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Exponential stabilization of distributed parameter switched systems under dwell time constraints

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Abstract: The exponential stabilization problem for finite dimensional switched systems is extended to the infinite dimensional distributed parameter systems in the Hilbert space. Based on the semigroup theory, by applying the multiple Lyapunov function method, the exponential stabilization conditions are derived. These conditions are given in the form of linear operator inequalities where the decision variables are operators in the Hilbert space; while the stabilization properties depend on the switching rule. Being applied to the two-dimensional heat switched propagation equations with the Dirichlet boundary conditions, these linear operator inequalities are transformed into standard linear matrix inequalities. Finally, two examples are given to illustrate the effectiveness of the proposed results.

Key words: distributed parameter switched systems; exponential stabilization; multiple Lyapunov function; linear operator inequalities; dwell time

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During the last decade, the study of switched systems has attracted considerable attention due to its significance in both theoretical research and practical applications[1]. A switched system is a dynamical system described by a family of continuous-time subsystems and a rule that governs the switching between them. In many real cases, switched systems can be described by partial differential equations (PDE) or a combination of ordinary differential equations (ODE) and PDE, such as in chemical industry processes and biomedical engineering. We refer to these switched systems as distributed parameter switched systems (DPSS) or infinite dimensional switched systems[2-3]. The results of infinite dimensional dynamical switched systems are usually not straightforward, and they frequently require further analysis. Based on the fact that switched systems described by the PDE are more common in general, there is a realistic need to discuss such systems.

Analysis of switching sequences is a main research topic in the field of switched systems, and it plays an important role in the study of problems such as stability analysis and control design. The stability issues of switched systems include several interesting phenomena. It is well known and easy to demonstrate that switching between stable subsystems may lead to instability[4-6]. This fact makes stability and stabilization analysis of switched systems an important and challenging problem, which has received great attention[4-13]. Among them, there has been considerable growth of interest in using the dwell time approach to deal with switched systems[4, 11, 13].

On the other hand, there are several works concerning the infinite dimensional DPSS[14-21]. For example, Farra et al. [14] used Galerkin's method to control synthesis for a quasi-linear parabolic equation, in which the state equation is fixed and the controller is switched. Sasane[15] generalized the finite dimensional switched system[8] to the infinite dimensional Hilbert space. Ref. [15] shows that when all the subsystems are stable and commutative pairwise, the switched linear system is stable under arbitrary switching via the common Lyapunov function. Hante et al. [18-19] gave necessary and sufficient conditions in terms of the the existence of the common Lyapunov function for the DPSS. Ouzahra[20] considered the feedback stabilization of the fixed distributed semilinear systems using switching controls which does not require the knowledge of the state of the system. Although much research has been done on stability and stabilization for switched systems, to the best of our knowledge, the control synthesis problem for the DPSS has not been extensively investigated.

Motivated by the above considerations, in this paper, we investigate control synthesis of the DPSS via the multiple Lyapunov function method. We use the semigroup theory due to the fact that it plays a central role and provides a unified and powerful tool for the study of the PDE systems[22]. The control design problem concentrates on the state feedback design problem. The main contribution of this paper is twofold. First, the controller is designed

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for the DPSS by applying the linear operator inequalities (LOIs) framework for the first time. Secondly, the sufficient conditions for exponential stabilization are derived in terms of the LOIs where the decision variables are operators in the Hilbert space, and the stabilization properties depend on the switching rule, while the existing work aims at unswitched distributed parameter systems. Being applied to heat switched propagation equations with the Dirichlet boundary conditions, the LOIs are subsequently reduced to standard linear matrix inequalities (LMIs), which has the advantage of being numerically well tractable by using the Matlab software. Compared with Ref. [21], it should be pointed out that our stabilization conditions completely depend on the system parameters and boundary data.

1 Preliminaries and Problem Formulation

Let H, U be separable Hilbert space with the inner product $\langle \cdot , \cdot \rangle$. Notation $\| \cdot \|$ denotes the usual norm on H. Let $L(U,H)$ denote the space of the bounded linear operator from U to H, and $L(H)$ denotes the space of the bounded linear operator from H to H. I stands for the identity operator on H or appropriate dimensional identity matrix.

