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Uniform asymptotics for finite-time ruin probability in some dependent compound risk models with constant interest rate

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Abstract: Consider two dependent renewal risk models with constant interest rate. By using some methods in the risk theory, uniform asymptotics for finite-time ruin probability is derived in a non-compound risk model, where claim sizes are upper tail asymptotically independent random variables with dominatedly varying tails, claim inter-arrival times follow the widely lower orthant dependent structure, and the total amount of premiums is a nonnegative stochastic process. Based on the obtained result, using the method of analysis for the tail probability of random sums, a similar result in a more complex and reasonable compound risk model is also obtained, where individual claim sizes are specialized to be extended negatively dependent and accident inter-arrival times are still widely lower orthant dependent, and both the claim sizes and the claim number have dominatedly varying tails.

Key words: compound and non-compound risk models; finite-time ruin probability; dominatedly varying tail; uniform asymptotics; random sums; dependence structure

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Consider a compound renewal risk model, in which the claims at each accident moment are aggregated from a number of individual claims. Meanwhile, in a non-compound risk model one claim at each accident time appears. More precisely, the compound renewal risk model satisfies the following assumptions: the individual claim sizes $\{X_k, k\geqslant 1\}$ form a sequence of nonnegative random variables (r. v. s) with a common distribution F, and the accident inter-arrival times $\{\theta_k, k\geqslant 1\}$ are nonnegative r. v. s. The individual claim sizes and the claim number caused by the n-th accident at the accident time $\tau_n = \theta_1+\theta_2+\cdots+\theta_n$ are $\{X_k^{(n)}, k\geqslant 1\}$ and N_n, respectively. Here, $\{X_k^{(n)}, k\geqslant 1\}$ are independent copies of $\{X_k, k\geqslant 1\}$, and $\{N_k, k\geqslant 1\}$ are independent and identically distributed (i. i. d) integer-valued r. v. s with common distribution G. $\{N_k, k\geqslant 1\}$, $\{\theta_k, k\geqslant 1\}$, and $\{\{X_k^{(n)}, k\geqslant 1\}, n\geqslant 1\}$ are mutually independent. This model was introduced in Ref. [1]. If each claim number $N_k=1$, it reduces to the non-compound one.

In such a compound model, the total claim amount at time τ_n and the total claim amount up to time $t\geqslant 0$ are, respectively, $S_{N_n}^{(n)} = \sum_{k=1}^{N_n} X_k^{(n)}$ and $\sum_{n=1}^{N(t)} S_{N_n}^{(n)}$, where $N(t) = \sup\{n\geqslant 0: \tau_n \leqslant t\}$ with $\lambda(t) = EN(t)$. The total amount of premiums accumulated before time $t\geqslant 0$, denoted by $C(t)$ with $C(0)=0$ and $C(t)<\infty$ almost surely (a. s.), is a nonnegative and non-decreasing stochastic process, independent of $\{N_k, k\geqslant 1\}$, $\{\theta_k, k\geqslant 1\}$ and $\{\{X_k^{(n)}, k\geqslant 1\}, n\geqslant 1\}$. Assume that $\widetilde{C}(t) = \int_{0-}^{t} \mathrm{e}^{-\delta y} C(\mathrm{d}y) < \infty$ a. s. Let $\delta\geqslant 0$ be the constant interest rate (that is, after time t, the capital x becomes $x\mathrm{e}^{\delta t}$) and $x\geqslant 0$ be the initial capital reserve. Then, the finite-time ruin probability within any fixed time $T\geqslant 0$ is defined by

$$\psi(x,T) = P\Big(\sup_{0<t\leqslant T}\Big(\sum_{n=1}^{N(t)} S_{N_n}^{(n)} \mathrm{e}^{-\delta\tau_n} - \int_{0-}^{t} \mathrm{e}^{-\delta s} C(\mathrm{d}s)\Big) > x\Big) \tag{1}$$

