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Stability analysis of time-varying systems via parameter-dependent homogeneous Lyapunov functions

Zhang Huasheng[1, 2, 3] Zhang Kanjian[1, 2]

([1]Key Laboratory of Measurement and Control of Complex Systems of Engineering of Ministry of Education, Southeast University, Nanjing 210096, China)

([2]School of Automation, Southeast University, Nanjing 210096, China)

([3]School of Mathematical Sciences, Liaocheng University, Liaocheng 252000, China)

Abstract: This paper considers the stability analysis of linear continuous-time systems, and that the dynamic matrices are affected by uncertain time-varying parameters, which are assumed to be bounded, continuously differentiable, with bounded rates of variation. First, sufficient conditions of stability for time-varying systems are given by the commonly used parameter-dependent quadratic Lyapunov function. Moreover, the use of homogeneous polynomial Lyapunov functions for the stability analysis of the linear system subject to the time-varying parametric uncertainty is introduced. Sufficient conditions to determine the sought after Lyapunov function is derived via a suitable paramenterization of polynomial homogeneous forms. A numerical example is given to illustrate that the stability conditions are less conservative than similar tests in the literature.

Key words: linear time-varying systems; polytopic uncertainty; robust stability; linear matrix inequality

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The study of linear time-varying system stability has been an important issue in control theory for many years. It is well known that quadratic stability is a sufficient condition for the stability of linear systems with arbitrarily fast time-varying parameters. This condition is appealing from a numerical point of view mainly because of its simplicity, and it has been widely used for robust control and robust filter design, in most cases through convex problems formulated in terms of linear matrix inequalities (LMIs)[1−2]. In order to reduce conservativeness, more general classes of Lyapunov functions have been considered, including polyhedral Lyapunov functions[3−4], piecewise quadratic Lyapunov functions[5], and homogeneous polynomial Lyapunov functions (HPLFs)[6−7].

Homogeneous polynomial Lyapunov functions are a viable alternative to the above classes of Lyapunov functions. In fact, that this class of Lyapunov functions can improve the robust stability results provided by quadratic Lyapunov functions has been recognized for a long time[8]. Recently, it has been shown that for these systems, robust stability is equivalent to the existence of a smooth Lyapunov function that turns out to be the sum of the squares of homogeneous polynomial forms[9−10].

This paper focuses on the stability analysis of linear systems where dynamic matrices are affected by uncertain time-varying parameters with a bounded variation rate. The problem can be tackled by HPLFs and constructing HPLFs can be formulated in terms of special convex optimization techniques based on linear matrix inequalities (LMI). An example here is shown, which proves that HPLFs are powerful tools for stability analysis.

1 Problem Formulation and Preliminaries

Consider the linear time-varying system

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}(\boldsymbol{\alpha}(t))\boldsymbol{x}(t) \tag{1}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state and $\boldsymbol{A}(\boldsymbol{\alpha}(t))\in\mathbf{R}^{n\times n}$ is an uncertain time-varying matrix belonging to the polytope Λ given by

$$\Lambda=\left\{\boldsymbol{A}(\boldsymbol{\alpha}(t)):\boldsymbol{A}(\boldsymbol{\alpha}(t))=\sum_{j=1}^{N}\alpha_j(t)\boldsymbol{A}_j,\ \sum_{j=1}^{N}\alpha_j(t)=1,\ \alpha_j(t)>0\right\}$$

In other words, for all $t>0$ with components, $\alpha_j(t)$ represents time varying unknown parametric perturbations such that $\boldsymbol{A}(\boldsymbol{\alpha}(t))\in\mathrm{co}\{\boldsymbol{A}_1,\ldots,\boldsymbol{A}_N\}$, where $\mathrm{co}\{\cdot\}$ denotes the convex hull.