Finally, let $\Omega \subset \mathbf{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$, and $\mathbf{R}^n$ denotes the n-dimensional Euclidean space. $L_2(\Omega,H)$ is the Hilbert space of square integrable functions x: $\Omega \rightarrow H$ with norm $\| x \|_{L_2(\Omega,H)} = \left(\int_\Omega | x |^2 \mathrm{d}x \right)^{1/2}$. $H^2(\Omega)$ and $H_0^2(\Omega)$ denote the classical Sobolev spaces and they are defined as

$$H^2(\Omega) = \left\{ u \in L^2(\Omega) \;\middle|\; \frac{\partial^2 u}{\partial x^2} \in L^2(\Omega) \right\}$$

$$H_0^2(\Omega) = \left\{ u \in L^2(\Omega) \;\middle|\; \frac{\partial u}{\partial x} \in L^2(\Omega), u(\partial\Omega, t) = 0 \right\}$$

Definition 1[23] Let $P: H \rightarrow H$ with a dense domain $D(P) \subset H$ be self-adjoint, then $P \geqslant 0$ (positive) if

$$\langle Px, x \rangle \geqslant 0 \qquad \forall x \in D(P) \tag{1}$$

where $P > 0$ (strictly positive), iff it is self-adjoint in the sense that $P^* = P$ and there exists a constant $m > 0$, such that

$$\langle Px, x \rangle \geqslant m \| x \|^2 \qquad \forall x \in D(P) \tag{2}$$

$A_0 \leqslant 0, A_0 < 0$ mean that $-A_0 \geqslant 0$, $-A_0 > 0$, respectively.

Definition 2 An operator $M \in L(H)$ is called invertible if there exists an operator $N \in L(H)$ such that $MN = NM = I$. We write $N = M^{-1}$ to denote the inverse of operator M.

Lemma 1 (Poincare inequality)[24] Let scalar function $u \in H_0^1(\overline{\Omega}, R)$. Moreover, $\Omega \subseteq \Omega_1$, then we have

$$\int_\Omega u^2 \mathrm{d}x \leqslant \gamma^2 \int_\Omega \sum_{i=1}^n \left(\frac{\partial u}{\partial x_i} \right)^2 \mathrm{d}x = \gamma^2 \int_\Omega | \nabla u |^2 \mathrm{d}x \tag{3}$$

where $\Omega_1: 0 \leqslant x_i \leqslant \delta \ (i = 1, 2, \cdots, n)$, $\gamma = \delta/\sqrt{n}$, $\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \cdots, \frac{\partial}{\partial x_n} \right)$.

We consider a general form of the linear distributed parameter switched control system

$$\dot{x}(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t) \qquad t \geqslant t_0 \tag{4}$$

with the initial condition

$$x(t_0) = x_0 \tag{5}$$

where $x \in H$ is the state of the system; $u \in U$ is the control. $\sigma: [t_0, \infty) \rightarrow \Theta$ is the switching signal mapping time to some finite index set $\Theta = \{1, 2, \cdots, m\}$, and the switching signal σ is a piecewise constant. The discontinuities of σ are called switching times or switches.

Let $\{x_0; (i_0, t_0), (i_1, t_1) \cdots, (i_k, t_k), \cdots, \mid i_k \in \Theta, k \in \mathbf{N}\}$ denote the switching sequence. Switching time $t_0 < t_1 < \cdots < t_{k-1} < t_k < \cdots$ with $\lim_{k\rightarrow\infty} t_k = \infty$, when $t \in [t_k, t_{k+1})$, the i_k-th subsystem is active. The switching time $t_1, t_2, \cdots$ satisfies the following inequality

$$t_k - t_{k-1} \geqslant \tau_{\mathrm{d}} \qquad \forall k \in \mathbf{N} \tag{6}$$

where $\tau_{\mathrm{d}} > 0$ is the dwell time.

The objective of this paper is concerned with the control synthesis problem for switched systems (4) and (5). The control synthesis is related to the design of a switched state feedback control

$$u(t) = K_{\sigma(t)} x(t) \tag{7}$$

which ensures the exponential stability of the closed-loop DPSS

$$\dot{x}(t) = [A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}] x(t) \tag{8}$$

under some switching law, where $K_1, K_2, \cdots, K_m$ are a family of gain operators to be determined.

2 Exponential Stabilization Analysis for DPSS under Dwell Time Constraints

In this section, the exponential stabilization condition for the switched control system is extended to the distributed parameter system in the Hilbert space.

Without loss of generality, we make the following assumptions.