In the non-compound model, where $N_k=1$, $k\geqslant 1$, the finite-time ruin probability can be simplified as

$$\psi_1(x,T) = P\Big(\sup_{0<t\leqslant T}\Big(\sum_{n=1}^{N(t)} X_n \mathrm{e}^{-\delta\tau_n} - \int_{0-}^{t} \mathrm{e}^{-\delta s} C(\mathrm{d}s)\Big) > x\Big) \tag{2}$$

This paper aims to investigate the asymptotics for the finite-time ruin probabilities in Eqs. (1) and (2) holding uniformly for all t such that $\lambda(t)$ is positive. Define the set $\Lambda = \{t: \lambda(t) > 0\}$.

1 Preliminaries

Hereafter, all the limit relationships hold for $x\rightarrow\infty$. For two positive bivariate functions $a(x,t)$ and $b(x,t)$, we write $a(x,t) \lesssim b(x,t)$ (or, equivalently, $b(x,t) \gtrsim$

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$a(x,t)$) holds uniformly for all t in a nonempty set A, if $\limsup \sup_{t\in A} a(x,t)/b(x,t) \leqslant 1$; we write $a(x,t) \sim b(x,t)$ holds uniformly for all $t\in A$, if $a(x,t) \lesssim b(x,t)$ and $a(x,t) \gtrsim b(x,t)$. For real y, the greatest integer smaller than or equal to y is denoted by $[y]$.

We will restrict the claim-size distribution F to be heavy-tailed. Throughout this paper, assume that $\overline{V}(x) = 1 - V(x) > 0$ for all x. A distribution V is said to belong to the class of distributions with dominatedly varying tails, denoted by $V \in D$, if for any $0 < y < 1$, $\limsup \overline{V}(xy)/\overline{V}(x) < \infty$. For a distribution V, its upper and lower Matuszewska indices are denoted, respectively, by $J_V^+ = -\lim_{y\to\infty} \log \overline{V}_*(y)/\log y$ with $\overline{V}_*(y) = \liminf \overline{V}(xy)/\overline{V}(x)$ and $J_V^- = -\lim_{y\to\infty} \log \overline{V}^*(y)/\log y$ with $\overline{V^*}(y) = \limsup \overline{V}(xy)/\overline{V}(x)$. In addition, we define a parameter $L_V = \lim_{y\downarrow 1} \overline{V}_*(y)$.

We next introduce some dependence structures. Ref. [2] proposed a pairwise dependence structure. A sequence of real-valued r. v. s $\{\xi_n, n\geqslant 1\}$ is said to be upper tail asymptotically independent (UTAI), if for each pair of indices (i,j), for all $i\neq j$, $\lim_{\min\{x_i,y_j\}\to\infty} P(\xi_i > x_i \mid \xi_j > y_j) = 0$. Such a dependence structure is wider than the following widely upper orthant dependence (WUOD), which was recently proposed in Ref. [3]. A sequence of r. v. s $\{\xi_n, n\geqslant 1\}$ is said to be widely upper orthant dependent if there exist $\{g_U^\xi(n), n\geqslant 1\}$ such that, for each $n\geqslant 1$ and all $x_1, x_2, \cdots, x_n$,

$$P(\xi_1 > x_1, \cdots, \xi_n > x_n) \leqslant g_U^\xi(n) \prod_{k=1}^n P(\xi_k > x_k) \tag{3}$$

They are said to be widely lower orthant dependent (WLOD) if there exist $\{g_L^\xi(n), n\geqslant 1\}$ such that

$$P(\xi_1 \leqslant x_1, \cdots, \xi_n \leqslant x_n) \leqslant g_L^\xi(n) \prod_{k=1}^n P(\xi_k \leqslant x_k) \tag{4}$$

and they are said to be widely orthant dependent (WOD) if they are both WUOD and WLOD.

