at the front and the rear tires; a and b are the distances from the center of gravity to the front and the rear axles.

The relationship between s and ϕ for the front and rear tires can be written as

$$\phi_f = \frac{F_{xf}}{F_{zf}} = \mu \sin[C \arctan(Bs_f)] \tag{8}$$

$$\phi_r = \frac{F_{xr}}{F_{zr}} = \mu \sin[C \arctan(Bs_r)] \tag{9}$$

3.3 Preliminary estimates of μ

Assuming that the front and rear tires are under the same road surface condition, which is true for many driving situations, the total longitudinal force is

$$F_x = F_{xf} + F_{xr} = \phi_f F_{zf} + \phi_r F_{zr} = \mu\{F_{zf}\sin[C\arctan(Bs_f)] + F_{zr}\sin[C\arctan(Bs_r)]\} \tag{10}$$

Eq. (10) can be rewritten into a standard parameter identification format as

$$y(k) = \boldsymbol{\phi}^{\mathrm{T}}(k)\boldsymbol{\theta}(k) + e(k) \tag{11}$$

where k denotes the discrete time; $y(k) = F_x$ is the system output; $\boldsymbol{\theta}(k) = \mu$ is the unknown parameter of interest; $\boldsymbol{\phi}(k) = \{F_{zf}\sin[C\arctan(Bs_f)] + F_{zr}\sin[C\arctan(Bs_r)]\}$ is the measured regression vector; $e(k)$ is the identification error. Then the only unknown parameter $\boldsymbol{\theta}(k) = \mu$ can be identified in real-time using the RLS method with the forgetting factor as follows:

1) Measure the system output $y(k)$ and calculate the regression vector $\boldsymbol{\phi}(k)$.

2) Calculate the identification error $e(k)$,

$$e(k) = y(k) - \boldsymbol{\phi}^{\mathrm{T}}(k)\boldsymbol{\theta}(k-1)$$

3) Calculate the updated gain vector $\boldsymbol{K}(k)$ as

$$\boldsymbol{K}(k) = \frac{\boldsymbol{N}(k-1)\boldsymbol{\phi}(k)}{\lambda + \boldsymbol{\phi}^{\mathrm{T}}(k)\boldsymbol{N}(k-1)\boldsymbol{\phi}(k)}$$

And calculate the covariance matrix $\boldsymbol{N}(k)$ by

$$\boldsymbol{N}(k) = \frac{1}{\lambda}\left[\boldsymbol{N}(k-1) - \frac{\boldsymbol{N}(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^{\mathrm{T}}(k)\boldsymbol{N}(k-1)}{\lambda + \boldsymbol{\phi}^{\mathrm{T}}(k)\boldsymbol{N}(k-1)\boldsymbol{\phi}(k)}\right]$$

The parameter λ is called the forgetting factor, which is used to effectively reduce the influence of old data which may no longer be relevant to the model, and, therefore, prevents a covariance wind-up problem.

4) Update the parameter estimate vector $\boldsymbol{\theta}(k)$,

$$\boldsymbol{\theta}(k) = \boldsymbol{\theta}(k-1) + \boldsymbol{K}(k)e(k)$$

The road friction coefficient μ can be preliminary estimated in real-time.

4 Longitudinal Tire-Road Friction Coefficient Identification based on EKF

In the tire-road friction coefficient estimation process described above, the model parameters B and C are assumed to be known and constant. However, during vehicle operation, B and C cannot be directly measured and they are time-varying, which may affect the accuracy of the estimation of the tire-road friction coefficient. In order to real-time update B and C, and filter μ, the EKF model is established based on the longitudinal dynamic model using Eq. (3).

The discretized state equation and measurement equation can be written as

$$\left.\begin{aligned} \boldsymbol{X}(k) &= f(\boldsymbol{X}(k-1)) + \boldsymbol{W}(k-1) \\ \boldsymbol{Z}(k) &= h(\boldsymbol{X}(k)) + \boldsymbol{V}(k) \end{aligned}\right\} \tag{12}$$

where k refers to the discrete-time step; the state vector $\boldsymbol{X} = \{v, \mu, B, C\}^{\mathrm{T}}$; the measurement vector $\boldsymbol{Z} = \{a_x, v, \mu\}^{\mathrm{T}}$; $\boldsymbol{W}$ and $\boldsymbol{V}$ are the system and measurement noise vectors, respectively; $f(\cdot)$ and $h(\cdot)$ are the nonlinear system and measurement functions which can be deduced from Eq. (3).

