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Multiple solutions and fermion mass effect in QED_3

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Abstract: Due to the negligible non-perturbation effect in the low-energy region, quantum chromodynamics (QCD) is limited to be applied to hadron problems in particle physics. However, QED has mature non-perturbation models which can be applied to Fermi acting-energy between quark and gluon. This paper applies quantum electrodynamics in 2 + 1 dimensions (QED_3) to the Fermi condensation problems. First, the Dyson-Schwinger equation which the fermions satisfy is constructed, and then the Fermi energy gap is solved. Theoretical calculations show that within the chirality limit, there exist three solutions for the energy gap; beyond the chirality limit, there are two solutions; all these solutions correspond to different fermion condensates. It can be concluded that the fermion condensates within the chirality limit can be used to analyze the existence of antiferromagnetic, pseudogap, and superconducting phases, and two fermion condensates are discovered beyond the chirality limit.

Key words: Dyson-Schwinger equation; chiral limit; beyond chiral limit; fermion; condensate; multiple solutions; quantum electrodynamics in 2 + 1 dimensions (QED_3)

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Nowadays, it is widely accepted that quantum chromodynamics (QCD) in 3 + 1 dimensions is a basic theory for hadron physics. Especially in high energy regions, the theoretical results of QCD coincide with the corresponding hadron experiment. However, QCD is very difficult to deal with in the low-energy region because here the nonperturbative effects cannot be ignored. In addition, since the full propagators including high-order corrections contain all the physical information about QCD, the theoretical results depend on the form of the propagators. Up to now, the investigation of the propagators in low energy is still a challenging domain in field theory. Using the Lorenz analysis, we write the inverse quark propagator as

$$S_q^{-1}(m,\boldsymbol{p}) = i\boldsymbol{\gamma}\cdot\boldsymbol{p}A_q(m,p^2) + B_q(m,p^2)$$

where $A_q(m,p^2)$ is the vector part and $B_q(m,p^2)$ is the scaler part. As is well known, there are two solutions to the Dyson-Schwinger equation (DSE) for the quark propagator, i. e. the Numbu-Goldstone (NG) solution and the Wigner (WN) solution. In chiral limit, the NG solution corresponds to the chirally broken phase while the WN solution corresponds to the chirally symmetric phase. One accepts that the NG solution is the solution realized in the real world. So the investigation of the NG solution is an interesting problem.

In principle, we should solve the DSEs for the full quark and gluon propagators to study those problems, but it is very difficult to do even though we adopt the rainbow approximation. However, we can use a nonperturbative model in quantum electrodynamics in 2 + 1 dimensions (QED_3) to study them[1-3]. This model is considered with four-component spinors and is chirally symmetric in the absence of a bare fermion mass term, $m_0\bar{\psi}\psi$.

QED_3 as a field-theoretical model has been extensively studied in recent years. It has many features similar to quantum chromdynamics (QCD) in 3 + 1 dimensions. This is because QED_3 is known to have a phase where the initial chiral symmetry of the theory is spontaneously broken and it is also known that the fermions are confined in this phase. Moreover, QED_3 is superrenormalizable, so it is not plagued with the ultraviolet divergences which are present in QED_4. These are the basic reasons why QED_3 is regarded as an interesting toy model. It is far more simple in theoretical structure to study color confinement and dynamical chiral symmetry breaking (DCSB)[4-7] by QED_3 than by QCD. Therefore, we can use the DSEs for the fermion and photon propagators in QED_3 to investigate our problems.

1 Dyson-Schwinger Equation for Fermion Propagator

The Lagrangian of QED_3 in a general covariant gauge in the Euclidean space, ignoring the issues discussed in Ref. [8], can be written as

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$$L = \bar{\psi}(\partial + ie\boldsymbol{A} - m)\psi + \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \tag{1}$$

where the 4×1 spinor ψ is the fermion field. In chiral limit, $m = 0$; i. e., this Lagrangian is chiral symmetric. But the broken symmetry makes the fermion gain a nonzero mass due to the nonperturbative effect. Then the chiral phase transition occurs. The order parameter is trivially defined as

$$\langle\bar{\psi}\psi\rangle = \mathrm{Tr}[S(x \equiv 0)] = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{4B(p^2)}{A^2(p^2)p^2 + B^2(p^2)} \tag{2}$$

where $A(p^2)$, $B(p^2)$ are obtained in the following. The infrared value of $B(p^2)$ is also considered as the order parameter. At $m \neq 0$, the last definition should be changed[9].

