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Sequencing method for dual-shuttle flow-rack automated storage and retrieval systems

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Abstract: The dual-retrieval (DR) operation sequencing problem in the flow-rack automated storage and retrieval system (AS/RS) is modeled as an assignment problem since it is equivalent to pairing outgoing unit-loads for each DR operation. A recursion symmetry Hungarian method (RSHM), modified from the Hungarian method, is proposed for generating a DR operation sequence with minimal total travel time, in which symmetry marking is introduced to ensure a feasible solution and recursion is adopted to break the endless loop caused by the symmetry marking. Simulation experiments are conducted to evaluate the cost effectiveness and the performance of the proposed method. Experimental results illustrate that compared to the single-shuttle machine, the dual-shuttle machine can reduce more than 40% of the total travel time of retrieval operations, and the RSHM saves about 5% to 10% of the total travel time of retrieval operations compared to the greedy-based heuristic.

Key words: dual-shuttle; sequencing; flow rack; automated storage and retrieval system (AS/RS)

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Unit-load warehouses, in which items are organized as pallet quantities (unit-loads) to be stored and retrieved, usually serve as distribution centers in logistic networks so that a high throughput and high storage capacity are required. Automated storage and retrieval systems (AS/RSs) are deployed in unit-load warehouses to improve throughput, speed up response, reduce labor cost, and decrease errors[1–2]. Multi-deep AS/RSs, such as double-deep AS/RSs[3], 3D compact AS/RSs[4], and flow-rack AS/RSs[5–6], are designed to obtain high floor-space utilization.

The flow-rack AS/RS is a low-cost multi-deep AS/RS, in which unit-loads are stored onto the storage face of the flow-rack and are retrieved from the retrieval face. The slide of stored unit-loads is driven by gravity. Unit-loads follow a first-in-first-out (FIFO) mode, which implies that an outgoing unit-load may be blocked by the unit-loads stored in front of it. These blocking unit-loads must be removed to a restoring conveyor before retrieving the requested one. A handling machine (HM) is deployed to the retrieval face to retrieve outgoing unit-loads and remove blocking ones[7], which makes it difficult to improve the retrieval performance.

A dual-shuttle machine carries two unit-loads simultaneously[8–9], which reduces retrieval time by conducting dual-retrieval (DR) operations (two retrievals in one operation) in shift-based AS/RSs[10]. Since a dual-shuttle machine can remove two blocking unit-loads each time and retrieve two outgoing unit-loads within a single operation, it is reasonable to assume that it will provide a higher retrieval performance than that gained by the normal HM for a flow-rack AS/RS. However, the investment of a dual-shuttle machine requires that the DR machine, i.e., the dual-shuttle machine on the retrieval face, must bring enough retrieval improvement to achieve cost effectiveness. Therefore, high-performance DR operations are generated and sequenced to take advantage of the DR machine.

In this paper, the DR sequencing problem is modeled as an assignment problem because it is equivalent to finding a one-on-one matching of outgoing unit-loads with minimal travel time. A recursion symmetry Hungarian method (RSHM), modified from the Hungarian method[11], is proposed for DR sequencing, in which symmetry marking ensures the feasible DR operation sequence and recursion breaks the endless loop caused by the symmetry marking.

1 Dual-Retrieval Sequencing Problem in Dual-Shuttle Flow-Rack AS/RSs

A flow-rack is shown in Fig. 1, which consists of L columns and H rows of bins. Each bin contains M segments to store at most M unit-loads. The drop-off station is deployed at $(0, 1)$ and the restoring conveyor is arranged at $(L+1, 1)$ as shown in Fig. 1. Outgoing unit-loads are retrieved to the drop-off station while blocking unit-loads are removed to the restoring conveyor. A DR machine is deployed to the retrieval face, which can remove two blocking unit-loads each time and retrieve two outgoing unit-loads within a single DR operation.