The parameters of this system are assumed to have bounded time-derivatives, i.e.,

$$\begin{aligned}&|\dot{\alpha}_i(t)|\leqslant\rho_i\qquad i=1,2,\ldots,N-1\\&|\dot{\alpha}_N(t)|\leqslant\left|\sum_{i=1}^{N-1}\dot{\alpha}_i(t)\right|\leqslant\sum_{i=1}^{N-1}\rho_i\end{aligned} \tag{2}$$

Notice that the constraint (2) comes from $\sum_{i=1}^{N-1}\alpha_i(t)=0$ and it will be used throughout the analysis without loss of

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generality.

The function $f_m(x)$ is a homogeneous form of degree m in $\boldsymbol{x} \in \mathbf{R}^n$ if

$$f_m(x) = \sum_{i_1+i_2+\ldots+i_n=m} c_{i_1 i_2 \ldots i_m} x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n}$$

where $i_1, i_2, \ldots, i_n$ are non-negative integers, and $c_{i_1 i_2 \ldots i_m}$ are the weighting coefficient. The form $f_m(x)$ is said to be positive if $f_m(x) > 0, \ \forall x \neq 0$.

The following result provides a sufficient condition for establishing the existence of an HPLF of degree $2m$ for system (1).

Lmma 1[11] Let $\boldsymbol{A}_{j,\{m\}}$ denote the extended matrix of $\boldsymbol{A}_j$. If the system of the LMIs

$$\boldsymbol{V} > 0, \quad -\boldsymbol{V}\boldsymbol{A}_{j,\{m\}} - \boldsymbol{A}_{j,\{m\}}^{\mathrm{T}} \boldsymbol{V} > 0$$

admits a feasible solution $\boldsymbol{V} = \boldsymbol{V}^{\mathrm{T}} \in \mathbf{R}^{n \times n}$, then $v_{2m}(x) = x^{\{m\}\mathrm{T}} \boldsymbol{V} \boldsymbol{x}^{\{m\}}$ is an HPLP for Eq. (1).

The next lemma presents a sufficient LMI condition for the robust stability of linear time-varying systems in the polytopic form of Eq. (1).

Lmma 2[12] For given real scalars $\rho_i > 0, \ i = 1, 2, \ldots, N-1$, if there exist symmetric positive definite matrices $\boldsymbol{P}_j \in \mathbf{R}^{n \times n}, \ j = 1, 2, \ldots, N$, satisfying

$$\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_j + \boldsymbol{P}_j \boldsymbol{A}_j + \sum_{i=1}^{N-1} \pm \rho_i (\boldsymbol{P}_i - \boldsymbol{P}_N) < 0 \qquad (3)$$

$$\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_k + \boldsymbol{P}_k \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_j + \boldsymbol{P}_j \boldsymbol{A}_k + \sum_{i=1}^{N-1} \pm \rho_i (\boldsymbol{P}_i - \boldsymbol{P}_N) < 0$$
$$j = 1, 2, \ldots, N-1; \ k = j+1, \ldots, N \qquad (4)$$

then the system (1) is asymptotically stable for all time-varying uncertain parameters inside the polytope Λ respecting the time-derivative constraints (2) with the parameter dependent Lyapunov matrix given by

$$\boldsymbol{P}(\boldsymbol{\alpha}(t)) = \sum_{j=1}^{N} \alpha_j(t) \boldsymbol{P}_j, \quad \sum_{j=1}^{N} \alpha_j(t) = 1 \qquad \alpha_j(t) \geqslant 0$$

2 Main Results

The main result of the paper is a sufficient condition to determine the sought Lyapunov function, which amounts to solving an LMI feasibility problem, derived via a suitable parameterization of homogeneous polynomial forms.