Generally, the fermion propagator with nonzero mass can be divided into two parts, the massless part and the mass part. We suppose that the inverse fermion propagator is analytic about fermion mass.

$$S^{-1}(m, \boldsymbol{p}) = S^{-1}(\boldsymbol{p}) + \left.\frac{\partial S^{-1}(m, \boldsymbol{p})}{\partial m}\right|_{m=0} m + \ldots \tag{3}$$

At $m \to 0$, the high-order terms can be ignored. Based on the Lorentz analysis, the mass term in the last function is written as

$$\left.\frac{\partial S^{-1}(m, \boldsymbol{p})}{\partial m}\right|_{m=0} = i\boldsymbol{\gamma} \cdot \boldsymbol{p}C(p^2) + D(p^2) \tag{4}$$

We aim to obtain the involved four functions, A, B, C, D. The functions can be obtained by the application of the Dyson-schwinger (DS) equation for the fermion propagator. In chiral limit, the full inverse fermion propagator and the full photon propagator satisfy the following DS equation,

$$S^{-1}(\boldsymbol{p}) = S_0^{-1}(\boldsymbol{p}) + \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Gamma_\nu(\boldsymbol{p}, \boldsymbol{k})D_{\rho\nu}(\boldsymbol{p} - \boldsymbol{k}) \tag{5}$$

where $S_0^{-1}(\boldsymbol{p})$ is the bare inverse propagator for the massless fermion; $\Gamma_\nu(\boldsymbol{p}, \boldsymbol{k})$ is the full fermion-photon vertex. From $S^{-1}(\boldsymbol{p}) = i\boldsymbol{\gamma} \cdot \boldsymbol{p}A(p^2) + B(p^2)$ and Eq. (5), we obtain the equation satisfied by $A(p^2)$ and $B(p^2)$,

$$A(p^2) = 1 - \frac{1}{4p^2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}[i(\boldsymbol{\gamma} \cdot \boldsymbol{p})\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Gamma_\nu(\boldsymbol{p}, \boldsymbol{k})D_{\rho\nu}(\boldsymbol{p} - \boldsymbol{k})] \tag{6}$$

$$B(p^2) = \frac{1}{4}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}[\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Gamma_\nu(\boldsymbol{p}, \boldsymbol{k})D_{\rho\nu}(\boldsymbol{p} - \boldsymbol{k})] \tag{7}$$

where the notation Tr denotes trace over the Dirac indices. If the coupled functions have nontrivial solutions (the NG solution, $B(p^2) \neq 0$), then the original massless fermion will acquire nonzero mass and chiral symmetry is broken spontaneously.

In addition, the DS equation satisfied by the photon vacuum polarization tensor can be written as

$$\Pi_{\rho\nu}(q^2) = -\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}[S(\boldsymbol{k})\boldsymbol{\gamma}_\rho S(\boldsymbol{q} + \boldsymbol{k})\Gamma_\nu(\boldsymbol{p}, \boldsymbol{k})(\boldsymbol{q} + \boldsymbol{k}, \boldsymbol{k})] \tag{8}$$