The set of outgoing unit-loads is denoted as $R = \{1, 2, \dots, m\}$, in which outgoing unit-load i is located at bin

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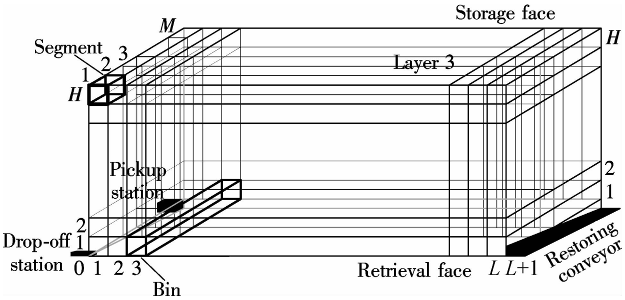

Fig. 1 Sketch of flow-rack

(x_i, y_i), where $i \in \{1, 2, \ldots, m\}$, $x_i \in \{1, 2, \ldots, L\}$, and $y_i \in \{1, 2, \ldots, H\}$. If m is odd, a fictitious outgoing unit-load located at $(0, 1)$ is inserted into R to make sure that m is even. (i, j) represents a DR operation, in which the DR machine leaves the drop-off station, removes all blocking unit-loads of i and j to the restoring conveyor, retrieves i and j, and returns to the drop-off station where $i, j \in R$. T_{ij} is the travel time of the DR operation. Let $T_{ii} = \infty$, where $i = 1, 2, \ldots, m$. X_{ij} is the assignment variable, which is assigned to 1 if the DR operation (i, j) is generated. This is because (i, j) and (j, i) represent the same DR operation, $X_{ij} = X_{ji}$ where $\forall\ i, j \in R$.

The DR sequencing problem can be described as finding a sequence of DR operations with a minimized total travel time from a given set of even numbered outgoing unit-loads. Finding the sequence is equivalent to pairing outgoing unit-loads for each DR operation. Therefore, the DR operation generation problem is modeled as an assignment problem, which has the following mathematical formulation:

$$\min \quad Z = \frac{1}{2} \sum_i \sum_j T_{ij} X_{ij} \tag{1}$$

$$\text{s. t.} \quad \sum_i X_{ij} = 1 \qquad j = 1, 2, \ldots, m \tag{2}$$

$$\sum_j X_{ij} = 1 \qquad i = 1, 2, \ldots, m \tag{3}$$

$$X_{ij} = X_{ji} \qquad \forall\, i, j \tag{4}$$

$$X_{ij} \in \{0, 1\} \qquad \forall\, i, j \tag{5}$$

Eq. (1) is the optimization objective of the problem. Eqs. (2) and (3) guarantee that each outgoing unit-load must be contained by one generated DR operation. Eq. (4) ensures that the DR operations must be feasible. (5) specifies the assignment variable.

2 Recursion Symmetry Hungarian Method

Greedy heuristics have been successfully employed for dual-command sequencing in AS/RSs[12-13], which can be applied to DR sequencing as well. In this paper, SDRT (shortest dual-retrieval time) is introduced as shown in Algorithm 1, of which the time complexity is $O(m^3)$.

Algorithm 1 SDRT

Input: R.
Output: DR operations.
  while ($R \neq \varnothing$) do
    $(i', j') \leftarrow \arg \min_{i,j \in R} \{T_{ij}\}$;
    Output (i', j');
    $R \leftarrow R - \{i', j'\}$;
  return.

Example 1 Assume that there are six outgoing unit-loads and the travel times of all the possible DR operations are illustrated as

$$\boldsymbol{W} = \begin{bmatrix} \infty & 21.8 & 18.6 & 14 & 26.3 & 27 \\ 21.8 & \infty & 20.4 & 15.4 & 35.5 & 37.4 \\ 18.6 & 20.4 & \infty & 14.6 & 32.3 & 34.2 \\ 14 & 15.4 & 14.6 & \infty & 27.7 & 29.6 \\ 26.3 & 35.5 & 32.3 & 27.7 & \infty & 37.7 \\ 27 & 37.4 & 34.2 & 29.6 & 37.7 & \infty \end{bmatrix}$$

A DR operation sequence ((1, 4), (2, 3), (5, 6)) is obtained by SDRT, of which the total travel time is 72.1. Although SDRT generates the DR operation sequence, room is still left for improvement.

If Eq. (4) is relaxed, the DR sequencing problem becomes a minimization classic assignment problem, which can be solved by the Hungarian method. Let $\boldsymbol{W}$ be the weight matrix, in which the elements are equal to the travel times of DR operations. Therefore, $\boldsymbol{W}$ is symmetrical. The execution of the Hungarian method (runs in Example 1) is shown in Fig. 2, in which " * " marks zeros and " _ " labels the uncovered minima.

