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Optimal quasi-periodic maintenance policies for two-unit series system

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Abstract: To investigate the effects of various random factors on the preventive maintenance (PM) decision-making of one type of two-unit series system, an optimal quasi-periodic PM policy is introduced. Assume that PM is perfect for unit 1 and only mechanical service for unit 2 in the model. PM activity is randomly performed according to a dynamic PM plan distributed in each implementation period. A replacement is determined based on the competing results of unplanned and planned replacements. The unplanned replacement is trigged by a catastrophic failure of unit 2, and the planned replacement is executed when the PM number reaches the threshold N. Through modeling and analysis, a solution algorithm for an optimal implementation period and the PM number is given, and optimal process and parametric sensitivity are provided by a numerical example. Results show that the implementation period should be decreased as soon as possible under the condition of meeting the needs of practice, which can increase mean operating time and decrease the long-run cost rate.

Key words: maintenance policy optimization; quasi-periodic preventive maintenance; two-unit series system

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As one of some major maintenance policies, preventive maintenance (PM) policy is widely used to reduce downtime and breakdown risk in areas such as industry, military, health and the environment. As such, the effective implementation of PM policies has been extensively studied[1]. Most research on PM models is concerned with the optimal maintenance policies for a machine or a mono-unit system[2]. In contrast, the PM models of two-unit systems have received less attention, and many of these existing works focused on redundant systems[3-4] or condition-based maintenance[5]. However, one type of two-unit series system is broadly existent in practice, in which the lifetime of unit 1 (U_1) is always stochastically much smaller than that of unit 2 (U_2), and PM for U_1 is nearly perfect while for U_2 is only mechanical service and the system will be replaced if a catastrophic failure of U_2 occurs, for example, a cylinder and its cylinder liner, a nozzle matching parts of a diesel engine, plunger matching parts, and so on. Meanwhile, PM plans of the system are often modified by some random factors, such as production task, the transport cycle of a locomotive, which make the PM intervals become a limited stochastic value rather than a fixed value.

Various approaches have been developed to optimize the PM policy[6]. Most of them ignore the influence of random factors on PM policy, and thus regard that the maintenance period is a fixed value. Actually, it should be a limited random value in practice because of random factors that make the maintenance actions unable to be performed as soon as a planned PM period is reached. Two typical examples of these cases are PM on some Chinese diesel locomotives being 2 300 to 2 600 km[7] and on some Japanese planes for 12 to 18 months[8]. Only a few of them discussed the maintenance optimization for a system with a stochastic PM interval considering a job cycle or a failure occurrence[9-11]. Sheu et al.[9] examined a generalized age replacement policy. In Ref. [9], if a unit fails at age $y < t$, it is subject to a perfect repair with $p(y)$ or undergoes a minimal repair with probability $q(y) = 1 - p(y)$; otherwise, the unit is replaced when the first failure after t occurs or the total operating time reaches age T ($0 < t < T$), whichever occurs first. Castro et al.[10] analyzed a maintenance policy for a repairable system with delay repairs. In their research, if the system fails in $[0, T^*]$, then it is repaired, whereas if it fails in $(T^*, T]$, the repair is not performed; the system is replaced when the non-repairable failures reach N over $(T^*, T]$. Chen[11] considered an age replacement policy for a system, which can continuously service for multiple jobs with random working times, and can undergo minimal repairs upon failures. The planned replacement is postponed at the first completion of the working time or when a job incurs some damage to the system over a planned time T. In this model, the PM actions are delayed and determined by the job cycles.