Theorem 1 For given real scalars $\rho_i > 0, \ i = 1, 2, \ldots, N-1$, if there exist symmetric positive definite matrices $\boldsymbol{P}_{ij} \in \mathbf{R}^{n \times n}, \ j = 1, 2, \ldots, N$, satisfying

$$\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{jj} + \boldsymbol{P}_{jj} \boldsymbol{A}_j + 2 \sum_{i=1}^{N-1} \pm \rho_i (\boldsymbol{P}_{ij} - \boldsymbol{P}_{Nj}) < 0 \qquad (5)$$

$$\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jk} \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_{jj} + \boldsymbol{P}_{jj} \boldsymbol{A}_k + 2 \sum_{i=1}^{N-1} \pm \rho_i (\boldsymbol{P}_{ik} - \boldsymbol{P}_{Nk} + 2(\boldsymbol{P}_{ij} - \boldsymbol{P}_{Nj})) < 0 \qquad (6)$$

$$\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{kk} + \boldsymbol{P}_{kk} \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jk} \boldsymbol{A}_k + 2 \sum_{i=1}^{N-1} \pm \rho_i (\boldsymbol{P}_{ij} - \boldsymbol{P}_{Nj} + 2(\boldsymbol{P}_{ik} - \boldsymbol{P}_{Nk})) < 0 \qquad (7)$$

$$\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{kl} + \boldsymbol{P}_{kl} \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_{jl} + \boldsymbol{P}_{jl} \boldsymbol{A}_k + \boldsymbol{A}_l^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jk} \boldsymbol{A}_l + 2 \sum_{i=1}^{N-1} \sum_{q=1}^{N} \pm \rho_i (\boldsymbol{P}_{iq} - \boldsymbol{P}_{Nq}) < 0 \qquad (8)$$

where $k = 2, \ldots, N; \ l = 2, \ldots, N; \boldsymbol{P}(\boldsymbol{\alpha}(t)) = \sum_{j=1}^{N} \alpha_j^2(t) \boldsymbol{P}_{jj} + 2 \sum_{k>j} \alpha_j(t) \alpha_k(t) P_{jk}$, then the origin $x = 0$ is a globally asymptotically stable equilibrium point of system (1) for the bounded rates of parametric variation (2).

Proof Consider the quadratically parameter-dependent Lyapunov function $\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}} \boldsymbol{P}(\boldsymbol{\alpha}) \boldsymbol{x}$ with

$$\boldsymbol{P}(\boldsymbol{\alpha}(t)) = \sum_{j=1}^{N} \alpha_j^2(t) \boldsymbol{P}_{jj} + 2 \sum_{k>j} \alpha_j(t) \alpha_k(t) \boldsymbol{P}_{jk}$$
$$i, \ j = 1, 2, \ldots, N$$

where $\boldsymbol{P}(\boldsymbol{\alpha}(t)) = \boldsymbol{P}(\boldsymbol{\alpha}(t))^{\mathrm{T}}$. It is clear that $\boldsymbol{P}(\boldsymbol{\alpha}(t))$ is a positive definite parameter dependent Lyapunov matrix.

The time-derivative $\dot{\boldsymbol{v}}(t)$ can be given as

$$\dot{\boldsymbol{v}}(t) = \boldsymbol{x}^{\mathrm{T}} (\boldsymbol{A}(\boldsymbol{\alpha}(t))^{\mathrm{T}} \boldsymbol{P}(\boldsymbol{\alpha}(t)) + \boldsymbol{P}(\boldsymbol{\alpha}(t)) \boldsymbol{A}(\boldsymbol{\alpha}(t)) + \dot{\boldsymbol{P}}(\boldsymbol{\alpha}(t))) \boldsymbol{x} = \boldsymbol{x}^{\mathrm{T}} \boldsymbol{Q} \boldsymbol{x}$$

