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Sensitivity analysis for stochastic user equilibrium with elastic demand assignment model

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Abstract: This paper puts forward a rigorous approach for a sensitivity analysis of stochastic user equilibrium with the elastic demand (SUEED) model. First, proof is given for the existence of derivatives of output variables with respect to the perturbation parameters for the SUEED model. Then by taking advantage of the gradient-based method for sensitivity analysis of a general nonlinear program, detailed formulae are developed for calculating the derivatives of designed variables with respect to perturbation parameters at the equilibrium state of the SUEED model. This method is not only applicable for a sensitivity analysis of the logit-type SUEED problem, but also for the probit-type SUEED problem. The application of the proposed method in a numerical example shows that the proposed method can be used to approximate the equilibrium link flow solutions for both logit-type SUEED and probit-type SUEED problems when small perturbations are introduced in the input parameters.

Key words: network modeling; stochastic user equilibrium; elastic demand; sensitivity analysis; first-order approximation

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Sensitivity analysis is defined as a technique used to determine how the different values of independent variables impact a particular dependent variable under a given set of assumptions. In network modeling, it is constantly used for estimating the changes of objectives at macroscopic levels (i. e., network performance) caused by the variations in the objectives in macrocosmic levels (i. e., signal splits). The possible application of this method includes, but is not just restricted to, the first-order equilibrium solution approximation, critical parameter identification, parameter uncertainty analysis and effectiveness diagnosis.

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The research on sensitivity analysis for traffic assignment models can be traced back to the work done by Hall[1]. He investigated the direction of change when perturbations are added to the inputs of a user equilibrium traffic assignment model. Tobin and Friesz[2] overcame the problem of the non-uniqueness of the user equilibrium path flows by introducing an equivalent restricted program that has the desired uniqueness properties. Following their work, Yang[3] derived a gradient-based sensitivity analysis formula for network equilibrium problems with elastic demand. However, this method has several significant deficiencies that it is only applicable to non-degenerated point (equilibrium flows should be strictly positive), and the assumption that the link travel cost increases monotonously with respect to the link flow is also stronger than necessary. A detailed description of those deficiencies and the corresponding technique for improvement can be found in Ref. [4]. For the sensitivity analysis of logit-based stochastic user equilibrium (SUE) problem, Ying and Miyagi[5] formulated a computationally efficient link-based algorithm by adopting Dial's algorithm[6]. Clark and Watling[7] proposed a more generous method for sensitivity analysis of the SUE assignment model, which is actually a direct application of the first-order sensitivity approximation method for a general nonlinear program proposed by Fiacco[8]. The method can not only observe the changes of the equilibrium link flows with respect to uncertainty parameters at logit-based SUE but also can be equally applied for probit-based SUE.

The aforementioned literature mainly focused on addressing the sensitivity analysis problem for UE and SUE assignment models as well as some combination models. Based on our knowledge, no research has been conducted on implementing the sensitivity analysis of SUE with the elastic demand (SUEED) traffic assignment model. This paper aims to formulate a method for a sensitivity analysis of SUEED models with the gradient-based method.

1 SUEED Problem

The SUEED model assumes that, in the equilibrium state, route or link choices should be such that an SUE is formed, and, meanwhile, the demand between each O-D pair (used in the SUE assignment) must be consistent with travel costs between each O-D pair[9]. Maher et al. [9] proposed an equivalent unconstrained mathematical program for the SUEED problem, which is formulated as

$$\min_{q,v} Z(\boldsymbol{q},\boldsymbol{v}) = \sum_a v_a t_a(v_a) - \sum_a \int_0^{v_a} t_a(x)\mathrm{d}x - \sum_r \sum_s S_{rs} D_{rs}(S_{rs}) + \sum_r \sum_s D_{rs}^{-1}(q_{rs}) D_{rs}(S_{rs}) + \sum_r \sum_s \int_0^{q_{rs}} D_{rs}^{-1}(q)\mathrm{d}q - \sum_r \sum_s q_{rs} D_{rs}^{-1}(q_{rs}) \tag{1}$$

where a is the link in the network; r is the origin node; s is the destination node; v_a is the link flow on link a; t_a is the travel cost on link a; q_{rs} is the O-D demand between O-D pair r-s; S_{rs} is the expected perceived O-D travel time between O-D pair r-s; $D_{rs}(S_{rs})$ is the demand function between O-D pair r-s, and it is a function of S_{rs} between O-D pair r-s; $D_{rs}^{-1}(q_{rs})$ is the inverse function of the O-D demand function.