Assuming that the system and measurement noises to be Gaussian with a zero mean and their covariance matrices are $\boldsymbol{Q}$ and $\boldsymbol{R}$, respectively, the EKF process consists of the following two phases.

1) Time update:

$$\hat{\boldsymbol{X}}(k,k-1) = f(\boldsymbol{X}(k-1))$$

$$\boldsymbol{P}(k,k-1) = \boldsymbol{A}(k,k-1)\boldsymbol{P}(k-1)\boldsymbol{A}'(k,k-1) + \boldsymbol{Q}(k-1)$$

2) Measurement update:

$$\boldsymbol{K}(k) = \boldsymbol{P}(k,k-1)\boldsymbol{H}'(k)[\boldsymbol{H}(k)\boldsymbol{P}(k,k-1)\boldsymbol{H}'(k) + \boldsymbol{R}(k)]^{-1}$$

$$\hat{\boldsymbol{X}}(k) = \hat{\boldsymbol{X}}(k,k-1) + \boldsymbol{K}(k)[\boldsymbol{Z}(k) - \boldsymbol{H}(k)\hat{\boldsymbol{X}}(k,k-1)]$$

$$\boldsymbol{P}(k) = [\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{H}(k)]\boldsymbol{P}(k,k-1)$$

where $\boldsymbol{I}$ is the identity matrix; $\boldsymbol{A}$ and $\boldsymbol{H}$ are the Jacobian matrices of the system function $f(\cdot)$ and the measurement function $h(\cdot)$ with respect to $\boldsymbol{X}$; i. e.,

$$\boldsymbol{A}_{[i,j]} = \frac{\partial f_i}{\partial x_j}(\hat{\boldsymbol{X}}(k,k-1)) \qquad i=1,2,3,4; j=1,2,3,4$$

$$\boldsymbol{H}_{[i,j]} = \frac{\partial h_i}{\partial x_j}(\hat{\boldsymbol{X}}(k,k-1)) \qquad i=1,2,3; j=1,2,3,4$$

The model parameters B and C, estimated by the EKF, are feedbacks to the tire model, so the estimated values by the RLS can be updated in real-time. Therefore, the estimation accuracy of the tire-road friction coefficient can be improved, and the estimated values can respond to the road state changes. The μ output by the EKF is the final estimation result.

5 Simulation Results and Discussion

To evaluate the performance of the proposed estimation method of the longitudinal friction coefficient, numerical simulations are performed using Carsim in Matlab/Simulink. According to Ref. [12], the initial values of model parameters B and C are 14 and 1.3, respectively. The forgetting factor λ is set to be 0.995. The proposed algorithm is validated under the high and the low slip ratio conditions with the tire-road friction coefficient changing, and the estimation results are compared with the conventional slip-slope algorithm. Simulation results show that the proposed algorithm can be applied to both the high and the low slip ratios; the estimation results are accurate and robust, and they can quickly respond to the changes in road conditions.

5.1 Simulation under low slip ratio condition

The main vehicle parameters used in the simulations are: $k_t = 230$ N/mm, $m = 1\,220$ kg, $r_s = 310.8$ mm, $r_w = 304$ mm, $a = 1.04$ mm, $b = 1.56$ mm. Fig. 3 and Fig. 4 are the simulation results. The figures show that the values of the slip ratio are small, and the proposed method can quickly identify the road friction coefficient with high accuracy; the error is less than 0.1. From Fig. 4, we can see that the proposed method can converge to the true value within 2 s when the tire-road friction coefficient jumps, which meets the real-time requirements.