Observing that

$$\dot{\boldsymbol{P}}(\boldsymbol{\alpha}(t)) = 2 \sum_{k=1}^{N} \alpha_k^2(t) \Big(\sum_{i=1}^{N} \alpha_i(t) \dot{\alpha}_i(t) \boldsymbol{P}_{ii} + \sum_{i=1}^{N-1} \sum_{j=2}^{N} \alpha_i(t) \alpha_j(t) P_{ij} \Big) + 2 \sum_{p=1}^{N-1} \sum_{q=1+p}^{N} \alpha_p(t) \dot{\alpha}_q(t) \Big(\sum_{i=1}^{N} \alpha_i(t) \dot{\alpha}_i(t) \boldsymbol{P}_{ii} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \dot{\alpha}_i(t) \alpha_j(t) P_{ij} \Big)$$

we have

$$\begin{aligned}\boldsymbol{Q}(\boldsymbol{\alpha}(t)) = &\sum_{j=1}^{N} \alpha_j^3(t) \Big(\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{jj} + \boldsymbol{P}_{jj} \boldsymbol{A}_j + 2 \sum_{l=1}^{N} \dot{\alpha}_l(t) \boldsymbol{P}_{lj} \Big) + \\ &\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \alpha_j^2(t) \alpha_k(t) \Big(\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jk} \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_{jj} + \boldsymbol{P}_{jj} \boldsymbol{A}_k + 2 \sum_{l=1}^{N} \alpha_l(t) (\dot{\boldsymbol{P}}_{lk} + 2\boldsymbol{P}_{lj}) \Big) + \\ &\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \alpha_j(t) \alpha_k^2(t) \Big(\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{kk} + \boldsymbol{P}_{kk} \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jjk} \boldsymbol{A}_k + 2 \sum_{l=1}^{N} \alpha_l(t) (\dot{\boldsymbol{P}}_{lj} + 2\boldsymbol{P}_{lk}) \Big) + \\ &2 \sum_{j=1}^{N-2} \sum_{k>j}^{N-1} \sum_{l>k}^{N} \alpha_j(t) \alpha_k(t) \alpha_l(t) \Big(\boldsymbol{A}_j^{\mathrm{T}} \boldsymbol{P}_{kl} + \boldsymbol{P}_{kl} \boldsymbol{A}_j + \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jk} \boldsymbol{A}_k + \boldsymbol{A}_l^{\mathrm{T}} \boldsymbol{P}_{jk} + \boldsymbol{P}_{jk} \boldsymbol{A}_k + 2 \sum_{p=1}^{N} \sum_{q=1}^{N} \dot{\alpha}_p(t) \boldsymbol{P}_{pq} \Big)\end{aligned} \qquad (9)$$

According to $\sum_{j=1}^{N} \alpha_j(t) = 1, \ \sum_{j=1}^{N} \dot{\alpha}_j(t) = 0$, and $\dot{\alpha}_N(t) = -\sum_{j=1}^{N-1} \dot{\alpha}_j(t)$, the term $\sum_{l=1}^{N} \dot{\alpha}_l(t) (\boldsymbol{P}_{lk} + 2\boldsymbol{P}_{lj})$ in $Q(\alpha(t))$ can be replaced by $\sum_{l=1}^{N-1} \dot{\alpha}_l(t) (\boldsymbol{P}_{ik} - \boldsymbol{P}_{Nk} + 2(\boldsymbol{P}_{ij} - \boldsymbol{P}_{Nj}))$. Taking into account the constraints $|\dot{\alpha}_j(t)| < \rho_j$, conditions (2) are sufficient to guarantee that Eq. (9) is a negative definite to all $\sum_{j=1}^{N} \alpha_j(t) = 1$.

A first remark on the conditions of Theorem 1 is that note scalar variables are used in the tests (5) to (8) than in the tests of Lemma 2, which can provide less conservative evaluations for stability at the price of a slightly higher numerical complexity. Notice that if the conditions of Lemma 1 are feasible, then $\boldsymbol{P}_1, \boldsymbol{P}_2, \dots, \boldsymbol{P}_N$ and $\boldsymbol{P}_{ij} = \frac{\boldsymbol{P}_i + \boldsymbol{P}_j}{2}$ yield a feasible LMIs of Theorem 1. Actually, in this case, the LMIs of Theorem 1 can be obtained as a linear combination of (3) and (4). The conditions of Theorem 1 can provide less conservative evaluations for stability than the conditions given in Ref. [13], which is also based on quadratically parameter-dependent Lyapunov functions, as shown in the sequel by numerical examples.