In the above mentioned research, the maintenance plan

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is modified by failures or the working cycle of a job, and yet it should be jointly affected by some random external and internal factors in a finite time period in practical. These random factors may result in a modification of the preconcerted PM period in $[T - w_1,\ T + w_2]$, where T is a planned PM period and w_1 and w_2 are constant given by engineers. For ease of research, herein set $T - w_1$ as T and $w_1 + w_2 = W$. Consequently, the dynamic PM schedule randomly distributes within $[T,\ T + W]$. Under this condition, the PM policy is different from the general periodic PM policy, and thus it is termed as a quasi-periodic PM policy in this paper. In our recent study[12], a quasi-periodic replacement policy for a mono-unit system considering the modifications of replacement plans is discussed. By comparison, the quasi-periodic PM for a two-unit series system is widely used in most real cases, although its optimization is more complex. Based on these considerations, an optimal quasi-periodic PM policy for the two-unit series system is introduced in this paper.

1 Model Description

For a quasi-periodic PM policy, a scheduled PM plan and a dynamic PM plan are presented. The scheduled PM plan is a long-term plan and is predetermined without considering the provisional effects of some random factors. According to the schedule plan, the system is preventively maintained at kT ($k = 1, 2, \dots, N - 1$) and replaced at NT. Nevertheless, the dynamic PM plan is a short-term plan considering the effect of some provisional external factors, in which the first $(N - 1)$ PM interval is divided into a planned PM period and an implementation period. The length of the planned PM period is a fixed value and made by the scheduled PM plan, while the length of the implementation period is given by engineers' experience considering some practical needs.

The quasi-periodic PM policy for a series system comprised of two units (denoted as U_1 and U_2) is considered. The lifetimes of U_1 and U_2 are random variables (marked as ζ_1 and ζ_2) with distribution function $F^{(i)}(t)$ ($i = 1, 2$), respectively, and it is assumed that ζ_1 is stochastically smaller than ζ_2. U_1 is assumed to be repairable and it undergoes minimal repair at failures. Thus, U_1 failures occur according to a non-homogeneous Poisson process with intensity rate $h^{(1)}(t)$.

Assume that failures of U_1 are minor failures which can be removed by minimal repair at a cost of $C_{\mathrm{m}}^{(1)}$. A failure of U_2 that occurs in the i-th PM interval is either a minor one with probability p_i or a catastrophic one with probability $q_i = 1 - p_i$, where $0 \leqslant q_i \leqslant 1$ and q_i is non-decreasing in i. Herein, minor and catastrophic failures of U_2 occurring in $(0, t]$ constitute a non-homogeneous Poisson process with intensity rate $p_i h^{(2)}(t)$ and $q_i h^{(2)}(t)$, respectively. Minor failures of U_2 can be rectified by minimal repairs at a cost of $C_{\mathrm{m}}^{(2)}$, and catastrophic failures can be removed by an unplanned replacement. Let $h^{(i)}(t)$ denote the hazard rate function and $H^{(i)}(t) = \int_0^t h^{(i)(u)\mathrm{d}u}$ be the mean value function of $U_i (i = 1, 2)$. Thus, the mean minimal repair cost of $U_i(i = 1, 2)$ in $[0, t)$ can be written as

$$A^{(i)}(t) = C_{\mathrm{m}}^{(i)} \int_0^t h^{(i)}(u)\,\mathrm{d}u = C_{\mathrm{m}}^{(i)} H^{(i)}(t) \qquad (1)$$

PM to U_1 is perfect and to U_2 is only mechanical service with a cost of C_{p} (including the PM cost of U_1 and the mechanical service cost of U_2), and it often involves lubricating, adjusting load carried to the mating parts, cleaning the jam and rust, etc. Ref. [13] presented that maintenance service can only improve the extrinsic state of the system, and the hazard rate of a unit after the $(i - 1)$-th mechanical service is $h_i(t) = \sum_{j=0}^{i-1} h(x_j) + h(t)$. In this model, x_i is a random variable over $[T, T + W]$, and thus it is difficult to compute expected maintenance cost since $h_i(t)$ is a random function. Herein we set $a_i(T) = \sum_{j=0}^{i-1} E[h(x_j)] = (i - 1)E[h(x_j)]$, which is the initial hazard rate value just at the beginning of the $(i - 1)$-th PM. Thus, the reliability of a unit after the $(i - 1)$-th mechanical service in $[0, t)$ becomes