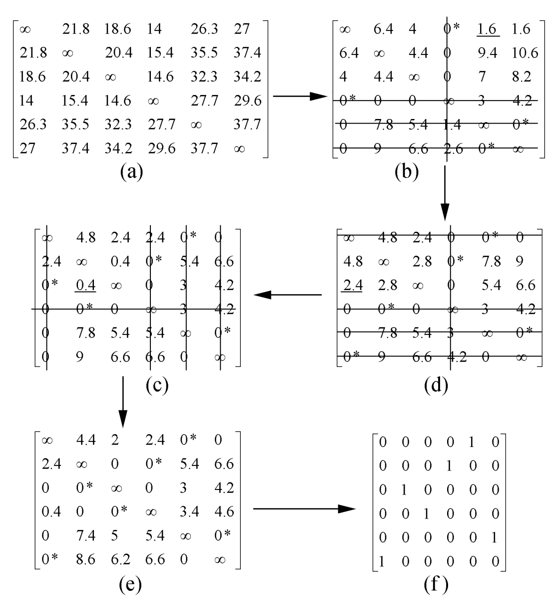

Fig. 2 Execution of the Hungarian method on Example 1

The transformation makes $\boldsymbol{W}$ become asymmetrical as shown in Fig. 2(b). Therefore, W_{ij} may be marked where $W_{ij} = 0$ and $W_{ji} > 0$ by the Hungarian method (e. g., W_{31} is marked and W_{13} is not in Fig. 2(c)). Finally, the assignment matrix obtained by the Hungarian method is illustrated in Fig. 2(f), in which Eq. (4) is unsatisfied and the corresponding DR operation sequence is unfeasible. Therefore, a novel marking mechanism is desirable for obtaining a feasible solution.

Based on Eq. (4), symmetry marking is proposed, in which only pairs of symmetrical independent zeros are simultaneously marked. A pair of symmetrical independent zeros represents two weights $W_{ij} = W_{ji} = 0$ and there are no marked zeros in the i-th and j-th rows and the i-th and j-th columns of $\boldsymbol{W}$. After marking W_{ij} and W_{ji}, all zeros in the corresponding rows and columns are deleted. The Hungarian method using symmetry marking is denoted as the symmetry Hungarian method (SHM).

Ideally, the SHM obtains feasible DR operation sequences. However, fewer zeros are marked by the SHM than those marked by the Hungarian method, which leads to the failure execution. A failure execution of the SHM (runs in Example 1) is illustrated in Fig. 3, in which " * " marks symmetrical independent zeros and " _ " labels the uncovered minima. Fig. 3(e) demonstrates that all the elements are covered, which causes $\boldsymbol{W}$ to remain unchanged, so an endless loop occurs.

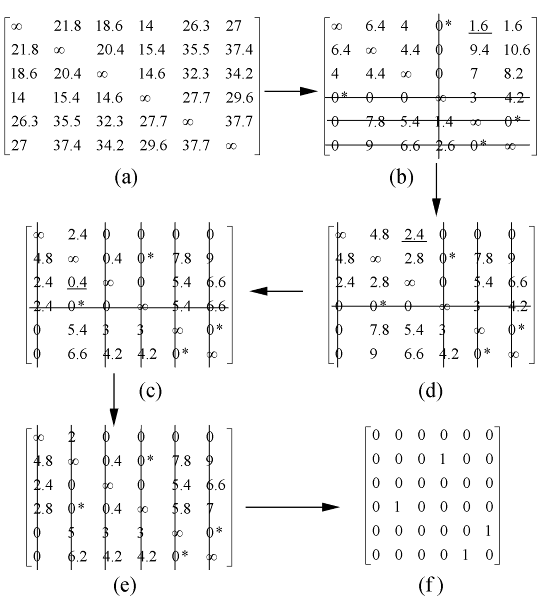

Fig. 3 A failure execution of the SHM

However, the assignment matrix $\boldsymbol{X}$ can be obtained by the SHM as shown in Fig. 3(f), from which a DR operation sequence is generated as shown in Algorithm 2.