In the case $N=2$ (two vertices) of Theorem 1, a simpler formulation can be obtained as

$$\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_{11} + \boldsymbol{P}_{11}\boldsymbol{A}_1 \pm 2\boldsymbol{\rho}_1(\boldsymbol{P}_{11} - \boldsymbol{P}_{21}) < 0$$
$$\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_{12} + \boldsymbol{P}_{12}\boldsymbol{A}_1 + \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{P}_{11} + \boldsymbol{P}_{11}\boldsymbol{A}_2 \pm 2\rho_1(2\boldsymbol{P}_{11} - \boldsymbol{P}_{21} - \boldsymbol{P}_{22}) < 0$$
$$\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_{22} + \boldsymbol{P}_{22}\boldsymbol{A}_1 + \boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{P}_{12} + \boldsymbol{P}_{12}\boldsymbol{A}_2 \pm 2\rho_1(\boldsymbol{P}_{11} + \boldsymbol{P}_{12} - 2\boldsymbol{P}_{22})) < 0$$
$$\boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{P}_{22} + \boldsymbol{P}_{22}\boldsymbol{A}_2 \pm 2\rho_1(\boldsymbol{P}_{12} - \boldsymbol{P}_{22}) < 0$$

Following the ideas of square matricial representations (SMR) of homogeneous forms and Lemma 2, a new sufficient condition based on homogeneous parameter-dependent Lyapunov functions is stated in the next theorem.

Theorem 2 Let $\boldsymbol{A}_{j,\{m\}}$ and $\boldsymbol{P}_{ij,\{m\}}$ denote the extended matrix of $\boldsymbol{A}_j$ and $\boldsymbol{P}_{ij}$, respectively. Then, the system (1) is asymptotically stable if $\boldsymbol{P}_{ij\text{ exists},\{m\}} > 0$ exists, such that the following set of LMIs is satisfied:

$$\boldsymbol{A}_{j,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{jj,\{m\}} + \boldsymbol{P}_{jj,\{m\}}\boldsymbol{A}_{j,\{m\}} + 2\sum_{i=1}^{N-1} \pm\rho_i(\boldsymbol{P}_{ij,\{m\}} - \boldsymbol{P}_{Nj,\{m\}}) < 0 \tag{10}$$

$$\boldsymbol{A}_{j,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{jk,\{m\}} + \boldsymbol{P}_{jk,\{m\}}\boldsymbol{A}_{j,\{m\}} + \boldsymbol{A}_{k,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{jj,\{m\}} + \boldsymbol{P}_{jj,\{m\}}\boldsymbol{A}_{k,\{m\}} + 2\sum_{i=1}^{N-1} \pm\rho_i(\boldsymbol{P}_{ik,\{m\}} - \boldsymbol{P}_{Nk,\{m\}} + 2(\boldsymbol{P}_{ij,\{m\}} - \boldsymbol{P}_{Nj,\{m\}})) < 0 \tag{11}$$

$$\boldsymbol{A}_{j,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{kl,\{m\}} + \boldsymbol{P}_{kl,\{m\}}\boldsymbol{A}_{j,\{m\}} + \boldsymbol{A}_{k,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{jl,\{m\}} + \boldsymbol{P}_{jl,\{m\}}\boldsymbol{A}_{k,\{m\}} + \boldsymbol{A}_{l,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{jk,\{m\}} + \boldsymbol{P}_{jk,\{m\}}\boldsymbol{A}_{l,\{m\}} + 2\sum_{i=1}^{N-1}\sum_{q=1}^{N} \pm\rho_i(\boldsymbol{P}_{iq,\{m\}} - \boldsymbol{P}_{Nq,\{m\}}) < 0 \tag{12}$$