Clearly, if r. v. s $\{\xi_n, n\geqslant 1\}$ are WUOD, then they are also UTAI. If $g_U^\xi(n) = g_L^\xi(n) = M$ for some positive constant $M > 0$ and all $n\geqslant 1$ in (3) and (4), the r. v. s $\{\xi_n, n\geqslant 1\}$ are said to be extended negatively dependent (END) (see, e. g., Ref. [4]).

2 Uniform Asymptotics for Finite-Time Ruin Probabilities

In this section, we first consider a non-compound risk model. Set $D_\delta(t) = \sum_{k=1}^\infty X_k e^{-\delta\tau_k} 1_{\{\tau_k \leqslant t\}}$.

Theorem 1 Consider a non-compound dependent risk model with constant interest rate $\delta\geqslant 0$ described in section 1. Assume that the claim sizes $\{X_n, n\geqslant 1\}$ are UTAI r. v. s with common distribution $F\in D$, and the inter-arrival times $\{\theta_n, n\geqslant 1\}$ are WLOD r. v. s with dominating coefficients $g_L^\theta(n)$ satisfying

$$\limsup_{n\to\infty} g_L^\theta(n) e^{-\varepsilon_0 n} < \infty \tag{5}$$

for some $\varepsilon_0 > 0$. Then for any $T\in\Lambda$, it holds that uniformly for all $t\in\Lambda\cap[0,T]$,

$$L_F^2 \int_{0-}^t \overline{F}(xe^{\delta y}) \lambda(dy) \lesssim \psi_1(x,T) \lesssim L_F^{-1} \int_{0-}^t \overline{F}(xe^{\delta y}) \lambda(dy) \tag{6}$$

Lemma 1 Under the conditions of Theorem 1, for all $T\in\Lambda$, it holds that uniformly for $t\in\Lambda\cap[0,T]$,

$$L_F \int_{0-}^t \overline{F}(xe^{\delta y}) \lambda(dy) \lesssim P(D_\delta(t) > x) \lesssim L_F^{-1} \int_{0-}^t \overline{F}(xe^{\delta y}) \lambda(dy) \tag{7}$$

Proof The proof of Lemma 1 follows the line of Theorem 1.1 in Ref. [5].

Proof of Theorem 1 Clearly, for any $t\in\Lambda\cap[0,T]$, $P(D_\delta(t) > x + \widetilde{C}(T)) \leqslant \psi_1(x,t) \leqslant P(D_\delta(t) > x)$. For any $\varepsilon > 0$, by Lemma 1, $F\in D$ and Fatou's lemma, it holds that uniformly for $t\in\Lambda\cap[0,T]$,

$$\liminf_{x\to\infty} \inf_{t\in\Lambda\cap[0,T]} \frac{\psi_1(x,t)}{\int_{0-}^t \overline{F} xe^{\delta y} \lambda(dy)} \geqslant (1-\varepsilon) L_F \overline{F}_*(1+\varepsilon) \int_{0-}^\infty P(\widetilde{C}(T) \in dy) \to L_F^2$$

as $\varepsilon \downarrow 0$, which implies that the desired lower bound in (6) holds. Again by Lemma 1, we obtain that uniformly for all $t\in\Lambda\cap[0,T]$, $\psi_1(x,t) \leqslant P(D_\delta(t) > x) \lesssim L_F^{-1} \int_{0-}^t \overline{F}(xe^{\delta s}) \lambda(ds)$.

In the following, we study the uniform asymptotics for the finite-time ruin probability in a compound renewal risk model by the investigation of the asymptotic tail behavior of random sums. Some related results can be found in Refs. [6-8].