Assumption 1 1) The state of the DPSS (8) does not jump at switching instants; i. e., the trajectory $x(t)$ is everywhere continuous. Switching signal $\sigma(t)$ has a finite switching number at any finite interval time.

2) Each operator $A_i (i = 1, 2, \cdots, m)$ generates analytical semigroup $T_i(t)$ and the domain $D(A_i) \subset H$ of the operator A_i is dense in H.

3) Operators satisfy the conditions $B_i \in L(U,H)$ and $K_i \in L(H,U)$.

Choose the following multiple Lyapunov function candidate

$$V(x,t) = V_{\sigma(t)}(x,t) = \langle P_{\sigma(t)}x(t), x(t)\rangle \qquad (9)$$

for (8) in the corresponding Hilbert space $D(A_i)$ $(i=1, 2,\cdots,m)$, where operators $P_i: D(A_i)\to H$ and $P_i>0$ satisfy

$$\gamma_{p_i}\langle x,x\rangle \leqslant \langle P_i x,x\rangle \leqslant \gamma_{P_i}\langle x,x\rangle, x\in D(A_i) \qquad (10)$$

for some positive constants $\gamma_{p_i}, \gamma_{P_i}$.

Theorem 1 For a given constant $\beta>0$, suppose that Assumption 1 holds, if there exist linear operators $X_i>0$ and Y_i such that the following LOIs

$$A_iX_i + X_iA_i^* + B_iY_i + Y_i^*B_i^* + \beta X_i \leqslant 0 \qquad \forall i\in\Theta \qquad (11)$$

hold in the Hilbert space $D(A_i)$ in the sense of (2), where $X_i = P_i^{-1}$ $(X_i: D(A_i)\to H)$, $Y_i = K_iP_i^{-1}$ $(Y_i \in L(H))$. Then the state feedback given by (7) with $K_i = Y_iP_i$ exponentially stabilizes the systems (4) and (5) for arbitrary switching signal $\sigma(t)$ under the dwell time, satisfying $\tau_d > \tau_d^* = \dfrac{\ln\mu}{\beta}$ where $\mu = \max\limits_{i\in\Theta}\left\{\dfrac{\gamma_{P_i}}{\gamma_{p_i}}\right\}$.

Proof Systems (4) and (5) with state feedback control (7) results in system (8). Suppose that Assumption 1 holds, from Corollary 5.2.4 of Ahmed [22], it can be proved that systems (5) and (8) have a unique classical solution for every $x_0\in H$, i.e., systems (5) and (8) turn to be well-posed on time interval $[t_0,\infty)$ because the state does not jump at the switching instants.

Choosing the multiple Lyapunov function candidate for system (8) as the form of (9), where $V_i = C(H\times[t_0, +\infty), R^+)$, and operators P_i satisfying (10) and the following inequalities

$$\Omega_i = P_iA_i + P_iB_iK_i + A_i^*P_i + K_i^*B_i^*P_i + \beta P_i \leqslant 0 \qquad \forall i\in\Theta \qquad (12)$$

For $t\in[t_{k-1},t_k)$, we can obtain

$$\langle P_{\sigma(t_{k-1})}x(t), x(t)\rangle = V(t) \leqslant \gamma_{P_{\sigma(t_{k-1})}} \| x(t_{k-1})\|^2$$

By using (10), it follows that

$$\|x(t)\|^2 \leqslant \frac{\gamma_{P_{\sigma(t_{k-1})}}}{\gamma_{p_{\sigma(t_{k-1})}}}\|x_{t_{k-1}}\|^2 \leqslant \mu\|x(t_{k-1})\|^2 \qquad \forall t\in[t_{k-1},t_k)$$

It is easy to calculate that

$$\begin{aligned}\|x(t)\|^2 &\leqslant \mu\|x(t_{k-1})\|^2 \leqslant \\ &\mu e^{-\beta(t-t_{k-1})}\|x(t_{k-1})\|^2 \leqslant \\ &\mu^2 e^{-\beta(t-t_{k-1})}e^{-\beta(t_{k-1}-t_{k-2})}\|x(t_{k-2})\|^2 \leqslant \\ &\mu^2 e^{-\beta(t-t_{k-2})}\|x(t_{k-1})\|^2 \leqslant \\ &\vdots \\ &\mu^k e^{-\beta(t-t_0)}\|x(t_0)\|^2\end{aligned}$$

for all $t\geqslant t_0$ and constant $\mu\geqslant 1$. Noticing the fact that $(k-1)\tau_d \leqslant t-t_0$, then

$$\begin{aligned}\|x(t)\|^2 &\leqslant \mu e^{-\beta(t-t_0)}e^{(k-1)\ln\mu}\|x(t_0)\|^2 \leqslant \\ &\mu e^{-(\beta-\ln\mu/\tau_d)(t-t_0)}\|x(t_0)\|^2\end{aligned} \qquad (13)$$