$$\boldsymbol{A}_{j,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{kk,\{m\}} + \boldsymbol{P}_{kk,\{m\}}\boldsymbol{A}_{j,\{m\}} + \boldsymbol{A}_{k,\{m\}}^{\mathrm{T}}\boldsymbol{P}_{jk,\{m\}} + \boldsymbol{P}_{jk,\{m\}}\boldsymbol{A}_{k,\{m\}} + 2\sum_{i=1}^{N-1} \pm\rho_i(\boldsymbol{P}_{ij,\{m\}} - \boldsymbol{P}_{Nj,\{m\}} + 2(\boldsymbol{P}_{ik,\{m\}} - \boldsymbol{P}_{Nk,\{m\}})) < 0 \tag{13}$$

where $k=2,3,\dots,N$; $l=2,3,\dots,N$.

Proof According to Lemma 1, let $v_{2m}(\boldsymbol{x}) = \boldsymbol{x}^{\{m\}\mathrm{T}} \cdot \boldsymbol{P}(\alpha)_{\{m\}}\boldsymbol{x}^{\{m\}}$ be the HPLF of the system (1).

By differentiating $v_{2m}(\boldsymbol{x})$ along the trajectories of the system and exploiting the properties of SMR, we can obtain

$$\dot{v}_{2m}(t) = \boldsymbol{x}^{\{m\}\mathrm{T}}(\boldsymbol{A}(\boldsymbol{\alpha}(t))_{\{m\}}^{\mathrm{T}}\dot{\boldsymbol{P}}(\boldsymbol{\alpha}(t))_{\{m\}} + \dot{\boldsymbol{P}}(\boldsymbol{\alpha}(t))_{\{m\}}\boldsymbol{A}(\boldsymbol{\alpha}(t))_{\{m\}} + \dot{\boldsymbol{P}}(\boldsymbol{\alpha}(t))_{\{m\}})\boldsymbol{x}^{\{m\}} = \boldsymbol{x}^{\{m\}\mathrm{T}}\boldsymbol{Q}(\boldsymbol{\alpha}(t))\boldsymbol{x}^{\{m\}}$$

With a similar approach as proof of Theorem 1, we can obtain (10), (11), (12) and (13).

Several remarks can be given on the results above.

Remark 1 The conditions of Theorem 2 are used more for scalar variables in the tests (10) to (13) than in the tests of Lemma 2 and Theorem 1, which can provide less conservative evaluations of stability at the price of a slightly higher numerical complexity.

Remark 2 The family of HPLFs in the case $m=1$, which have been considered by Theorem 1, can be reduced to quadratic Lyapunov functions with affine parameter dependence. The condition provided by Theorem 2 is based on the SMR of homogeneous polynomial forms.

3 Numberical Example

Example 1 The following second-order linear differential equation is considered:

$$\ddot{\boldsymbol{x}}(t) + \varsigma\dot{\boldsymbol{x}}(t) + (\omega^2 + \delta^2 p(t))\boldsymbol{x}(t) = 0 \tag{14}$$