Algorithm 2 ReturnList

Input: X.
Output: L_{DRO}.
$L_{\mathrm{DRO}} \leftarrow \varnothing$;
for ($i = 1$ to m) do
  for ($j = i + 1$ to m) do
    if ($X_{ij} = 1$) do
      $L_{\mathrm{DRO}} \leftarrow L_{\mathrm{DRO}} \cup \{(i, j)\}$;
return L_{DRO}

A partial solution with two DR operations, (2, 4) and (5, 6), can be generated. The remaining outgoing unit-loads combine the third DC operation (1, 3). Then, the DR operation sequence is ((2, 4), (5, 6), (1, 3)), in which the total travel time is 71.7. If more than two outgoing unit-loads are unpaired when the SHM falls into the endless loop, a recursion can be employed; in which a partial solution is generated from the marked zeros and the unpaired outgoing unit-loads are regrouped to be solved by calling the SHM again. Based on the above mechanism, a recursive process is proposed, which is denoted as the RSHM (recursion symmetry Hungarian method) and shown in Algorithm 3.

Algorithm 3 RSHM

Input: R.
Output: L_{DRO}.
1 $L_{\mathrm{DRO}} \leftarrow \varnothing$, $m \leftarrow |R|$;
2 Initialize $\boldsymbol{X}$: $X_{ij} \leftarrow 0$, $\forall i, j \in R$;
3 Establish $\boldsymbol{W}$: $W_{ij} \leftarrow T_{ij}$, $\forall i, j \in R$;
4 Initialize $\boldsymbol{W}$: perform row minima subtracting and column minima subtracting;
5 Mark symmetrical independent zeros in $\boldsymbol{W}$;
6 if (there are m marked zeros) then
7   $X_{ij} \leftarrow 1$ if W_{ij} is marked $\forall i, j \in \{1, 2, \dots, m\}$;
8 $L_{\mathrm{DRO}} \leftarrow$ ReturnList($\boldsymbol{X}$);
9 goto 19;
10 Use a minimal number of lines to cover all zeros in $\boldsymbol{W}$;
11 if (all weights are covered) then
12 $X_{ij} \leftarrow 1$ if W_{ij} is marked $\forall i, j \in \{1, 2, \dots, m\}$;
13 $L_{\mathrm{DRO}} \leftarrow$ ReturnList($\boldsymbol{X}$);
14 $R \leftarrow R - \{i\}$ if $\sum_j X_{ij} = 1 \ \forall i \in \{1, 2, \dots, m\}$;
15 $L'_{\mathrm{DRO}} \leftarrow$ RSHM(R), $L_{\mathrm{DRO}} \leftarrow L_{\mathrm{DRO}} \cup L'_{\mathrm{DRO}}$;
16 goto 19;
17 Update $\boldsymbol{W}$: uncovered weights minus the uncovered minima and weights at intersections of lines plus the uncovered minima;
18 goto 5;
19 return L_{DRO}.

Extremely, only one DR operation can be generated in each recursion, of which the time complexity is $O(m^3)$. Therefore, the time complexity of RSHM is $O(m^4)$.

3 Performance Evaluation

In this section, simulation experiments are conducted to analyze the retrieval improvement of the DR mechanism, which is evaluated by the average travel time of retrieving a single outgoing unit-load. All the experiments are programmed in C + + and run on a PC with 3 GHz CPU and 4 GB RAM.

Assume that the flow-rack contains L columns and H rows of bins, each of which has $M = 10$ segments. Let t_{h} and t_{v} be the horizontal travel time and vertical travel time between a pair of adjacent bins, respectively. Let $T = \max\{Lt_{\mathrm{h}}, Ht_{\mathrm{v}}\}$ and $b = \min\{Lt_{\mathrm{h}}/T, Ht_{\mathrm{v}}/T\}$ be the normalization factor and the shape factor of the retrieval face, respectively. Without loss of generality, we assume that the HM and the DR machine have the same moving capacity, both of which move faster in a horizontal direction than in a vertical direction. Therefore, $t_h = 0.1$ and $t_v = 0.2$. A total of N stored unit-loads are uniformly distributed on the flow-rack and have the same probability of being retrieved. Let $\rho = N/(LHM)$ represent the load rate.

For evaluating the cost effectiveness of deploying the DR machine, SR (single-retrieval) is proposed to generate normal retrieval operations for a normal HM. Therefore, the total travel time of the retrieval operation sequence obtained by SR is Z_S and the average retrieval time of a single unit-load is $z_S = Z_S/|R|$, which remains unchanged in different retrieval operation sequences.