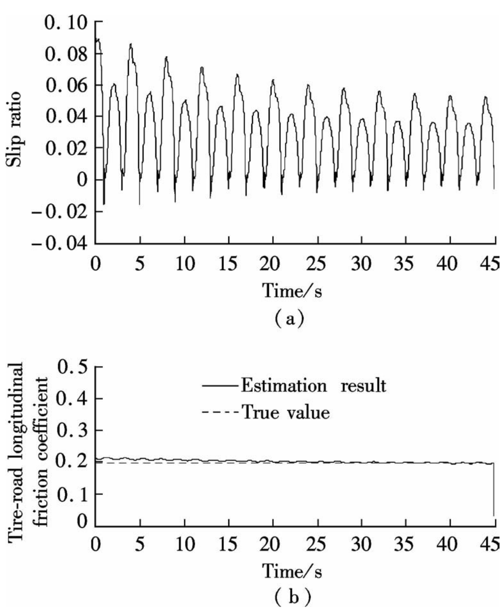

Fig. 3 Simulation results of low slip ratios. (a) Slip ratio; (b) Tire-road friction coefficient

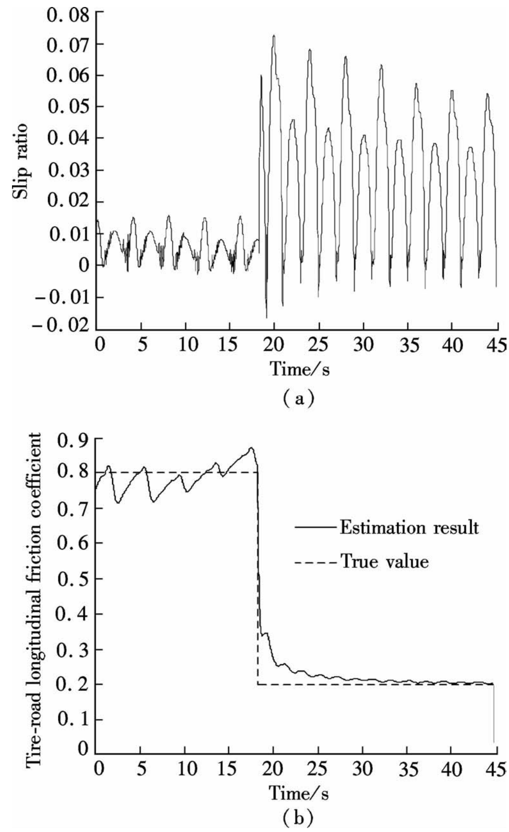

Fig. 4 Simulation results of low slip ratios with friction coefficient changing. (a) Slip ratio; (b) Tire-road friction coefficient

5.2 Simulation under high slip ratio condition

The conventional slip-slope algorithm is no longer suitable for the high slip ratio condition because the relationship between s and ϕ is not linear. Fig. 5 and Fig. 6 are the simulation results. The figures show that estimation results by the slip-slope algorithm produce a great error. The proposed method can quickly identify the road friction coefficient with high accuracy at high slip ratios and quickly respond to the changes in road conditions.

6 Conclusion

Simulation results show that the proposed algorithm can quickly and accurately estimate the tire-road friction coefficient under both the high and the low slip ratio conditions, which can meet the requirements of the vehicle longitudinal active safety system. And the proposed method only needs the existing sensors in commercial vehicles, so the proposed method is suitable for on-board applications with low computational complexity.

The key of the proposed algorithm is to obtain an accurate s-ϕ curve. The s-ϕ curve can be obtained by the bench test, but the friction conditions on an actual road is different from the bench test, and the accuracy of the real-time tire-road friction coefficient is also reduced due