$$R_i^{(2)}(t) = \exp\Big(- \int_0^t h(u)\,\mathrm{d}u - a_i(T)t \Big)$$

A replacement of the system contains two cases: planned and unplanned activities. The former occurs when there are no catastrophic failures in each PM interval and when the PM number reaches the threshold N determined by the PM plan. The replacement cycle is shown in Fig. 1, where the first $(N - 1)$-th PM actions are executed at the time $t_i (i = 1, 2, \dots, N - 1)$ following dynamic PM plans, and a planned replacement is performed at $t_N (t_N - t_{N-1} = T,$ or $Y_{s,N} > T)$.

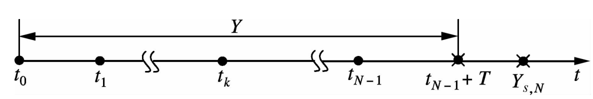

Fig. 1 Planned replacement

The later is trigged by a catastrophic failure of U_2. The replacement cycle is shown in Fig. 2, where the first $(i - 1)$-th PM actions are executed at the time t_{i-1} ($i = 1, 2, \dots, N$) according to the dynamic PM plans $(0 < Y_{\mathrm{sc}} < W)$. In Fig. 2(a), an unplanned replacement is performed at t_i $(0 < i < N)$, and Fig. 2(b) shows the case when $i = N$. A replacement of the system brings a cost of C_{r} (including the replacement cost of U_1 and U_2, $C_{\mathrm{r}} > C_{\mathrm{m}}$).

Although there are some random factors affecting the PM activities, they may be forecasted in a relative short implementation period in practice. Thus, the system can be completely arranged to be preventively maintained during each implementation period . According to this fact ,

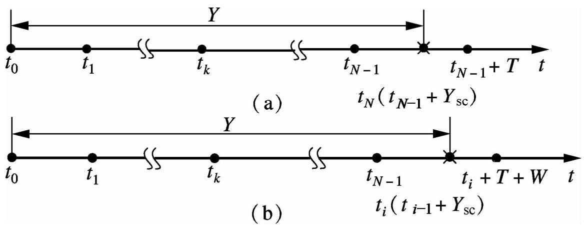

Fig. 2 Unplanned replacement

we assume that the occurring time of a dynamic PM plan, Y_{sc}, is a random variable and follows the uniform distribution with a probability density function $1/W$ in each implementation period.

According to the above maintenance description, three model assumptions are given as below:

1) Failures of U_1 and U_2 are independent. All failures can be instantly detected and repaired.

2) The hazard rate function of U_1 and U_2 are continuous, positive, concave and non-decreasing in t. After a replacement, the system can be restored to "as good as new" state.

3) All minimal repair, PM, and replacement time are negligible.

2 Maintenance Optimization

A replacement cycle is defined as a time interval between an installation of the system and the first replacement or between two consecutive replacements. Let Y and C denote the length of a renewal cycle and the operational cost over a replacement cycle, respectively. Thus, $\{Y, C\}$ constitutes a renewal reward process. If $C(t)$ denotes the operational cost over the time interval $[0, t)$, then we have

$$g(T, N) = \lim_{t\to\infty}\frac{C(t)}{t} = \frac{E[C]}{E[Y]} \tag{2}$$

where $E[C]$ and $E[Y]$ denote the s-expected maintenance cost and the s-expected renewal cycle, respectively. An unplanned replacement may occur in each PM period or ahead of the dynamic PMs in each implementation period if $y_{s,i} < T + y_{\mathrm{sc}}$, where $y_{s,i}$ denotes an occurring time of a catastrophic failure and y_{sc} denotes the scheduled time of the dynamic PM plan in the i-th PM interval. Then, the length of the renewal cycle Y can be described as