Theorem 2 Consider a compound risk model with constant interest rate $\delta\geqslant 0$ described in section 1. Assume that the individual claim sizes $\{X_k, k=1\}$ are END r. v. s with common distribution $F\in D$, and the inter-arrival times $\{\theta_k, k\geqslant 1\}$ are WLOD r. v. s with dominating coefficients $g_L^\theta(n)$ satisfying (5) for some $\varepsilon_0 > 0$. If $G\in D$, then for any finite $T\in\Lambda$, it holds that uniformly for all $t\in\Lambda\cap[0,T]$,

$$\min\{L_F^6, L_G^6\} \int_{0-}^t (\mu_G L_F \overline{F}(xe^{\delta s}) + L_G \overline{G}(\mu_F^{-1} xe^{\delta s})) \lambda(ds) \lesssim \psi(x,t) \lesssim \max\{L_F^{-3}, L_G^{-3}\} \int_{0-}^t (\mu_G L_F^{-1} \overline{F}(xe^{\delta s}) + L_G^{-1} \overline{G}(\mu_F^{-1} xe^{\delta s})) \lambda(ds)$$

Denote the partial sum by $S_n = X_1 + X_2 + \cdots + X_n, n \geqslant 1$.

Lemma 2 Let $\{X_n, n=1\}$ be END nonnegative r. v. s with common distribution $F \in D$ and mean $\mu_F > 0$, and N be an integer-valued r. v., independent of $\{X_n, n=1\}$, with distribution $G \in D$ and mean $\mu_G > 0$. Then

$$\mu_G L_F \overline{F}(x) + L_G \overline{G}(\mu_F^{-1} x) < P(S_N > x) < \mu_G L_F^{-1} \overline{F}(x) + L_G^{-1} \overline{G}(\mu_F^{-1} x) \tag{8}$$

Proof For any $0 < \varepsilon < 1$ and integer m, we divide the tail probability of S_N into three parts:

$$P(S_N > x) = \Big(\sum_{i=1}^{m} + \sum_{m < i \leqslant (1-\varepsilon)\mu_F^{-1}} + \sum_{i > (1-\varepsilon)\mu_F^{-1} x}\Big) P(S_i > x) \cdot P(N = i) =: L_1 + L_2 + L_3 \tag{9}$$

By $F \in D$ and Theorem 1 in Ref. [9], we have that

$$L_F \mu_G \leqslant \lim_{m \uparrow \infty} \liminf_{x \to \infty} \frac{L_1}{\overline{F}(x)} \leqslant \lim_{m \uparrow \infty} \limsup_{x \to \infty} \frac{L_1}{\overline{F}(x)} \leqslant L_F^{-1} \mu_G \tag{10}$$

For any $m < i \leqslant (1-\varepsilon) x/\mu_F$, by using Lemma 2. 1 in Ref. [10] and $F \in D$, we have that there exists a positive constant C_0, when m is sufficiently large,

$$P(S_i > x) = P(S_i - i\mu_F > x - i\mu_F) \leqslant C_0 i \overline{F}(x - (i-1)\mu_F) \leqslant C i \overline{F}(x)$$

where the last step uses $F \in D$ and C is a positive constant irrespective to i. By using Theorem 1 in Ref. [9] and the dominated convergence theorem, we obtain that

$$\lim_{m \uparrow \infty} \limsup_{x \to \infty} \frac{L_2}{\overline{F}(x)} \leqslant L_F^{-1} \lim_{m \uparrow \infty} \sum_{i=m+1}^{\infty} i P(N = i) = 0 \tag{11}$$

$$\lim_{\varepsilon \downarrow 0} \limsup_{x \to \infty} \frac{L_3}{\overline{G}(\mu_F^{-1} x)} \leqslant \lim_{\varepsilon \downarrow 0} \limsup_{x \to \infty} \frac{\overline{G}((1-\varepsilon)\mu_F^{-1} x)}{\overline{G}(\mu_F^{-1} x)} = L_G^{-1} \tag{12}$$

Thus, combining (9) to (12), we can obtain the upper bound in (8).