2 Sensitivity Analysis for SUEED Assignment Problem

2.1 Sensitivity analysis for general nonlinear programming

Fiacco[8] made a significant contribution to the sensitivity analysis of nonlinear programming.

$$\begin{aligned} &\min_x z(\boldsymbol{x}, \varepsilon) \\ \text{s. t. } &p(\varepsilon) \\ &g_i(\boldsymbol{x}, \varepsilon) \geqslant 0 \qquad i = 1, 2, \ldots, m \\ &h_j(\boldsymbol{x}, \varepsilon) = 0 \qquad j = 1, 2, \ldots, n \end{aligned}$$

He proved that for any nonlinear programming with above form $p(\varepsilon)$, if at an optimal point $(\boldsymbol{x}^*, 0)$, the following conditions are held: 1) The objective function and the constraints are twice continuous differentiable in the neighborhood of $(\boldsymbol{x}^*, 0)$; 2) The optimal solution has a strict local uniqueness (the second-order sufficient condition); 3) Strict complimentary slackness condition is satisfied; 4) The gradient $\nabla_x g_i(\boldsymbol{x}, \varepsilon)$ ($g_i(\boldsymbol{x}, \varepsilon) = 0$, $u_i > 0$) and $\nabla_x h_j(\boldsymbol{x}, \varepsilon)$, $\forall j$ are linearly independent, then the designed variables are directional derivable with respect to perturbation parameter ε at $(\boldsymbol{x}^*, 0)$, and the formulation can be obtained as

$$[\nabla_\varepsilon \boldsymbol{x} \quad \nabla_\varepsilon \boldsymbol{u} \quad \nabla_\varepsilon \boldsymbol{w}]^{\mathrm{T}} = -\boldsymbol{M}(\varepsilon)^{-1} \boldsymbol{N}(\varepsilon) \tag{2}$$

where u_i is the Lagrangian multiplier associated with the i-th inequality constraint $g_i(\boldsymbol{x}, \varepsilon)$; $\nabla_\varepsilon \boldsymbol{x}$ is the vector of the derivative of solutions with respect to parameter ε (dimension n); $\nabla_\varepsilon \boldsymbol{u}$ is the m-dimentional vector of derivative of Lagrangian multipliers with respect to parameter ε; $\boldsymbol{w}$ is the vector of Lagrangian equality multipliers corresponding to equality constraints in $p(\varepsilon)$; $\nabla_\varepsilon \boldsymbol{w}$ is the n-dimentional vector of the derivative of Lagrangian equality multipliers with respect to parameter ε. The close form of matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ (as function of ε) in Eq. (2) can be found in Ref. [7].