$$\begin{aligned} Y = &\sum_{k=1}^{N-1}\Big\{ I_{\{T+y_{\mathrm{sc},1}<y_{s,1}^{(2)},\ \dots,\ T+y_{\mathrm{sc},k-1}<y_{s,k-1}^{(2)},\ y_{s,k}^{(2)}<T+y_{\mathrm{sc},k}\}}\Big(\sum_{j=0}^{k-1}x_j + y_{s,k}^{(2)}\Big) + \\ &I_{\{T+y_{\mathrm{sc},1}<y_{s,1}^{(2)},\ \dots,\ T+y_{\mathrm{sc},N-1}<y_{s,N-1}^{(2)},\ y_{s,N}^{(2)}<T\}}\Big(\sum_{j=0}^{N-1}x_j + y_{s,N}^{(2)}\Big) + \\ &I_{\{T+y_{\mathrm{sc},1}<y_{s,1}^{(2)},\ \dots,\ T+y_{\mathrm{sc},N-1}<y_{s,N-1}^{(2)},\ y_{s,N}^{(2)}>T\}}\Big(\sum_{j=0}^{N-1}x_j + T\Big)\Big\} \end{aligned} \tag{3}$$

where x_j is the length of the j-th PM interval; $I_B(Z)$ is an indicator function of set B, i. e.,

$$I_B(Z) = \begin{cases} 1 & Z \in B \\ 0 & \text{otherwise} \end{cases}$$

Let $A_i^{(1)}(t)$ and $A_i^{(2)}(t)$ denote the mean minimal repair cost of U_1 and U_2 over $[0, t)$ in the i-th PM interval that can be stated using Eq. (1), and $K_{\mathrm{m}} = C_{\mathrm{m}}^{(1)}/C_{\mathrm{m}}^{(2)}$, $K_{\mathrm{r}} = C_{\mathrm{r}}/C_{\mathrm{m}}^{(2)}$, $K_{\mathrm{p}} = C_{\mathrm{p}}/C_{\mathrm{m}}^{(2)}$. Then, the maintenance cost R of a replacement cycle can be described as

$$\begin{aligned} C = C_{\mathrm{m}}^{(2)}&\sum_{k=1}^{N-1}\Big\{ I_{\{T+y_{\mathrm{sc},1}<y_{s,1}^{(2)},\ \dots,\ T+y_{\mathrm{sc},k-1}<y_{s,k-1}^{(2)},\ y_{s,k}^{(2)}<T+y_{\mathrm{sc},k}\}}\Big(\sum_{j=0}^{k-1}(K_{\mathrm{m}}H^{(1)}(x_j) + p_j(a_{j-1}(T)x_j + H^{(2)}(x_j)) + K_{\mathrm{p}}) + \\ &K_{\mathrm{m}}H^{(1)}(y_{s,k}^{(2)}) + p_k(a_{k-1}(T)y_{s,k}^{(2)} + H^{(2)}(y_{s,k}^{(2)})) + K_{\mathrm{r}}\Big) + I_{\{T+y_{\mathrm{sc},1}<y_{s,1}^{(2)},\ \dots,\ T+y_{\mathrm{sc},N-1}<y_{s,N-1}^{(2)},\ y_{s,N}^{(2)}<T\}}\Big(\sum_{j=0}^{k-1}(K_{\mathrm{m}}H^{(1)}(x_j) + \\ &p_j(a_j(T)x_j + H^{(2)}(x_j)) + K_{\mathrm{p}}) + K_{\mathrm{m}}H^{(1)}(y_{s,k}^{(2)}) + p_N(a_{N-1}(T)y_{s,k}^{(2)} + H^{(2)}(y_{s,N}^{(2)})) + K_{\mathrm{r}}\Big) + \\ &I_{\{T+y_{\mathrm{sc},1}<y_{s,1}^{(2)},\ \dots,\ T+y_{\mathrm{sc},N-1}<y_{s,N-1}^{(2)},\ y_{s,N}^{(2)}>T\}}\Big(\sum_{j=0}^{k-1}(K_{\mathrm{m}}H^{(1)}(x_j) + p_j(a_j(T)x_j + H^{(2)}(x_j)) + K_{\mathrm{p}}) + K_{\mathrm{m}}H^{(1)}(T) + \\ &p_N(a_N(T)T + H^{(2)}(T)) + K_{\mathrm{r}}\Big)\Big\} \end{aligned} \tag{4}$$