For analyzing performance, the RSHM, the Hungarian method, and the SDRT are employed. Let Z_D, Z_H, and Z_R be the total travel time of DR operation sequences generated by the SDRT, the Hungarian method, and the RSHM, respectively. Therefore, $z_D = Z_D/|R|$, $z_H = Z_H/|R|$, and $z_R = Z_R/|R|$ are the average travel time of retrieving single unit-load obtained by the SDRT, the Hungarian method, and the RSHM, respectively.

Eq. (4) is not considered in the Hungarian method, which implies that the obtained DR operation sequences may be unfeasible. Therefore, the successful rate of the Hungarian method must be analyzed, in which $m \in \{6, 8, 10, 20, 30, 40, 50, 60, 70\}$, $b = 1.00$, and $\rho = 0.5$. For each value of m, the SDRT, the Hungarian method, and the RSHM, respectively, repeat 100 times to generate DR operation sequences for the same R where $|R| = m$. Experimental results are illustrated in Tab. 1.

Tab. 1 Retrieval performance obtained by the SDRT, the Hungarian method and the RSHM

m	n_S	Retrieval time/s			CPU time/s		
		SDRT	Hungarian	RSHM	SDRT	Hungarian	RSHM
6	61	21.04	17.34	17.48	0.00	0.00	0.00
8	47	19.48	16.44	16.53	0.00	0.00	0.00
10	32	19.47	16.67	16.77	0.00	0.00	0.00
20	1	20.49	17.89	17.89	0.00	0.00	0.01
30	0				0.01	0.00	0.00
40	0				0.00	0.03	0.00
50	0				0.00	0.06	0.01
60	0				0.00	0.81	0.00
70	0				0.01	2.64	0.02

Note: n_S represents the number of feasible DR operation sequences obtained by the Hungarian method in 100 repetitions.

Tab. 1 illustrates that the success rate of the Hungarian method decreases with the increase in m. $z_H < z_R < z_D$ shows that the Hungarian method obtains the best solution if it returns a feasible one. z_R is much closer to z_H than to z_D. Tab. 1 clarifies that the Hungarian method hardly obtains feasible DR operation sequences when $m \geqslant 20$. CPU times demonstrate that the Hungarian method needs a much longer computation time than the SDRT and the RSHM when $m \geqslant 60$. The Hungarian method is not employed in the following experiments because of its low success rate.

The following experiments are conducted for evaluating the cost effectiveness of the DR machine and analyzing the performance of the RSHM. The former is measured by z_R/z_S while the latter is ranked by z_R/z_D. The efficiency of the RSHM is measured by CPU time.

Let $(L, H) \in \{(40, 20), (47, 17), (57, 14), (80, 10)\}$, i. e., $b \in \{1.00, 0.72, 0.49, 0.25\}$, to simulate different configurations, $m \in \{6, 8, 10, 20, 30, 40, 50, 60, 70\}$ to simulate different workloads, and $\rho = 0.5$. There are a total of 36 parameter combinations of b and m, in each of which the simulation experiment repeats 100 times to obtain average results. In each repetition, a set of outgoing unit-loads R is generated where $|R| = m$. SR gains a retrieval operation sequence while the SDRT and the RSHM generate DR operation sequences, respectively. The experimental results are illustrated in Tab. 2.

Tab. 2 shows that z_R and z_D are much smaller than z_S because the DR machine removes two blocking unit-loads to the restoring conveyor each time and retrieves two outgoing unit-loads within a single DR operation. z_R/z_S shows that the DR operations obtained by the RSHM save more than 40% travel time to retrieve the same outgoing unit-loads when compared against retrieval operations gained by SR. In all combinations, z_R/z_D shows that the DR operations obtained by the RSHM have shorter travel time than those obtained by the SDRT. The RSHM reduces by more than 15% travel time in nine combinations, saves about 10% to 15% travel time in six combinations, and shortens 4% to 10% travel time in 21 combinations when compared against the SDRT. With the decrease of b, z_R/z_S and z_R/z_D decrease, which implies that the RSHM saves more travel time with a smaller b. However, z_S, z_D, and z_R increase with the decrease in b, which demonstrates that the square-in-time configuration, i. e., $b = 1.00$, is optimal. CPU times demonstrate that the RSHM is an efficient method.