2.2 Sensitivity analysis method for SUEED problem

Before developing the formulae for the sensitivity analysis of the SUEED model, we assume that $t_a(\cdot)$ in SUEED problem (1) is a monotonic increasing function and once continuously differentiable with respect to link flows. With this assumption, it is very clear that condition 1) in section 2.1 is satisfied. Maher et al.[9] proved that the Hessian matrix of the SUEED function is a positive definite; therefore, condition 2) is met. Since the SUEED model is an unconstraint optimization problem, the solution will automatically satisfy the flow conservation and nonnegativity of route flow constraints. Therefore, condition 3) and condition 4) are then satisfied simultaneously. From the above discussions, it can be seen that Fiacco's[8] method is applicable for calculating the sensitivity of the output parameters in the SUEED with respect to input parameters. Since SUEED is formulated as an unconstrained nonlinear program, the terms in the matrices of $\boldsymbol{M}$ and $\boldsymbol{N}$ will be reduced to $\nabla^2 z(\boldsymbol{q}, v)$ for matrices $\boldsymbol{M}$ and $\nabla_{x\varepsilon}^2 z(\boldsymbol{q}, \boldsymbol{v})$ for matrices $\boldsymbol{N}$, respectively. Here, $\boldsymbol{x}$ denotes the vector of all input variables; $\boldsymbol{q}$ is a vector of all O-D demands and $\boldsymbol{v}$ is a vector of all link flows. In the SUEED model, the solution parameters are links that flow together with O-D demands; therefore, we have $\boldsymbol{x} = (\boldsymbol{q}, \boldsymbol{v})$. The explicit expressions for the second-order derivatives of $Z(\boldsymbol{q}, \boldsymbol{v})$ with respect to the O-D demands and link flows formulated by Maher et al.[9] are given by

$$\boldsymbol{M}(\varepsilon) = \nabla_x^2 Z = \begin{bmatrix} \left[\dfrac{\partial^2 Z}{\partial q_{rs} \partial q_{rs}}\right] & \left[\dfrac{\partial^2 Z}{\partial v_b \partial q_{rs}}\right] \\ \left[\dfrac{\partial^2 Z}{\partial q_{rs} \partial v_b}\right] & \left[\dfrac{\partial^2 Z}{\partial v_b \partial v_a}\right] \end{bmatrix} = \begin{bmatrix} \nabla_q^2 Z & \nabla_{q,v}^2 Z \\ \nabla_{v,q}^2 Z & \nabla_v^2 Z \end{bmatrix} = \begin{bmatrix} \nabla_q^2 Z & \nabla_{q,v}^2 Z \\ \nabla_{q,v}^2 Z^{\mathrm{T}} & \nabla_v^2 Z \end{bmatrix} \tag{3}$$

where

$$\nabla_q^2 Z = \operatorname{diag}\left[-\frac{\mathrm{d}D_{rs}^{-1}}{\mathrm{d}q_{rs}}\right] \tag{4}$$

$$\nabla_{q,v}^2 Z = \nabla_v \boldsymbol{S} \tag{5}$$

$$\nabla_v^2 Z = \nabla_v \boldsymbol{t} + \sum_{rs} q_{rs} [(\nabla_v \boldsymbol{t} \cdot \Delta^{rs})(-\nabla_c \boldsymbol{P}^{rs})(\nabla_v \boldsymbol{t} \cdot \Delta^{rs})^{\mathrm{T}}] + 2(\nabla_v \boldsymbol{S})^{\mathrm{T}} \left(\operatorname{diag}\left[-\frac{\mathrm{d}D_{rs}}{\mathrm{d}S_{rs}}\right]\right)(\nabla_v \boldsymbol{S}) \tag{6}$$

where $\boldsymbol{S}$ is the vector of all expected perceived O-D travel times; $\boldsymbol{t}$ is the vector of all link travel times. Assume that ε is a perturbed parameter in the SUEED problem. Matrix $\boldsymbol{N}(\varepsilon)$ in Eq. (2) is then extended as

$$\boldsymbol{N}(\varepsilon) = \nabla_{x\varepsilon} Z = \frac{\partial \nabla_x Z(\boldsymbol{q}, \boldsymbol{v})}{\partial \varepsilon} = \left[\frac{\partial Z}{\partial \boldsymbol{q} \partial \varepsilon} \quad \frac{\partial Z}{\partial \boldsymbol{v} \partial \varepsilon}\right]^{\mathrm{T}} \tag{7}$$