Let $\lambda = \beta - \dfrac{\ln\mu}{\tau_d} > 0$, where $\tau_d > \dfrac{\ln\mu}{\beta} = \tau_d^*$.

We now derive that $\|x(t)\| \leqslant \sqrt{\mu}e^{-\lambda/2(t-t_0)}\|x(t_0)\|$. This shows that the overall DPSS (8) is exponentially stable for arbitrary switching signal with the dwell time $\tau_d > \tau_d^* = \ln\mu/\beta$. The proof is completed.

The inequalities (12) for (P_i, K_i) is not LOIs. Here, in a similar manner as that in the ODE case, we use the operator transformation method. Let LOIs (12) be transformed into LOIs (11). It is easily shown by left-and right-multiplying P_i^{-1} and $(P_i^{-1})^*$; moreover, $(P_i^{-1})^* = P_i^{-1}$, where $X_i = P_i^{-1}$ $(X_i: D(A_i)\to H)$, $Y_i = K_iP_i^{-1}$ $(Y_i \in L(H))$.

Remark 1 From the proof of Theorem 1, we find that $\dfrac{\lambda}{2} = \dfrac{\beta}{2} - \dfrac{\ln\mu}{2\tau_d}$; thus $\dfrac{\lambda}{2} \leqslant \dfrac{\beta}{2}$. This demonstrates that the decay rate of the overall DPSS (8) is smaller than that of its subsystems. In particular, when there is no switching, we have $\tau_d\to\infty$, in this case, the decay rate λ is equal to β, which is just the decay rate of the subsystems.

3 Application of Two Dimensional Switched Heat Propagation Systems

For the following switched heat propagation control system:

$$\begin{aligned}&\boldsymbol{y}_t(x,y,t) = \boldsymbol{D}_{\sigma(t)}\nabla^2\boldsymbol{y}(x,y,t) + \boldsymbol{B}_{\sigma(t)}u(t) \\ &(x,y,t)\in[0,\sqrt{2}]\times[0,\sqrt{2}]\times[t_0,+\infty)\end{aligned} \qquad (14)$$

Let the boundary value condition be

$$\boldsymbol{y}(x,y,t) = \boldsymbol{0} \qquad (x,y,t)\in\partial\Omega\times[t_0,+\infty) \qquad (15)$$

The initial condition is

$$\boldsymbol{y}(x,y,t_0) = \boldsymbol{y}_0 \qquad (16)$$

We consider that the static state feedback is

$$\boldsymbol{u}(t) = \boldsymbol{K}_{\sigma(t)}\boldsymbol{y}(t) \qquad (17)$$

Ensure the exponential stability of the closed-loop DPSS to be

$$\left.\begin{aligned}&\boldsymbol{y}_t(x,y,t) = \boldsymbol{D}_{\sigma(t)}\nabla^2\boldsymbol{y}(x,y,t) + \boldsymbol{B}_{\sigma(t)}\boldsymbol{K}_{\sigma(t)}\boldsymbol{y}(x,y,t) \\ &(x,y,t)\in[0,\sqrt{2}]\times[0,\sqrt{2}]\times[t_{k-1},t_k) \\ &\boldsymbol{y}(x,y,t) = \boldsymbol{0} \qquad (x,y,t)\in\partial\Omega\times[t_0,+\infty) \\ &\boldsymbol{y}(x,y,t_0) = \boldsymbol{y}_0\end{aligned}\right\} \qquad (18)$$

where $\boldsymbol{y} = (y_1, y_2, \cdots, y_n) \in \mathbf{R}^n$ is an n-dimensional state vector (the state vector of the control system). $\boldsymbol{u} = (u_1, u_2, \cdots, u_l) \in \mathbf{R}^l$ is an l-dimensional control vector. $\Omega = [0, \sqrt{2}] \times [0, \sqrt{2}] \subset \mathbf{R}^2$ is a bounded domain with smooth boundary $\partial\Omega$ and state variable vector$(x, y) \in \Omega$. $t \in [t_0, \infty)$ is the time, $k \in \mathbf{N}$. Each $\boldsymbol{D}_i = \operatorname{diag}(d_{i1}, d_{i2}, \cdots, d_{in})$ is a positive diagonal matrix. $\boldsymbol{B}_i, \boldsymbol{K}_i (i \in \theta)$ are appropriate dimensional constant matrices. $\theta = \{1, 2\}$, ∇^2 denotes the Laplace operator; i. e., $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and ∇ denote the gradient; i. e., $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$.