where ω, ς and δ are the parameters and $\alpha(t)$ is a time-varying parameter not exactly specified by such that $|p(t)| \leqslant 1$ and $|\dot{\boldsymbol{\alpha}}(t)| \leqslant \boldsymbol{v}$ for all $t>0$, which was studied in Refs. [14 - 15]. It is interesting to see that this differential equation reduces to the celebrated Mathieu's equation with damping for $\boldsymbol{\alpha}(t) = \cos\omega t$. In order to rewrite (14) in the form of (1), it suffices to set $N=2$, defining the parameter vector $\boldsymbol{\alpha}(t) := \begin{bmatrix} 0.5 + 0.5p(t) \\ 0.5 - 0.5p(t) \end{bmatrix}$, $\forall t \geqslant 0$, the extreme matrices $\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ -(\omega^2 + \delta^2) & -\varsigma \end{bmatrix}$, $\boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 \\ -(\omega^2 - \delta^2) & -\varsigma \end{bmatrix}$, and $r = \omega/2$. Our goal is to determine the region of the plane $\omega \times \delta$ with $0 \leqslant \omega \leqslant 4$ and $0 \leqslant \delta \leqslant 1$ such that global asymptotical stability is preserved. For numerical calculations, we have considered $\omega = 1$ and a small damping represented by $\varsigma = 0.05$. Since $\boldsymbol{H} = [h_1 \quad h_2] = \begin{bmatrix} -r & r \\ r & -r \end{bmatrix}$, then Theorem 2 states that for each pair (ω, δ) satisfying the LMIs (10) to (13), the equilibrium solution of the differential equation (1) is globally asymptotically stable for all $\boldsymbol{\alpha}(t)$. Clearly, due to the fact that $\boldsymbol{\alpha}(t) = \cos\omega t$ is a feasible trajectory, the region of the plane (ω, δ) determined from the stability condition of Theorem 2 is a subset of the region of global asymptotical stability of the Mathieu equation.

Fig. 1 shows the regions of stability below of each curve provided by Ref. [14] (dashed line), and Theorem 2 in the case $m=2$ (solid line), respectively, calculated with an LMI solver that verifies the feasibility of each stability condition. The same example was also considered in Ref. [15], which gives the limits of stability corresponding to the region of the plane ($\omega = 4$ and $\delta = 0.23$). The limits of stability corresponding to the region of the plane are: $\omega = 4$ and $\delta = 0.27$. It is to be noted that an important improvement is when the present result is compared with Ref. [15].

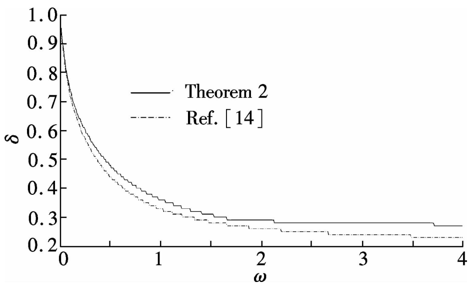

Fig. 1 Stability regions

4 Conclusion

In this paper, we introduce some new stability conditions for time-varying continuous-time polytopic systems using homogeneous Lyapunov functions. With respect to previous work on this class of Lyapunov functions, better results have been obtained by exploiting a complete parameterization of homogeneous forms of a given degree. Compared with some previous stability conditions, the main results via HPDFs in this paper have less conservatism. An numerical example is proposed to show less conservativeness with some existing results.

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基于参数依赖齐次多项式的时变系统稳定性分析

张化生[1,2,3]　　张侃健[1,2]

([1]东南大学复杂工程系统测量与控制教育部重点实验室,南京 210096)

([2] 东南大学自动化学院,南京 210096)

([3]聊城大学数学科学学院,聊城 252000)

摘要:基于齐次多项式 Lyapunov 函数这一新工具研究了时变不确定系统鲁棒稳定性问题. 针对常见的含参数时变且有界连续可微线性系统的最大稳定区域问题,首先构造常用的参数依赖二次 Lyapunov 函数,进而给出一个时变系统稳定的充分条件. 然后,通过构造适合的参数依赖齐次 Lyapunov 函数,并利用齐次多项式矩阵表示方法,最终以线性不等式的形式给出系统全局渐近稳定的一个充分条件. 数值仿真结果表明齐次 Lyapunov 函数方法得到的结论对于某些系统比之前类似文献具有更小的保守性.

关键词:线性时变系统;多面体不确定性;鲁棒稳定性;线性不等式

中图分类号:TP202.1;TP271.7