Using the relationship between the vacuum polarization $\Pi(q^2)$ and $\Pi_{\rho\nu}(q^2)$,

$$\Pi_{\rho\nu}(q^2) = (q^2\boldsymbol{\delta}_{\rho\nu} - \boldsymbol{q}_\rho\boldsymbol{q}_\nu)\Pi(q^2) \tag{9}$$

we can obtain an equation for $\Pi(q^2)$ which has ultraviolet divergence. Fortunately, it is present only in the longitudinal part. One can remove this divergence by applying the projection operator

$$\boldsymbol{P}_{\rho\nu} = \boldsymbol{\delta}_{\rho\nu} - 3\frac{\boldsymbol{q}_\rho\boldsymbol{q}_\nu}{q^2} \tag{10}$$

and obtain a finite vacuum polarization. However, the full vertex is unknown. Although several works have attempted to solve it [10-16], none of them have turned up trumps. Following some previous works[17-18], we choose the first part in the BC vertex,

$$\Gamma_\nu(\boldsymbol{p}, \boldsymbol{k}) = \frac{1}{2}[A(p^2) + A(k^2)]\boldsymbol{\gamma}_\nu \tag{11}$$

One argues that this tenser destroys Ward-Takahashi identity(WTI), whereas self-consistent investigation (more details can be found in Ref. [8]) shows that numerical results of the DSE for the fermion propagator obtained by adopting this vertex are almost equivalent to those by the application of BC and CP vertices[19]. Inspired by this, we choose the ansatze.

Finally, we have to choose a gauge. Since the Landau gauge is the most convenient and commonly used gauge, we use it. Using the rainbow approximation for the full vertex, the coupled DS equations for the fermion propagator and photon vacuum polarization reduce to the following form,

$$A(p^2) = 1 + \frac{1}{p^2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \frac{A(k^2)[A(p^2) + A(k^2)](\boldsymbol{pq})(\boldsymbol{kq})/q^2}{q^2[A^2(k^2)k^2 + B^2(k^2)][1 + \Pi(q^2)]} \tag{12}$$

$$B(p^2) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \frac{[A(p^2) + A(k^2)]B(k^2)}{q^2[A^2(k^2)k^2 + B^2(k^2)][1 + \Pi(q^2)]} \tag{13}$$

$$\Pi(q^2) = \frac{N}{q^2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \frac{A(k^2)A(p^2)[A(p^2) + A(k^2)]}{A^2(k^2)k^2 + B^2(k^2)} \frac{2k^2 - 4(\boldsymbol{kq}) - 6(\boldsymbol{kq})^2/q^2}{A^2(p^2)p^2 + B^2(p^2)} \tag{14}$$

where $\boldsymbol{p} = \boldsymbol{q} + \boldsymbol{k}$.

The next step is to obtain mass functions $C(p^2)$ and $D(p^2)$. In order to obtain the two functions, we start from propagators with massive fermion, where we will meet $\Gamma_\nu(m;\boldsymbol{p},\boldsymbol{k})$. Since the Lagrangian of QED_3 is translation invariant, the WTI remains. We write the identity at small mass,

$$(\boldsymbol{p} - \boldsymbol{k})_\nu \Gamma_\nu(m;\boldsymbol{p},\boldsymbol{k}) = S^{-1}(\boldsymbol{p}) - S^{-1}(\boldsymbol{k}) + m[\Gamma(\boldsymbol{p},0) - \Gamma(\boldsymbol{k},0)] \tag{15}$$

Based on the freeness of kinematic singularities, the vertex is given as

$$\Gamma_\nu(m;\boldsymbol{p},\boldsymbol{k}) = \Gamma_\nu^{BC}(\boldsymbol{p},\boldsymbol{k}) + m\Gamma_\nu^{M}(\boldsymbol{p},\boldsymbol{k}) \tag{16}$$

The structure of the second part (Γ_ν^M) should be the same as the BC vertex. One can obtain Γ_ν^M if he only displaces A, B in the BC vertex as C, D. Following the same approximation in chiral limit, we also choose the first part in $\Gamma_\nu^M(\boldsymbol{p},\boldsymbol{k})$, and the ansatze for the full vertex in the DS equation for the massive fermion propagator can be written as

$$\Gamma_\nu^m(m;\boldsymbol{p},\boldsymbol{k}) = \left[\frac{A(p^2) + A(k^2)}{2} + m\frac{C(p^2) + C(k^2)}{2}\right]\boldsymbol{\gamma}_\nu \tag{17}$$