For a precise characterization of the class of the PDE systems considered in this paper, we formulate the system of Eq. (14) as an infinite dimensional system in the Hilbert space $H = L_2(\Omega, \boldsymbol{R}^n)$ with H being the space of sufficiently smooth n-dimensional vector functions defined on Ω that satisfy the boundary condition (16).

Define the state function x on H as

$$x(t) = \boldsymbol{y}(\cdot, \cdot, t) \qquad t \geqslant t_0 \tag{19}$$

the operators $A_1 = \boldsymbol{D}_1 \nabla^2 = \boldsymbol{D}_1 \frac{\partial}{\partial x^2} + \boldsymbol{D}_1 \frac{\partial}{\partial y^2}$, $A_2 = \boldsymbol{D}_2 \nabla^2 = \boldsymbol{D}_2 \frac{\partial}{\partial x^2} + \boldsymbol{D}_2 \frac{\partial}{\partial y^2}$; then Eq. (14) can be rewritten in the form of Eq. (4) and the first equation of (18) can be rewritten as Eq. (8), respectively, where the operator A_i has the dense domain

$$D(A_i) = \{\boldsymbol{y} \in H^2(\Omega, \mathbf{R}^n) \cap H_0^1(\Omega, \mathbf{R}^n) : \boldsymbol{y}(x, y, t) = 0, (x, y) \in \partial\Omega\} \tag{20}$$

It is easily known that operators A_1 and A_2 generate analytical semigroups $T_1(t)$ and $T_2(t)$, respectively, and system (18) has a unique classical solution[2].

The multiple Lyapunov function is chosen as

$$V(t) = V_{\sigma(t)}(t) = \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} \boldsymbol{y}^{\mathrm{T}}(x, y, t) \boldsymbol{P}_{\sigma(t)} \boldsymbol{y}(x, y, t) \, \mathrm{d}x\mathrm{d}y \tag{21}$$

with positive constant diagonal matrices $\boldsymbol{P}_i$.

Differentiating (21), we find that

$$\langle (A_i^* P_i + P_i A_i) x, x \rangle = \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} (\nabla^2 \boldsymbol{y}^{\mathrm{T}} \boldsymbol{D}_i \boldsymbol{P}_i \boldsymbol{y} + \boldsymbol{y}^{\mathrm{T}} \boldsymbol{P}_i \boldsymbol{D}_i \nabla^2 \boldsymbol{y}) \mathrm{d}x\mathrm{d}y$$

for $x \in D(A_i)$.

Because $\boldsymbol{P}_i, \boldsymbol{D}_i$ are constant diagonal matrices, then $\boldsymbol{P}_i \boldsymbol{D}_i = \boldsymbol{D}_i \boldsymbol{P}_i$.

Noticing that $\boldsymbol{P}_i, \boldsymbol{D}_i$ are positive diagonal matrices, we have

$$\begin{aligned}
\langle (A_i^* P_i + P_i A_i) x, x \rangle &\leqslant 2\lambda_{\max}(\boldsymbol{P}_i \boldsymbol{D}_i) \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} \nabla^2 \boldsymbol{y}^{\mathrm{T}} \boldsymbol{I} \boldsymbol{y} \mathrm{d}x\mathrm{d}y \leqslant \\
&2\lambda_{\max}(\boldsymbol{P}_i \boldsymbol{D}_i) \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} [\nabla^2 y_1, \cdots, \nabla^2 y_n] \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \mathrm{d}x\mathrm{d}y \leqslant \\
&2\lambda_{\max}(\boldsymbol{P}_i \boldsymbol{D}_i) \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} (y_1 \nabla^2 y_1 + \cdots + y_n \nabla^2 y_n) \mathrm{d}x\mathrm{d}y
\end{aligned}$$

hold in corresponding $x \in D(A_i)$ $(i = 1, 2)$.