Let $f^{(i)}(t)$ denote the pdf of $U_i(i = 1, 2)$ failures. Furthermore, according to Eqs. (3) and (4), the expected renewal cycle and the maintenance cost can be described as

$$\begin{aligned} E[Y] = &\sum_{k=1}^{N-1}W^{-k}\int_0^W\int_0^{T+u}\int_T^{+\infty}\int_T^{y_{k-1}}\cdots\int_T^{+\infty}\int_T^{y_k}\Big(\sum_{j=0}^{k-1}x_j + y_{s,k}^{(2)}\Big)\prod_{l=1}^{k}f_l^{(2)}(y_l)\,\mathrm{d}x_1\mathrm{d}y_1\ldots\mathrm{d}x_{k-1}\mathrm{d}y_{k-1}\mathrm{d}y_k\mathrm{d}u + \\ &W^{1-N}\Big\{\int_0^T\int_T^{+\infty}\int_T^{y_{N-1}}\cdots\int_T^{+\infty}\int_T^{y_1}\Big(\sum_{j=1}^{N-1}x_j + y_{s,N}^{(2)}\Big)\prod_{l=1}^{N}f_l^{(2)}(y_l)\,\mathrm{d}x_1\mathrm{d}y_1\ldots\mathrm{d}x_{N-1}\mathrm{d}y_{N-1}\mathrm{d}y_N\mathrm{d}u + \\ &\int_T^{+\infty}\int_T^{+\infty}\int_T^{y_{N-1}}\cdots\int_T^{+\infty}\int_T^{y_1}\Big(\sum_{j=1}^{N-1}x_j + T\Big)\prod_{l=1}^{N}f_l^{(2)}(y_l)\,\mathrm{d}x_1\mathrm{d}y_1\ldots\mathrm{d}x_{N-1}\mathrm{d}y_{N-1}\mathrm{d}y_N\Big\} \end{aligned} \tag{5}$$

$$\begin{aligned} E[C] = C_{\mathrm{m}}^{(2)}\Big\{&\sum_{k=1}^{N-1}W^{-k}\int_0^W\int_0^{T+u}\int_T^{+\infty}\int_T^{y_{k-1}}\cdots\int_T^{+\infty}\int_T^{y_1}\Big(\sum_{j=0}^{k-1}(KH^{(1)}(x_j) + p_j(a_{j-1}(T)x_j + H^{(2)}(x_j)) + C_p) + \\ &KH^{(1)}(y_{s,k}^{(2)}) + p_k(a_{k-1}(T)y_{s,k}^{(2)} + H^{(2)}(y_{s,k}^{(2)})) + C_r\Big)\prod_{l=1}^{k}f_l^{(2)}(y_l)\,\mathrm{d}x_1\mathrm{d}y_1\ldots\mathrm{d}x_{k-1}\mathrm{d}x_{k-1}\mathrm{d}y_k\mathrm{d}u + \end{aligned}$$