Note that

$$\frac{\partial Z}{\partial q_{rs} \partial \varepsilon} = \frac{\partial D_{rs}(S_{rs})}{\partial S_{rs}} \sum_{k \in R_{rs}} \left(\frac{S_{rs}}{c_k} \frac{\partial c_k^{rs}}{\partial \varepsilon}\right) \frac{\mathrm{d}D_{rs}^{-1}}{\mathrm{d}q_{rs}} =$$

$$\frac{\partial D_{rs}(S_{rs})}{\partial S_{rs}}\sum_{k\in R_{rs}}\left(P_k^{rs}\frac{\partial c_k^{rs}}{\partial \varepsilon}\right)\frac{\mathrm{d}D_{rs}^{-1}}{\mathrm{d}q_{rs}} \tag{8}$$

where c_k^{rs} denotes the route travel cost on the k-th route between the O-D pair r-s; R_{rs} is the set of all routes between the O-D pair r-s; P_k^{rs} denotes the probability that a traveler from r to s chooses path k. Eq. (8) can be rewritten in a vector form as

$$\nabla_{q\varepsilon}Z = \mathrm{diag}\left[\frac{\mathrm{d}D_{rs}}{\mathrm{d}S_{rs}}\right]\left(\sum_{k\in R_{rs}}(\boldsymbol{P}^{rs}\,\nabla_{\varepsilon}c_k^{rs})\right)_{r,s}\nabla_{\boldsymbol{q}}\boldsymbol{D}^{-1}(\boldsymbol{q}) \tag{9}$$

where $\boldsymbol{P}^{rs}$ is the vector of all the route choice probabilities between the O-D pair r-s; $\boldsymbol{D}^{-1}(\boldsymbol{q})$ denotes the vector of all inverse functions of the O-D demand function. The mixed derivatives of $Z(\boldsymbol{q},\varepsilon)$ with respect to link flow v_b and perturbed parameter ε is

$$\frac{\partial^2 Z}{\partial v_b\partial\varepsilon} = v_b\frac{\mathrm{d}^2 t_b}{\mathrm{d}v_b\mathrm{d}\varepsilon} - \sum_{r,s}\left(\frac{\partial^2 S_{rs}}{\partial v_b\partial\varepsilon}D_{rs}(S_{rs}) + \frac{\partial S_{rs}}{\partial v_b}\frac{\partial D_{rs}(S_{rs})}{\partial\varepsilon}\right) - \sum_{r,s}\frac{\partial S_{rs}}{\partial\varepsilon}\frac{\partial D_{rs}(S_{rs})}{\partial v_b} + \sum_{r,s}(D_{rs}^{-1}(q_{rs}) - S_{rs})\frac{\partial^2 D_{rs}(S_{rs})}{\partial v_b\partial\varepsilon} \tag{10}$$

Simplifying Eq. (10) yields

$$\frac{\partial^2 Z}{\partial v_b\partial\varepsilon} = -\sum_{r,s}q_{rs}\frac{\mathrm{d}t_b}{\mathrm{d}v_b}\sum_{k\in R_{rs}}\delta_{bk}^{rs}\sum_{l\in R_{rs}}\frac{\partial P_k^{rs}}{\partial c_l^{rs}}\frac{\mathrm{d}c_l^{rs}}{\mathrm{d}\varepsilon} - 2\sum_{r,s}\sum_{r\in R_{rs}}\left(\frac{\partial S_{rs}}{\partial c_l^{rs}}\frac{\mathrm{d}c_l^{rs}}{\mathrm{d}\varepsilon}\right)\frac{\partial D_{rs}(S_{rs})}{\partial S_{rs}}\frac{\partial S_{rs}}{\partial v_b} \tag{11}$$

where δ_{bk}^{rs} is the link-route indicator, $\delta_{bk}^{rs}=1$ if b is a link on route k; else $\delta_{bk}^{rs}=0$. Eq. (11) can be formulated in a vector form as

$$\nabla_{x\varepsilon}Z = \sum_{r,s}q_{rs}[(\Delta_{\boldsymbol{v}}\boldsymbol{t}\cdot\Delta^{rs})(-\nabla_{\boldsymbol{c}}\boldsymbol{P}^{rs})\nabla_{\varepsilon}\boldsymbol{c}] + 2(\boldsymbol{P}^{rs}\,\nabla_{\varepsilon}\boldsymbol{c})_{r,s}\left(\mathrm{diag}\left[-\frac{\mathrm{d}D_{rs}(S_{rs})}{\mathrm{d}S_{rs}}\right]\right)(\nabla_{\boldsymbol{v}}\boldsymbol{S}) \tag{12}$$