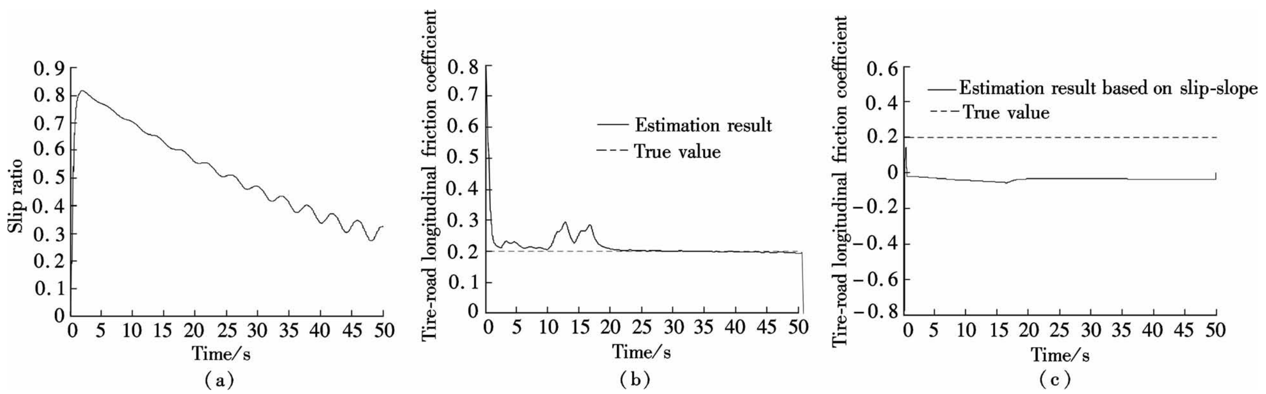

Fig. 5 Simulation results of high slip ratios. (a) Slip ratio; (b) Friction coefficient estimated by the proposed method; (c) Friction coefficient estimated by the slip-slope method

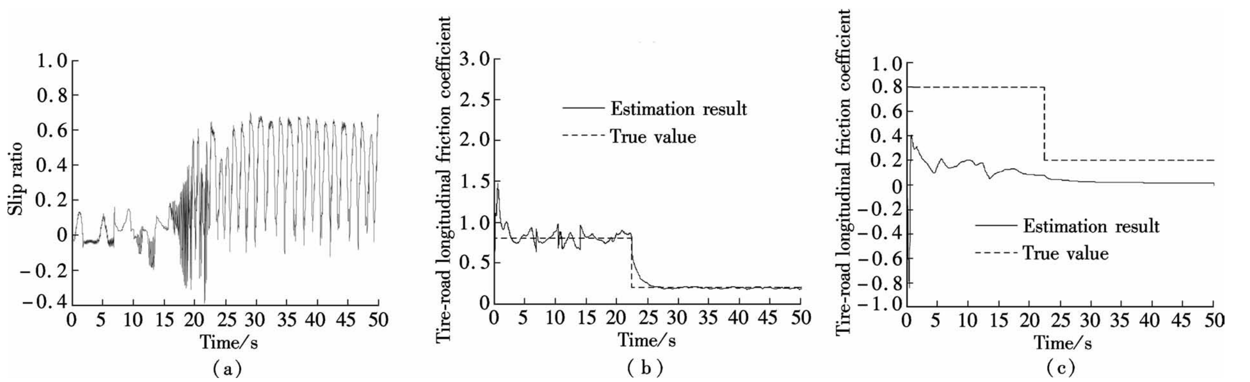

Fig. 6 Simulation results of high slip ratios with friction coefficient changing. (a) Slip ratio; (b) Friction coefficient estimated by the proposed method; (c) Friction coefficient estimated by the slip-slope method

to the high dynamic characteristics and noises. So the further work must focus on building s-ϕ relationships in different roads by a lot of vehicle tests on the common road, and then the proposed method can be applied to practice and achieves mass-market applications.

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一种道路纵向附着系数估计方法

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摘要：针对汽车纵向安全辅助系统道路自适应的要求，提出了一种道路纵向附着系数估计方法. 该方法能够同时适应高滑移率和低滑移率工况. 首先基于简化魔术轮胎模型，利用递归最小二乘方法实时初步估计出纵向附着系数，然后将所估计出的附着系数与轮胎模型参数作为扩充状态，利用扩展卡尔曼滤波算法，滤除信号噪声，实现轮胎模型系数的自适应调整，最终实时获取准确的道路纵向附着系数估计，并通过车辆动力学软件 Carsim 仿真验证了算法的有效性和可行性. 结果表明该算法优于传统算法，在高滑移率和低滑移率工况下都能够快速、准确地估计出道路附着系数，误差小于 0.1，响应时间小于 1 s，满足车辆纵向安全辅助系统的需要.

关键词：道路附着系数；递归最小二乘法；扩展卡尔曼滤波；汽车纵向安全辅助系统

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