The corresponding boson propagator in Landau gauge can be written as

$$D_{\rho\nu}(m,\boldsymbol{q}) = \frac{\boldsymbol{\delta}_{\rho\nu} - \boldsymbol{q}_\rho\boldsymbol{q}_\nu/q^2}{q^2[1 + \Pi(m,q^2)]} \tag{18}$$

Based on the same idea for the fermion propagator, we also express this propagator as $O(m^1)$,

$$D_{\rho\nu}(m,\boldsymbol{q}) \doteq D_{\rho\nu}(\boldsymbol{q}) + \left.\frac{\partial D_{\rho\nu}(m,\boldsymbol{q})}{\partial m}\right|_{m=0} m = D_{\rho\nu}(\boldsymbol{q}) - \frac{\boldsymbol{\delta}_{\rho\nu} - \boldsymbol{q}_\rho\boldsymbol{q}_\nu/q^2}{q^2[1 + \Pi(m,q^2)]^2}\left.\frac{\partial\Pi(m,q^2)}{\partial m}\right|_{m=0} m = D_{\rho\nu}(\boldsymbol{q}) - D_{\rho\nu}(\boldsymbol{q})\frac{\Pi'(q^2)}{1 + \Pi(q^2)}m$$

where

$$\Pi'(q^2) = \left.\frac{\partial\Pi(m,q^2)}{\partial m}\right|_{m=0} \tag{19}$$

Substituting Eqs. (4) and (5) into the DS equation for the fermion propagator, we can obtain

$$S^{-1}(m,\boldsymbol{p}) = S_0^{-1}(m,\boldsymbol{p}) + \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\boldsymbol{\gamma}_\rho S(m,\boldsymbol{k})\Gamma_\nu(\boldsymbol{p},\boldsymbol{k})D_{\rho\nu}(m,\boldsymbol{q}) \tag{20}$$

and then obtain

$$\begin{aligned} S^{-1}(\boldsymbol{p}) + \left.\frac{\partial S^{-1}(m,\boldsymbol{p})}{\partial m}\right|_{m=0} m = S_0^{-1}(\boldsymbol{p}) + \left.\frac{\partial S_0^{-1}(m,\boldsymbol{p})}{\partial m}\right|_{m=0} m + \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\boldsymbol{\gamma}_\rho\left[S(\boldsymbol{k}) + \left.\frac{\partial S(m,\boldsymbol{k})}{\partial m}\right|_{m=0} m\right]\times \\ \left[\frac{A(p^2) + A(k^2)}{2} + m\frac{C(p^2) + C(k^2)}{2}\right]\boldsymbol{\gamma}_\nu\left[D_{\rho\nu}(\boldsymbol{q}) + \left.\frac{\partial D_{\rho\nu}(m,\boldsymbol{q})}{\partial m}\right|_{m=0} m\right] \end{aligned} \tag{21}$$

From the DS equation of the fermion propagator at $m \neq 0$, ignoring the high order of mass $O(m^2)$, we can obtain the linear part of fermion mass as follows:

$$\begin{aligned} i\boldsymbol{\gamma}\cdot\boldsymbol{p}C(p^2) + D(p^2) = 1 - \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\left[\frac{A(p^2) + A(k^2)}{2}\right]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q})\frac{\Pi'(q^2)}{1 + \Pi(q^2)} + \\ \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\boldsymbol{\gamma}_\rho\left.\frac{\partial S(m,\boldsymbol{k})}{\partial m}\right|_{m=0}\left[\frac{A(p^2) + A(k^2)}{2}\right]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q}) + \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\left[\frac{C(p^2) + C(k^2)}{2}\right]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q}) \end{aligned} \tag{22}$$