Integrating by part, according to the famous Green's first identity and boundary condition (15), we can obtain the following inequalities

$$\langle (A_i^* \boldsymbol{P}_i + \boldsymbol{P}_i A_i) x, x \rangle \leqslant -2\lambda_{\max}(\boldsymbol{P}_i \boldsymbol{D}_i) \cdot \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} (|\nabla y_1|^2 + \cdots + |\nabla y_n|^2) \mathrm{d}x\mathrm{d}y$$

According to Poincare's inequality (3), we can obtain

$$\begin{aligned}
&\langle (A_i^* \boldsymbol{P}_i + \boldsymbol{P}_i A_i) x, x \rangle \leqslant \\
&-2\lambda_{\max}(\boldsymbol{P}_i \boldsymbol{D}_i) \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} (|y_1|^2 + \cdots + |y_n|^2) \mathrm{d}x\mathrm{d}y \leqslant \\
&-2\lambda_{\max}(\boldsymbol{P}_i \boldsymbol{D}_i) \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} \boldsymbol{y}^{\mathrm{T}} \boldsymbol{I} \boldsymbol{y} \mathrm{d}x\mathrm{d}y \leqslant \int_0^{\sqrt{2}} \int_0^{\sqrt{2}} \boldsymbol{y}^{\mathrm{T}} (-2\boldsymbol{P}_i \boldsymbol{D}_i) \boldsymbol{y} \mathrm{d}x\mathrm{d}y
\end{aligned}$$

Then we have

$$\langle \Omega_i x(t), x(t) \rangle \leqslant \langle (-2\boldsymbol{P}_i \boldsymbol{D}_i + \boldsymbol{P}_i \boldsymbol{B}_i \boldsymbol{K}_i + \boldsymbol{K}_i^{\mathrm{T}} \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P}_i + \beta \boldsymbol{P}_i) x(t), x(t) \rangle < \boldsymbol{0}$$

provided that the following inequalities

$$-2\boldsymbol{P}_i \boldsymbol{D}_i + \boldsymbol{P}_i \boldsymbol{B}_i \boldsymbol{K}_i + \boldsymbol{K}_i^{\mathrm{T}} \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P}_i + \beta \boldsymbol{P}_i < \boldsymbol{0} \tag{22}$$

are satisfied.

By the similar argument used in Theorem 1, it can be easily seen that (22) is equivalent to

$$-2\boldsymbol{D}_{\mathrm{i}} \boldsymbol{X}_{\mathrm{i}} + \boldsymbol{B}_{\mathrm{i}} \boldsymbol{Y}_{\mathrm{i}} + \boldsymbol{Y}_{\mathrm{i}}^{\mathrm{T}} \boldsymbol{B}_{\mathrm{i}}^{\mathrm{T}} + \beta \boldsymbol{X}_{\mathrm{i}} < \boldsymbol{0} \qquad \forall i \in \Theta \tag{23}$$

So, the following result is obtained.

Theorem 2 For a given constant $\beta > 0$, if there exist diagonal constant matrices $\boldsymbol{X}_i > 0$ and matrices $\boldsymbol{Y}_i$, such that the LMIs (23) are feasible, where $\boldsymbol{X}_i = \boldsymbol{P}_i^{-1}, \boldsymbol{Y}_i = \boldsymbol{K}_i \boldsymbol{P}_i^{-1}$. Then the state feedback given by (7) with $\boldsymbol{K}_i = \boldsymbol{Y}_i \boldsymbol{P}_i$ exponentially stabilizes systems (14) to (16) for arbitrary switching signal $\sigma(t)$ with the dwell time satisfying $\tau_{\mathrm{d}} > \tau_{\mathrm{d}}^* = \frac{\ln \mu}{\beta}, \mu = \max_{i \in \Theta} \left\{\frac{\lambda_{\max}(\boldsymbol{P}_i)}{\lambda_{\min}(\boldsymbol{P}_i)}\right\}$.

Remark 2 The idea of the LOIs is first applied to the study of distributed parameter systems in Refs. [25 – 26]. As it is shown, these LOIs are subsequently reduced to standard LMIs, which provide a new insight into the control theory of distributed parameter systems. Inspired by the above works, we utilize LOIs to the DPSS for the first time, and generalize the stability result of ODE switched systems[4] to the DPSS.