From the above discussions, we can calculate $\nabla_x^2 Z$ by Eq. (4) to Eq. (6) and $\frac{\partial Z}{\partial\boldsymbol{x}\partial\varepsilon}$ by Eq. (12), and thus the derivatives of equilibrium link flows and O-D demands with respect to signal splits can be obtained by Eq. (2).

3 Numerical Application

The example network shown in Fig. 1 is used to demonstrate how to apply the sensitivity analysis method. This network has two O-D pairs, seven links and six nodes, of which nodes E and F are signal-controlled intersections. There are three paths for O-D pair AB, that is, route 1: AEB; route 2: AFB and route 3: $AEFB$, and only one path, $CEFD$ for O-D pair CD.

The current O-D demands are assumed to be 18 veh/min and 6 veh/min for O-D pairs AB and CD, respectively. The link travel cost and its corresponding input data are

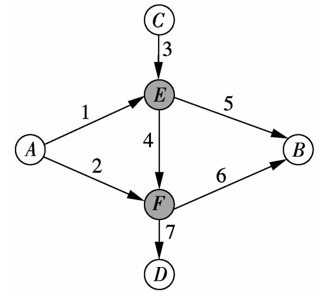

Fig. 1 The example road network

summarized in Tab. 1. Intersections E and F are supposed to be controlled by two independent signal splits, λ_1 and λ_2. The elastic demand functions for O-D pairs AB and CD in this numerical example are specified as

$$q_{AB} = D_{AB}(S_{AB}) = 50\exp(-0.5S_{AB}) \tag{13}$$

$$q_{CD} = D_{CD}(S_{CD}) = 30\exp(-0.2S_{CD}) \tag{14}$$

Tab. 1 Input data to the example network

Link number	1	2	3	4	5	6	7
Free-flow time t_a^0	2.0	1.0	2.0	3.0	1.0	2.0	1.0
Saturation flow s_a	24	30	30	35	24	30	30
Link travel cost	$t_a(v_a,\lambda_a)=t_a^0\left[1.0+0.5\left(\frac{v_a}{\lambda_a s_a}\right)^2\right]$						

3.1 Sensitivity of logit-type SUEED problem

The signal splits are set as $\lambda_1=\lambda_2=\lambda_3=\lambda_4=0.5$ to start a test. Let θ be the discrete parameter in the logit-type SUEED problem. The numerical results including equilibrium link flows, O-D demands and the Jacobian matrix of route choice probabilities calculated at $\theta=1$ are presented as

$$\boldsymbol{v}=\begin{bmatrix}6.507\\7.425\\7.960\\8.044\\6.424\\7.508\\7.960\end{bmatrix},\quad \boldsymbol{q}=\begin{bmatrix}13.932\\7.960\end{bmatrix}$$

$$\boldsymbol{P}_c^{AB}=\begin{bmatrix}-0.249 & 0.246 & 0.003\\ 0.246 & -0.249 & 0.003\\ 0.003 & 0.003 & -0.006\end{bmatrix},\quad \boldsymbol{P}_c^{CD}=[1]$$