The functions of C and D are obtained as

$$D(p^2) = 1 + \frac{1}{4}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}\Big\{\boldsymbol{\gamma}_\rho \frac{\partial S(m,\boldsymbol{k})}{\partial m}\Big|_{m=0}\Big[\frac{A(p^2)+A(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q}) - \boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Big[\frac{A(p^2)+A(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q})\frac{\Pi'(q^2)}{1+\Pi(q^2)} + \boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Big[\frac{C(p^2)+C(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q})\Big\} \tag{23}$$

$$C(p^2) = \frac{1}{4p^2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}\Big\{(i\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Big[\frac{A(p^2)+A(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q})\frac{\Pi'(q^2)}{1+\Pi(q^2)} - (i\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Big[\frac{C(p^2)+C(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q}) - (i\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{\gamma}_\rho\frac{\partial S(m,\boldsymbol{k})}{\partial m}\Big|_{m=0}\Big[\frac{A(p^2)+A(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu D_{\rho\nu}(\boldsymbol{q})\Big\} \tag{24}$$

And Boson polarization is also obtained in this approximation.

$$\begin{aligned}\Pi(m,q^2) &= \Pi(q^2) + \frac{\partial \Pi(m,q^2)}{\partial m}\Big|_{m=0} m = \Pi(q^2) + \Pi'(q^2)m = \\ &-\frac{N}{2q^2}\Big(\boldsymbol{\delta}_{\rho\nu} - 3\frac{\boldsymbol{q}_\rho\boldsymbol{q}_\nu}{q^2}\Big)\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}\{\boldsymbol{\gamma}_\rho S(m,\boldsymbol{k})\Gamma_\nu(\boldsymbol{p},\boldsymbol{k})S(m,\boldsymbol{p})\} = \\ &-\frac{N}{2q^2}\Big(\boldsymbol{\delta}_{\rho\nu} - 3\frac{\boldsymbol{q}_\rho\boldsymbol{q}_\nu}{q^2}\Big)\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}\Big\{\boldsymbol{\gamma}_\rho\Big[S(\boldsymbol{k}) + \frac{\partial S(m,\boldsymbol{k})}{\partial m}\Big|_{m=0} m\Big]\times \\ &\Big[\frac{A(p^2)+A(k^2)}{2} + m\frac{C(p^2)+C(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu\Big[S(\boldsymbol{p}) + \frac{\partial S(m,\boldsymbol{p})}{\partial m}\Big|_{m=0} m\Big]\Big\}\end{aligned}$$

$$\Pi'(q^2) = -\frac{N}{2q^2}\Big(\boldsymbol{\delta}_{\rho\nu} - 3\frac{\boldsymbol{q}_\rho\boldsymbol{q}_\nu}{q^2}\Big)\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\mathrm{Tr}\Big\{\boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Big[\frac{A(p^2)+A(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu\frac{\partial S(m,\boldsymbol{p})}{\partial m}\Big|_{m=0} + \boldsymbol{\gamma}_\rho S(\boldsymbol{k})\Big[\frac{C(p^2)+C(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu S(\boldsymbol{p}) + \boldsymbol{\gamma}_\rho\frac{\partial S(m,\boldsymbol{k})}{\partial m}\Big|_{m=0}\Big[\frac{A(p^2)+A(k^2)}{2}\Big]\boldsymbol{\gamma}_\nu S(\boldsymbol{p})\Big\} \tag{25}$$

According to Eqs. (20) to (24) and using the identity

$$\frac{\partial S(m,\boldsymbol{p})}{\partial m} \equiv -S(m,\boldsymbol{p})\frac{\partial S^{-1}(m,\boldsymbol{p})}{\partial m}S(m,\boldsymbol{p}) \tag{26}$$

we obtain the coupled equations for $C(p^2)$, $D(p^2)$, $\Pi'(q^2)$,

$$C(p^2) = \frac{1}{p^2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Big\{\frac{A(k^2)[[C(p^2)+C(k^2)][1+\Pi(q^2)] - \Pi'(q^2)[A(p^2)+A(k^2)]](\boldsymbol{pq})(\boldsymbol{kq})}{[A^2(k^2)k^2+B^2(k^2)]q^4[1+\Pi(q^2)]^2} - [A(p^2)+A(k^2)]\frac{[A^2(k^2)C(k^2)k^2 - B^2(k^2)C(k^2) + 2A(k^2)B(k^2)D(k^2)](\boldsymbol{pq})(\boldsymbol{kq})}{[A^2(k^2)k^2+B^2(k^2)]^2q^4(1+\Pi(q^2))}\Big\}$$