4 Examples

In this section we consider two examples to illustrate the proposed results.

Example 1 Utilize Theorem 2 for the switched heat propagation Eq. (18) with

$$\boldsymbol{D}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \ \boldsymbol{D}_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\boldsymbol{B}_1=\begin{bmatrix}0.1 & 0.2\\ 0.1 & 0\end{bmatrix},\ \boldsymbol{B}_2=\begin{bmatrix}0.1 & -0.1\\ 0.3 & 0.2\end{bmatrix}$$

Let $\beta=0.5$, by resolving LMIs (23), we obtain the state feedback matrices

$$\boldsymbol{K}_1=\begin{bmatrix}-0.2234 & 3.5533\\ 0.3389 & -1.555\end{bmatrix},\boldsymbol{K}_2=\begin{bmatrix}-0.4655 & 4.4633\\ -0.7438 & 3.1579\end{bmatrix}$$

Then $\tau_{\mathrm{d}}^*=\dfrac{\ln\mu}{2\beta}=\dfrac{\ln 2.2095}{2\times0.5}=0.7928$. We can choose the dwell time $\tau_{\mathrm{d}}=1.8>\tau_{\mathrm{d}}^*$. Thus the state decay for DPSS (18) is given by

$$\|\boldsymbol{y}(t)\|\leqslant 2.2095\mathrm{e}^{-0.0596(t-t_0)}\|\boldsymbol{y}_0\|$$

Example 2 Consider the switched heat propagation Eq. (18) with the following parameters:

$$\boldsymbol{D}_1=\begin{bmatrix}1&0&0\\0&2&0\\0&0&1\end{bmatrix},\ \boldsymbol{D}_2=\begin{bmatrix}2&0&0\\0&2&0\\0&0&1\end{bmatrix}$$
$$\boldsymbol{B}_1=\begin{bmatrix}1&2&1\\2&2&3\\1&3&2\end{bmatrix},\ \boldsymbol{B}_2=\begin{bmatrix}1&1&2\\1&0&3\\2&3&1\end{bmatrix}$$

Let $\beta=0.7$, by resolving LMIs(23), we obtain the state feedback matrices:

$$\boldsymbol{K}_1=\begin{bmatrix}-0.1935 & 0.1064 & 0.1000\\ 0.0603 & -0.0466 & -0.0023\\ 0.0383 & 0.1956 & -0.1814\end{bmatrix}$$
$$\boldsymbol{K}_2=\begin{bmatrix}-0.0844 & 0.0688 & -0.0467\\ 0.2104 & -0.0815 & -0.1152\\ -0.1650 & 0.1725 & -0.0070\end{bmatrix}$$

and $\mu=1.7432$. Then $\tau_{\mathrm{d}}^*=\dfrac{\ln\mu}{2\beta}=\dfrac{\ln 1.7432}{2\times0.7}=0.3969$. We can choose the dwell time $\tau_{\mathrm{d}}=2>\tau_{\mathrm{d}}^*$. Thus the state decay for DPSS (18) is given by

$$\|\boldsymbol{y}(t)\|\leqslant 1.7432\mathrm{e}^{-0.4222(t-t_0)}\|\boldsymbol{y}_0\|$$

5 Conclusion

In this paper, based on the semigroup and operator theory, some sufficient conditions of exponential stabilization for a class of linear DPSS are derived in a LOIs framework. We transform the LOIs into the LMIs, which has the advantage of being numerically well tractable by using the Matlab software. The control synthesis is investigated by means of the multiple Lyapunov approach. Finally, two examples are given to illustrate the effectiveness of the proposed results.

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基于驻留时间受限的分布参数切换系统的指数稳定性分析

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摘要:将有限维切换系统指数可稳性结果推广至无穷维 Hilbert 空间中的分布参数切换系统. 以半群理论为基础,通过利用多 Lyapunov 函数方法,推导了指数可镇定的充分条件. 这些条件以线性算子不等式的形式给出,其决定变量是 Hilbert 空间中的算子;同时系统的可稳性依赖于驻留时间受限的切换规则. 在应用到带 Dirichlet 边界条件的二维热传导切换系统时,这些线性算子不等式被转化成标准的线性矩阵不等式. 最后,通过 2 个例子说明给出结果的有效性.

关键词:分布参数切换系统;指数镇定;多李雅普若夫函数;线性算子不等式;驻留时间

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