In addition, m and ρ also influence the performance of DR operations since more blocking unit-loads require longer travel time for removal. Let $(L, H) \in \{(40, 20), (47, 17), (57, 14), (80, 10)\}$, i. e., $b \in \{1.00, 0.72, 0.49, 0.25\}$, $\rho \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, and $m = 40$. There are a total of 36 parameter combinat-

Tab. 2 The performance of the RSHM with different m and $\rho = 0.5$

m	b	Retrieval time/s					CPU time/s		
		z_S	z_D	z_R	z_R/z_S	z_R/z_D	SR	SDRT	RSHM
6	1.00	32.49	21.95	18.20	0.56	0.83	0.00	0.00	0.00
	0.72	33.59	23.11	18.30	0.54	0.79	0.00	0.00	0.00
	0.49	39.23	26.06	20.61	0.53	0.79	0.00	0.00	0.00
	0.25	49.20	32.65	25.06	0.51	0.77	0.00	0.00	0.00
8	1.00	30.33	19.83	16.82	0.55	0.85	0.00	0.00	0.00
	0.72	32.96	21.30	17.84	0.54	0.84	0.00	0.00	0.00
	0.49	38.54	24.34	20.16	0.52	0.83	0.00	0.00	0.00
	0.25	49.19	31.20	24.95	0.51	0.80	0.00	0.00	0.00
10	1.00	32.19	20.19	17.52	0.54	0.87	0.00	0.00	0.00
	0.72	34.18	21.18	18.07	0.53	0.85	0.00	0.00	0.00
	0.49	37.00	23.07	19.34	0.52	0.84	0.00	0.00	0.00
	0.25	48.03	29.41	24.04	0.50	0.82	0.00	0.00	0.00
20	1.00	32.02	18.93	17.32	0.54	0.92	0.00	0.00	0.01
	0.72	33.26	19.32	17.48	0.53	0.90	0.00	0.00	0.00
	0.49	36.48	20.73	18.37	0.50	0.89	0.00	0.00	0.00
	0.25	49.45	27.76	24.21	0.49	0.87	0.00	0.00	0.00
30	1.00	32.07	18.35	17.16	0.54	0.93	0.00	0.01	0.00
	0.72	33.16	18.59	17.23	0.52	0.93	0.00	0.00	0.00
	0.49	36.96	20.45	18.64	0.50	0.91	0.00	0.00	0.00
	0.25	49.47	26.87	24.03	0.49	0.89	0.00	0.00	0.00
40	1.00	32.57	18.27	17.28	0.53	0.95	0.00	0.00	0.00
	0.72	33.27	18.44	17.20	0.52	0.93	0.00	0.01	0.00
	0.49	37.18	20.13	18.64	0.50	0.93	0.00	0.00	0.00
	0.25	48.79	25.99	23.43	0.48	0.90	0.00	0.00	0.00
50	1.00	31.75	17.79	16.93	0.53	0.95	0.00	0.00	0.01
	0.72	33.40	18.33	17.26	0.52	0.94	0.00	0.00	0.01
	0.49	37.06	19.99	18.48	0.50	0.92	0.00	0.00	0.00
	0.25	49.56	26.22	23.74	0.48	0.91	0.00	0.00	0.00
60	1.00	31.65	17.61	16.85	0.53	0.96	0.00	0.00	0.00
	0.72	32.89	17.76	16.91	0.51	0.95	0.00	0.00	0.00
	0.49	37.26	19.93	18.56	0.50	0.93	0.00	0.00	0.03
	0.25	50.05	26.32	23.99	0.48	0.91	0.00	0.00	0.02
70	1.00	31.65	17.49	16.78	0.53	0.96	0.00	0.01	0.02
	0.72	32.51	17.56	16.74	0.51	0.95	0.00	0.00	0.01
	0.49	37.27	19.84	18.53	0.50	0.93	0.00	0.00	0.01
	0.25	50.08	26.06	23.85	0.48	0.92	0.00	0.01	0.03

ions of b and ρ, in each of which the simulation experiment is repeated 100 times to obtain average results. In each repetition, a set of outgoing unit-loads R is generated where $|R| = 40$. SR, the SDRT, and the RSHM are employed, in which SR obtains a retrieval operation sequence while the SDRT and the RSHM generate DR operation sequences, respectively. Experimental results are illustrated in Tab. 3.