$$D(p^2) = 1 + \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Big\{B(k^2)\Big[\frac{C(p^2)+C(k^2)-(A(p^2)+A(k^2))\Pi'(q^2)}{[A^2(k^2)k^2+B^2(k^2)]q^2[1+\Pi(q^2)]^2}\Big] + [A(p^2)+A(k^2)]\frac{B^2(k^2)D(k^2) + [2B(k^2)A(k^2)C(k^2) - D(k^2)A^2(k^2)]k^2}{[A^2(k^2)k^2+B^2(k^2)]^2q^2[1+\Pi(q^2)]}\Big\}$$

$$\begin{aligned}\Pi'(q^2) = \frac{N}{q^2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{[A^2(p^2)p^2+B^2(p^2)][A^2(k^2)k^2+B^2(k^2)]}\Big\{&A(k^2)A(p^2)[C(p^2)+C(k^2)]H(\boldsymbol{k},\boldsymbol{q}) - \\ &\frac{A(k^2)[A(p^2)+A(k^2)]}{[A^2(p^2)p^2+B^2(p^2)]}\Big[(B^2(p^2)C(p^2) - 2A(p^2)B(p^2)D(p^2))H(\boldsymbol{k},\boldsymbol{q}) + 2A^2(p^2)C(p^2)\boldsymbol{kq}p^2\Big(1+\frac{\boldsymbol{kq}}{q^2}\Big)\Big] - \\ &\frac{A(p^2)[A(p^2)+A(k^2)]}{[A^2(k^2)k^2+B^2(k^2)]}\Big[(B^2(k^2)C(k^2) - 2A(k^2)B(k^2)D(k^2))H(\boldsymbol{k},\boldsymbol{q}) + 2A^2(k^2)C(k^2)\boldsymbol{kq}k^2\Big(1+\frac{\boldsymbol{kq}}{q^2}\Big)\Big]\Big\}\end{aligned}$$

where $H(\boldsymbol{k},\boldsymbol{q}) = 2k^2 - 4\boldsymbol{kq} - 6(\boldsymbol{kq})^2/q^2$. By all appearances, all the above three functions are not the functions of the fermion mass. From them, we will numerically investigate $C(p^2)$, $D(p^2)$, $\Pi'(q^2)$. It is noted that the three functions have no effect on our model in chiral limit.

2 Numerical Results

In chiral limit, we first define the original value of each function as a plus value to iterate, till the DS equations (11) to (13) obtain stable results. We plot them in Fig. 1 and name this as a conventional or a plus solution. This solution was discussed in previous work[8].

Then, starting from $B<0$, we also iterate the equation group to obtain a stable result and show $A(p^2)$, $B(p^2)$ and $\Pi(q^2)$ in Fig. 1. We can see from Fig. 1 that the behaviors of $A(p^2)$ or $\Pi(q^2)$ are apparently different between the chirally symmetric phase and asymmetric phase, but are uniform when $B>0$ and $B<0$.