$$W^{1-N}\int_0^T\int_T^{+\infty}\int_T^{y_{N-1}}\cdots\int_T^{+\infty}\int_T^{y_k}\sum_{k=1}^{N-1}(KH^{(1)}(x_j)+p_j(a_{j-1}x_j+H^{(2)}(x_j))+C_{\mathrm{p}})+KH^{(1)}(y_{s,N}^{(2)}+p_N(a_{N-1}(T)y_{s,N}^{(2)}+$$
$$H^{(2)}(y_{s,N}^{(2)}))+C_{\mathrm{r}})\prod_{l=1}^{N}f_l^{(2)}(y_l)\,\mathrm{d}x_1\mathrm{d}y_1\ldots\mathrm{d}x_{N-1}\mathrm{d}x_{N-1}\mathrm{d}y_N+W^{1-N}\int_T^{+\infty}\int_T^{+\infty}\int_T^{y_{N-1}}\cdots\int_T^{+\infty}\int_T^{y_1}\sum_{j=1}^{N-1}(KH^{(1)}(x_j)+$$
$$p_j(a_{j-1}(T)x_j+H^{(2)}(x_j))+C_{\mathrm{p}})+KH^{(1)}(T)+p_N(a_{N-1}(T)T+H^{(2)}(T))+$$
$$C_{\mathrm{r}})\prod_{l=1}^{k}f_l^{(2)}(y_l)\,\mathrm{d}x_1\mathrm{d}y_1\ldots\mathrm{d}x_{N-1}\mathrm{d}x_{N-1}\mathrm{d}y_{N-1}\mathrm{d}y_N\Big\} \tag{6}$$

Submitting Eqs. (5) and (6) into Eq. (2), we can obtain the long-run cost rate function $g(T, N)$. In general, in order to obtain a unique solution T^* and N^* by minimizing Eq. (2) with the analytical method, we need to compute the second derivative and the second-order difference of $M(T, N)$, $P(T, N)$ and $S(T, N)$ with regard to T and N in Eq. (2); however it is difficult to compute them due to their complexity. As a practical application, it is unnecessary to determine a much more precise solution, while a feasible solution is acceptable by a simple method with an allowable deviation. Based on these considerations, the Hooke-Jeeves method is used in this model, which can be applied almost immediately to many nonlinear optimization problems[14]. The detailed steps of the Hooke-Jeeves method can be seen in Ref. [14].

3 Special Cases

According to the proposed model, there are three special cases with different W and p_i. The brief explanation and the long run expected per unit time are given for each case.

1) When $W=0$, $p_i\neq 1$, and $N>1$

In this case, the system is the same as presented in section 1, and the difference between the PM policy and the proposed policy is that the PM period is a fixed value T. The mean cost rate equation is shown as

$$g(T,N)=\frac{E[C]}{E[Y]} \tag{7}$$

where

$$E[Y]=\sum_{i=1}^{N-1}\prod_{j=1}^{i-1}\bar{F}_{q,j}^{(2)}(T)\left(\int_0^T yf_{q,i}^{(2)}(y)\,\mathrm{d}y+(i-1)T\right)+\prod_{j=1}^{N-1}\bar{F}_{q,j}^{(2)}(T)\left(\int_0^T\bar{F}_{q,N}^{(2)}(y)\,\mathrm{d}y+(N-1)T\right)$$

$$E[C]=C_{\mathrm{m}}^{(2)}\Big\{\sum_{i=1}^{N-1}\prod_{j=1}^{i-1}\bar{F}_{q,j}^{(2)}(T)\Big(\int_0^T f_{q,i}^{(2)}(y)(K_{\mathrm{m}}H^{(1)}(y)+p_iH^{(2)}(y)+p_ia_{i-1}y)\,\mathrm{d}y+\sum_{j=1}^{i-1}(K_{\mathrm{m}}H^{(1)}(T)+p_jH^{(2)}(T)+$$
$$p_ja_{j-1}(T)T+K_{\mathrm{p}})\Big)+\prod_{j=1}^{N-1}\bar{F}_{q,j}^{(2)}(T)\Big(\int_0^T\bar{F}_{q,N}^{(2)}(y)(K_{\mathrm{m}}h^{(1)}(y)+p_Nh^{(2)}(y)+p_Na_{N-1}(T))\,\mathrm{d}y+$$
$$\sum_{j=1}^{N-1}(K_{\mathrm{m}}H^{(1)}(T)+p_jH^{(2)}(T)+p_ja_{j-1}(T)T+K_{\mathrm{p}})\Big)\Big\}+C_{\mathrm{r}}$$