Assume that λ_1 is a perturbed parameter in the road network, and then according to Eqs. (3), (4), (5) and (12), matrix $\boldsymbol{M}$ and matrix $\boldsymbol{N}$ can be calculated correspondingly. Then using Eq. (2), the sensitivity of the equilibrium link flow and O-D demand solutions with respect to signal split λ_1 is obtained as

$$\begin{bmatrix}\nabla_{\lambda_1}\boldsymbol{q}\\ \nabla_{\lambda_1}\boldsymbol{v}\end{bmatrix} = -\boldsymbol{M}(\lambda_1)^{-1}\boldsymbol{N}(\lambda_1) = \begin{bmatrix}2.759 & -0.005 & 3.838\\ -1.079 & -0.005 & 0.048\\ 3.784 & -1.025 & -0.005\end{bmatrix}^{\mathrm{T}} \tag{15}$$

Eq. (15) has plentiful physical meanings and can be used to develop a first-order approximation of the perturbed solution for small changes in signal splits λ_1, given as

$$\begin{bmatrix} \boldsymbol{q}(\lambda_1+\Delta\lambda_1) \\ \boldsymbol{v}(\lambda_1+\Delta\lambda_1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{q}(\lambda_1) \\ \boldsymbol{v}(\lambda_1) \end{bmatrix} + \begin{bmatrix} \nabla_{\lambda_1}\boldsymbol{q} \\ \nabla_{\lambda_1}\boldsymbol{v} \end{bmatrix}\Delta\lambda_1 \qquad (16)$$

Fig. 2 describes the comparison of estimated link flow calculated by Eq. (16) and the exact link flow. The linear nature of the approximation procedure given here is clearly evident. The further the signal split from the initial solution value, the greater the deviation between the exact and approximate solutions.

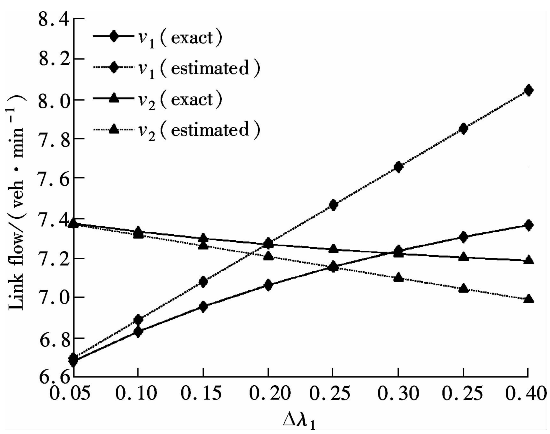

Fig. 2 Exact and approximated equilibrium solutions for the logit-type SUEED (signal split change)

3.2 Sensitivity of probit-type SUEED assignment problem

For the probit-type model, the link travel time is assumed to be normally distributed with a mean equal to the measured link travel time and with variance that is proportional to the measured link travel time. In other words,

$$T_a \sim N(t_a, \alpha t_a)$$

where α is the variance of the perceived travel time over a road segment of unit travel time. The covariance of route travel time then will be subject to a multivariate normal distribution as follows:

$$C_{rs} \sim MVN(\boldsymbol{t}\Delta^{rs}, \alpha\Delta^{rs}\boldsymbol{t}\Delta^{rs\mathrm{T}})$$

The equilibrium link flows, O-D demands and the Jacobian of route choice probabilities calculated with the MSA method at $\alpha = 1$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.5$ are specified as

$$\boldsymbol{v} = \begin{bmatrix} 7.445 \\ 8.292 \\ 7.957 \\ 8.108 \\ 7.298 \\ 8.443 \\ 7.957 \end{bmatrix}, \quad \boldsymbol{q} = \begin{bmatrix} 15.741 \\ 7.957 \end{bmatrix}$$

$$\boldsymbol{P}_c^{AB} = \begin{bmatrix} -0.211 & 0.203 & 0.008 \\ 0.204 & -0.216 & 0.013 \\ 0.008 & 0.013 & -0.021 \end{bmatrix}, \quad \boldsymbol{P}_c^{CD} = [1]$$