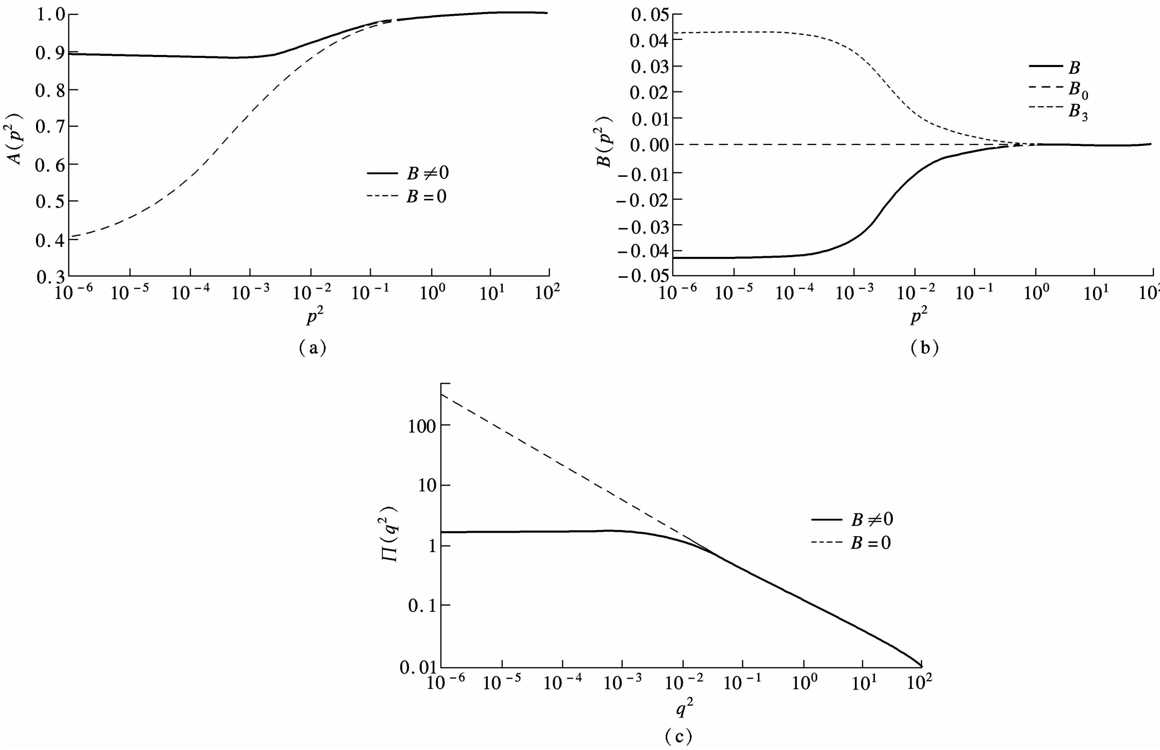

Fig. 1 Functions in the chirally symmetric phase and asymmetric phase. (a) $A(p^2)$; (b) $B(p^2)$; (c) $\Pi(q^2)$

One sees that the function of fermion self-energy is negative at all p^2 and its absolute value is similar to a plus solution while the other two functions A, Π do not reveal different behaviors between $B>0$ and $B<0$. Then, setting $B \equiv 0$, we numerically solve the coupled DS equations to obtain the Wigner solutions and they are also shown in Fig. 1. The trivial solution of fermion self-energy lies between $B<0$ and $B>0$. However, corresponding A, Π reveal different behaviors when $B \neq 0$. In infrared region, A diminishes when Π increases and diverges when $p^2 \to 0$.

Beyond chiral limit, we also study the linear effect of fermion mass on propagators. Using the above method, we obtain $C(p^2)$, $D(p^2)$ and $\Pi'(q^2)$ as shown in Fig. 2.