2) When $W=0$, $p_i=1$, and $N>1$

This model is a special case of the case 1 with $p_i=1$, in which failures of U_2 only include minor failures. Periodic PM for U_2 is mechanical service and for U_1 is perfect. A replacement is performed when the PM number reaches the threshold N. The mean cost rate equation is

$$g(T,N)=\frac{E[C]}{E[Y]}=\frac{C_{\mathrm{m}}^{(2)}\{\sum_{j=1}^{N}(K_{\mathrm{m}}H^{(1)}(T)+p_jH^{(2)}(T)+p_ja_{j-1}(T)T)\}+(N-1)C_{\mathrm{p}}+C_{\mathrm{r}}}{NT} \tag{8}$$

3) When $W\neq 0$, $p_i=1$, and $N>1$

In this case, a quasi-periodic PM policy is executed for one type of the two-unit system. Failures of the system are minor failures. Maintenance service for U_2 and a perfect PM for U_1 are stochastically performed in the implementation period. A replacement is performed when the PM number reaches the threshold N. The mean cost rate equation is described as

$$g(T,N)=\frac{E[C]}{E[Y]}=$$
$$\frac{C_{\mathrm{m}}^{(2)}\Big\{\sum_{j=1}^{N-1}\int_T^{T+W}(K_{\mathrm{m}}H^{(1)}(y)+H^{(2)}(y)+a_{j-1}(T)y)\,\mathrm{d}y+K_{\mathrm{m}}H^{(1)}(T)+a_{N-1}(T)T\Big\}+(N-1)C_{\mathrm{p}}+C_{\mathrm{r}}}{NT+0.5(N-1)W} \tag{9}$$

4 Numerical Example

In this section, the procedures and features of the model are illustrated using the following numerical examples.

Given that the lifetime distributions of U_1 and U_2 follow the Weibull distribution with cumulative distribution functions $R^{(1)}(t)=\mathrm{e}^{-(2t)^2}$ and $R^{(2)}(t)=\mathrm{e}^{-(0.5t)^2}$, respectively. Let $W=0.3$, $\alpha=0.1$, $\varepsilon=0.05$, $\beta=0.8$, $x_0=1$, $C_{\mathrm{m}}^{(2)}=2$, $K_{\mathrm{m}}=0.25$, $K_{\mathrm{p}}=0.6$, $K_{\mathrm{r}}=3$, $r=0.7$, and $\gamma=0.75$, $i=1,2,\ldots,N$.

Eq. (2) is computed in Matlab 2010b using the Hooke-Jeeves algorithms described above. We obtain the optimal

$T^* = 1.48$, $N^* = 6$, $C_{\min} = 5.664$, and $E[Y] = 5.648$. That is, the PM activities are stochastically performed in the time interval [1.48, 1.78]. The mean cost rate $C(N, T)$ is shown in Fig. 3.

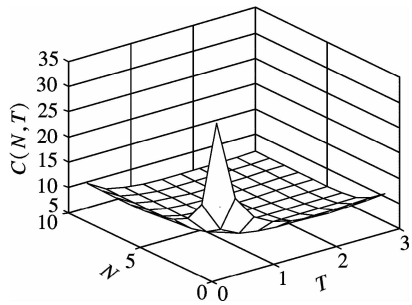

Fig. 3 Three-dimensional plot of (C, T, N)

In order to present the sensitivity of the maintenance and failure parameters to the mean cost rate, some numerical examples are given. Tabs. 1 and 2 exhibit the change tendency of T^*, N^* and $C_{\min}$ with different parameters K_p, K_r and W. As $W = 0.3$ and 0.5, the optimal results are shown in Tab. 1, respectively.