Now we estimate the lower bound of $P(S_N > x)$. For any $0 < \varepsilon < 1$ and integer m, we have that

$$P(S_N > x) \geqslant \Big(\sum_{i=1}^{m} + \sum_{i > (1+\varepsilon)\mu_F^{-1} x}\Big) P(S_N > x) P(N = i) =: L_1 + L_4 \tag{13}$$

For L_4, it holds that

$$L_4 \geqslant P(S_{[(1+\varepsilon)\mu_F^{-1} x]} > x) \overline{G}((1+\varepsilon)\mu_F^{-1} x) \geqslant P\left(\frac{S_{[(1+\varepsilon)\mu_F^{-1} x]}}{[(1+\varepsilon)\mu_F^{-1} x]} - \mu_F > -\frac{\varepsilon \mu_F}{1+\varepsilon}\right) \overline{G}((1+\varepsilon)\mu_F^{-1} x)$$

Hence, by the strong law of the large numbers of END r. v. s[11] and $F \in D$, we obtain that

$$\lim_{\varepsilon \downarrow 0} \liminf_{x \to \infty} \frac{L_4}{\overline{G}(\mu_F^{-1} x)} \geqslant L_G \tag{14}$$

Therefore, (13), (10) and (14) yield the lower bound in (8).

Proof of Theorem 2 Clearly, $\{S_{N_n}^{(n)}, n \geqslant 1\}$ are i. i. d. r. v. s with a common distribution, denoted by H. By Lemma 2, we have that

$$\mu_G L_F \overline{F}(x) + L_G \overline{G}(\mu_F^{-1} x) < \overline{H}(x) < \mu_G L_F^{-1} \overline{F}(x) + L_G^{-1} \overline{G}(\mu_F^{-1} x) \tag{15}$$

which, by $F \in D$ and $G \in D$, implies that $H \in D$. So, by (1), Theorem 1 and (15), for any fixed $T \in \Lambda$, we obtain that

$$\psi(x, T) < L_H^{-1} \int_{0-}^{t} \overline{H}(x e^{\delta s}) \lambda(ds) < L_H^{-1} \int_{0-}^{t} (\mu_G L_F^{-1} \overline{F}(x e^{\delta s}) + L_G^{-1} \overline{G}(\mu_F^{-1} x e^{\delta s})) \lambda(ds) \tag{16}$$

$$\psi(x, T) > L_H^2 \int_{0-}^{t} \overline{H}(x e^{\delta s}) \lambda(ds) > L_H^2 \int_{0-}^{t} (\mu_G L_F \overline{F}(x e^{\delta s}) + L_G \overline{G}(\mu_F^{-1} x e^{\delta s})) \lambda(ds) \tag{17}$$

hold uniformly for all $t \in \Lambda \cap [0, T]$. Note that by (15), it holds that for any $y > 1$,

$$\overline{H}_*(y) \geqslant \liminf \frac{\mu_G L_F \overline{F}(xy) + L_G \overline{G}(\mu_F^{-1} xy)}{\mu_G L_F^{-1} \overline{F}(x) + L_G^{-1} \overline{G}(\mu_F^{-1} x)} \geqslant \min\{L_F^2 \overline{F}_*(y), L_G^2 \overline{G}_*(y)\}$$

which implies that $L_H \geqslant \min\{L_F^3, L_G^3\}$. Substituting this into (16) and (17), the desired relation follows.

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带有常数利息率的相依复合风险模型中有限时破产概率的一致渐近性

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摘要:考虑了 2 个带有常数利息率的相依更新风险模型. 首先研究了非复合风险模型,其中索赔额是上尾渐近独立且带有控制变换尾分布的非负随机变量,索赔时间间隔是宽下象限相依的,保费收入过程是一个非负的随机过程,利用风险理论中的方法,得到了有限时破产概率在某个有界区间上的一致渐近性. 在此基础上,利用随机和尾渐近性的分析方法,进一步研究获得了更为复杂且合理的复合相依更新风险模型中有限时破产概率的一致渐近性公式,其中单个索赔额特殊化为广义负相依的,并且事故时间间隔仍然保持宽下象限相依的,索赔额和索赔次数均为控制变换尾的.

关键词:复合及非复合风险模型;有限时破产概率;控制变换尾;一致渐近性;随机和;相依结构

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