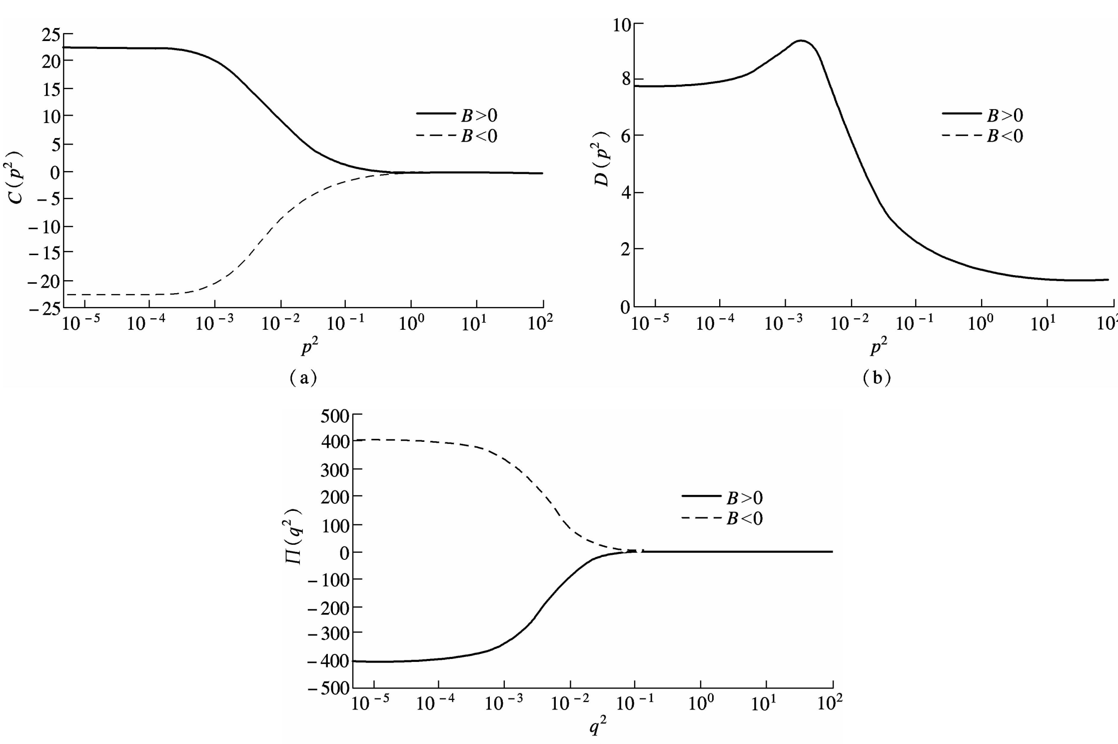

Fig. 2 Functions in the broken phase. (a) $C(p^2)$; (b) $D(p^2)$; (c) $\Pi'(q^2)$

One sees that there are two apparently different solutions for $C(p^2)$ and $\Pi'(q^2)$, while we find only one solution for $D(p^2)$. All solutions for the three functions hardly change in the infrared region, but each value of the solutions diminishes in large momentum. When $B>0$ which is studied frequently in previous works, the DS equation gives the plus solution for $C(p^2)$ and negative $\Pi'(q^2)$. The infrared value of vector function of the fermion propagator decreases with the increasing mass. Nevertheless, for $B<0$, the second infrared value for vector function gives the opposite movement, and the difference between two infrared values gets large with the increasing mass.

In principle, there should be two solutions for $D(p^2)$, corresponding to $C(p^2)$, $\Pi'(q^2)$, while we only find one and it is plus. Therefore, the two scalar functions of the fermion propagator tend to equal each other and this is in accordance with Ref. [9] where QCD is used.

3 Conclusion

We have investigated the coupled DS equation for the fermion propagator in the unquenched QED_3. In chiral limit, we find that, besides two familiar nonnegative solutions, there is a negative solution for this equation. Moreover, there exist two solutions for the DS equation beyond chiral limit. Since QED_3 may be relative to the theory of $d_{x^2-y^2}$ superconductivity, those solutions can be used to analyze the existence of three phases, antiferromagnet, pseudogap and superconductivity. We suppose that massless QED_3 is suitable in the underdoped region, especially near the area where the above mentioned three phases compete, and QED_3 with fermion mass does in the overdoped region in phase figure for YBCO.

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QED_3 中多重解和费米质量效应

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摘要：由于低能区域内的非微扰效应不能忽略，使得 QCD 在处理粒子物理中的强子问题时非常困难，而 QED 有较成熟的非微扰模型，可以很好地处理夸克和胶子传播子在费米作用能方面的相关问题，因此本文采用 QED_3 来处理费米凝聚问题. 首先构建费米子所满足的 Dyson-Schwinger 方程，然后求解费米能隙. 理论计算表明，在手征极限下该能隙方程存在 3 个解，而在超越手征极限下则存在 2 个解，这些解都对应于不同的费米凝聚. 手征极限下的费米凝聚可用来分析 3 个相的存在，即反铁磁相、赝能级和超导态. 而在超越手征极限下，发现存在 2 个费米子凝聚.

关键词：Dyson-Schwinger 方程；手征极限；超越手征极限；费米子；凝聚；多重解；三维量子电动力学

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