Tab. 1 Optimal results with $K_p = 0.6$, $r = 0.7$

W	K_r	T^*	N^*	$C_{\min}$	W	K_r	T^*	N^*	$C_{\min}$
	3	1.48	6	5.664		3	1.40	6	5.671
0.3	4	1.63	7	6.007	0.5	4	1.56	7	6.016
	5	1.78	7	6.334		5	1.70	7	6.341

From Tab. 1 it can be found that T^*, N^* and $C_{\min}$ obviously increase with the increase of K_r under a given K_p, while T^* decreases and $C_{\min}$ keeps a slow growth trend with the increase of W under a given K_r and K_p. The optimal N^* has no obvious change with different W because it is mainly determined by C_R.

Tab. 2 shows that T^*, N^* and $C_{\min}$ also increase with the increase of K_p under a given K_r, while T^* and N^* decrease and $C_{\min}$ increases slowly with the increase of W under a given K_r and K_p.

Tab. 2 Optimal results with $K_r = 4$, $r = 0.7$

W	K_p	T^*	N^*	$C_{\min}$	W	K_p	T^*	N^*	$C_{\min}$
	0.4	1.50	6	5.877		0.4	1.40	6	5.886
0.3	0.5	1.59	7	5.944	0.5	0.5	1.48	6	5.952
	0.6	1.63	7	6.007		0.6	1.56	7	6.016

When $W = 0$, the proposed model is the general model described as the special case 1, of which the mean cost rate is given as Eq. (7). Assuming that $C_m^{(2)} = 2$, $K_m = 0.25$, $K_p = 0.6$, $K_r = 5$, $a_i = 0.75(i - 1)$, and $W = 0.8$, the failure parameters are the same as those of the numerical example. It can be found that the optimal $N^* = 7$ in the two models, and $C_{\min}$ in the proposed model is 6.34 and in the general model is 6.22. Their difference is about 2%. According to the optimal results of Tab. 1, with the increase of W, the minimal mean cost rate also increases, and thus the proposed policy can be replaced by the special case 1 with a lesser W, otherwise the proposed model should be given great attention when W is relatively great.

5 Conclusion

In this paper, a quasi-periodic PM policy for one type of two-unit series system considering the influence of some random factors is proposed. Based on our analysis and the examples above, the following conclusions are obtained:

1) The optimal T^* and N^* can be obtained although the model is complex, and the method used in this paper can also be extended to some special cases proposed above.

2) The increase in the length of the implementation period can cause an increase in the mean cost rate and a decrease in the mean operational time, and thus the choice of W should be decreased as soon as possible under the condition of meeting the requirements of practice.

3) The policy proposed in this paper is more convenient for production. It can be replaced by the general PM policy (the special case 3) for a lesser W, while the difference between the proposed policy and the general PM policy should not be ignored if W is relatively greater.

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两部件串联系统拟周期预防性维修策略优化

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摘要:为研究多种随机因素对一类两部件串联系统预防性维修决策的影响,提出一种拟周期预防性维修策略优化模型.假设预防性维修对部件 1 为完整修,对部件 2 为保养,预防性活动在实施区间内根据动态计划随机进行;意外更换和预防性更换的竞争结果决定系统的更换,当部件 2 发生严重故障时引发系统的意外更换,当预防性维修次数达到设定的阈值 N 时实施预防性更换.通过建模和优化分析,给出了最优预防性维修实施区间和最佳维修次数的求解算法,并通过数值案例对模型的优化过程和参数的敏感性做了说明.分析结果表明,在满足实际生产需求的前提下,尽可能地缩短实施区间能提高系统的平均运行时间和降低系统的长期运行成本率.

关键词:维修策略优化;拟周期预防性维修;两部件串联系统

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