Assume that there is a perturbation in the input parameter λ_1. Similarly, by using the derivatives of Eq. (2), the sensitivity of the link equilibrate solutions with respect to input parameter λ_1 can be calculated. The linear approximation to the solution for a small change in signal split λ_1 is thus presented as

$$\begin{bmatrix} q_{AB} \\ q_{CD} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} 15.741 \\ 7.957 \\ 7.449 \\ 8.292 \\ 7.957 \\ 8.108 \\ 7.298 \\ 8.443 \\ 7.957 \end{bmatrix} + \begin{bmatrix} 3.898 \\ -0.033 \\ 4.944 \\ -1.078 \\ -0.024 \\ 0.221 \\ 4.709 \\ -0.843 \\ -0.014 \end{bmatrix}\Delta\lambda_1 \qquad (17)$$

Eq. (17) denotes that an increase in signal splits λ_1 will lead to different change patterns in the output parameters. The demand between O-D pair *AB* and the equilibrium flow on link 1 and link 5 will benefit most from the increased signal splits λ_1 while the equilibrium flow on link 2 and link 6 are reduced significantly.

According to Eq. (17), the exact and estimated equilibrium route flow solutions are drawn in Fig. 3. As can be seen, the estimated route flows are very close to corresponding exact values, which implies significant potential use in practice. If high accuracy is not requested, the sensitivity analysis method will be a good alternative to approximate the equilibrium solutions when changes are introduced to the network input parameters, which also saves much effort being required to re-solve the assignment. But it should be noted that the accuracy of the estimated equilibrium patterns is significantly dependent on the perturbations itself. The larger the perturbation introduced, the greater the divergence between the exact and approximate solutions will be.

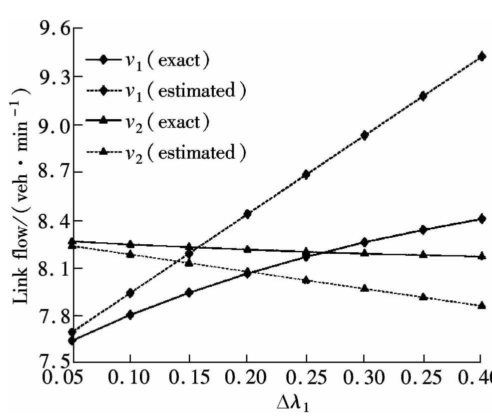

Fig. 3 Exact and approximated equilibrium solutions for probit-type SUEED (signal split change)

4 Conclusion

This paper develops a computationally efficient method for the sensitivity analysis of the SUEED assignment problem put forward by Maher et al[9]. Proof is given that the SUEED assignment problem satisfies all the conditions required for the sensitivity analysis of a general nonlinear problem. Explicit expressions are then given for obtaining the derivatives of equilibrium solutions with respect to input parameters by taking advantage of the gradient-based method. Those expressions are not only applicable for the sensitivity analysis of the logit-type SUEED problem but also can be equally applied to the probit-type SUEED problem. Numerical examples are presented to demonstrate how to obtain the derivatives of equilibrium solutions with respect to perturbed parameters with both logit and probit assumptions. These derivatives can be used to approximate the changes in solution variables when the network characteristics are changed slightly. Further research is to explore the applicability and efficiency of the propose method in various applications such as critical parameters identification, paradox and network uncertainty analysis.

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弹性需求下随机用户均衡分配问题敏感性分析

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摘要:提出了一种对弹性需求下随机用户均衡(SUEED)分配问题进行敏感性分析的方法. 首先,证明了SUEED模型在均衡解处输出变量对扰动参数的可导性. 其次,通过采用对一般非线性规划问题进行敏感性分析的梯度下降法,建立了SUEED模型中设计变量在均衡流量解处对扰动参数的计算公式. 这些公式不仅可以对logit型SUEED问题进行敏感性分析,而且同样适用于probit型SUEED问题. 算例路网的应用研究发现,所提出的方法可有效估计logit型SUEED问题和probit型SUEED问题中输入变量扰动后的均衡流量解.

关键词:网络建模;随机用户均衡;弹性需求;敏感性分析;一阶估计
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