Since $M = 10$, $\rho = 0.1$ implies that there are few blocking unit-loads. Tab. 3 shows that z_D and z_R are similar when $\rho = 0.1$, which implies that the SDRT and the RSHM have similar effectiveness when there is no blocking unit-load. With the increase in ρ, more blocking unit-loads exist. Therefore, z_S, z_D, and z_R increase because more blocking unit-loads require a longer travel time for removing to the restoring conveyor. When $\rho \geqslant 0.2$, z_R/z_D demonstrates that the RSHM can save more than 5% of total travel time when compared against the SDRT. With the decrease in b, z_R/z_D decreases, which implies that more travel time is reduced by the RSHM with a smaller b. However, z_D and z_R demonstrate that the retrieval performance reaches optimum when $b = 1.00$, i. e., the square-in-time configuration is optimal. z_S, z_D, and z_R highlight that the DR machine needs almost half the travel time to retrieve the same outgoing unit-loads as compared with a normal HM. z_R/z_S shows that more than 40% retrieval time is saved by the execution of DR operations. CPU times display the high efficiency of the RSHM as well.

In summary, the DR machine reduces the retrieval time when compared against normal HM, which proves that deploying the DR machine brings retrieval improvement in flow-rack AS/RSs. The retrieval time is mainly saved by removing two blocking unit-loads to the restoring con-

Tab. 3 The performance of the RSHM with different ρ and $m=40$

ρ	b	Retrieval time/s					CPU time/s		
		z_S	z_D	z_R	z_R/z_S	z_R/z_D	SR	SDRT	RSHM
0.1	1.00	10.52	5.79	5.74	0.55	0.99	0.00	0.00	0.01
	0.72	10.99	6.02	5.98	0.54	0.99	0.00	0.00	0.00
	0.49	12.35	6.72	6.65	0.54	0.99	0.00	0.00	0.01
	0.25	16.32	8.68	8.67	0.53	1.00	0.00	0.00	0.01
0.2	1.00	15.88	10.10	9.39	0.59	0.93	0.00	0.00	0.00
	0.72	16.38	9.93	9.33	0.57	0.94	0.00	0.00	0.01
	0.49	18.35	10.53	10.05	0.55	0.95	0.00	0.00	0.00
	0.25	24.83	13.55	12.93	0.52	0.95	0.00	0.00	0.02
0.3	1.00	20.85	11.95	11.22	0.54	0.94	0.00	0.00	0.01
	0.72	21.87	12.13	11.36	0.52	0.94	0.00	0.00	0.00
	0.49	25.07	13.30	12.39	0.49	0.93	0.00	0.00	0.00
	0.25	32.59	16.73	15.56	0.48	0.93	0.00	0.00	0.00
0.4	1.00	27.01	15.93	15.03	0.56	0.94	0.00	0.00	0.00
	0.72	27.51	15.70	14.65	0.53	0.93	0.00	0.00	0.00
	0.49	31.41	17.68	16.10	0.51	0.91	0.00	0.01	0.00
	0.25	41.31	22.62	20.30	0.49	0.90	0.00	0.00	0.01
0.5	1.00	31.84	17.93	16.99	0.53	0.95	0.00	0.00	0.00
	0.72	33.21	18.39	17.19	0.52	0.93	0.00	0.00	0.00
	0.49	37.30	20.29	18.61	0.50	0.92	0.00	0.00	0.00
	0.25	49.96	26.55	24.06	0.48	0.91	0.00	0.00	0.01
0.6	1.00	37.32	21.20	20.06	0.54	0.95	0.00	0.00	0.00
	0.72	38.93	21.81	20.38	0.52	0.93	0.00	0.00	0.01
	0.49	43.45	24.03	22.02	0.51	0.92	0.00	0.00	0.01
	0.25	56.99	30.71	27.80	0.49	0.91	0.00	0.00	0.00
0.7	1.00	42.73	23.74	22.51	0.53	0.95	0.00	0.00	0.00
	0.72	45.46	24.89	23.40	0.51	0.94	0.00	0.00	0.00
	0.49	50.11	27.08	25.10	0.50	0.93	0.00	0.00	0.01
	0.25	64.63	34.57	31.43	0.49	0.91	0.00	0.00	0.00
0.8	1.00	48.26	26.97	25.61	0.53	0.95	0.00	0.00	0.00
	0.72	49.15	26.98	25.38	0.52	0.94	0.00	0.00	0.01
	0.49	55.87	30.44	28.09	0.50	0.92	0.01	0.01	0.01
	0.25	73.17	39.37	35.87	0.49	0.91	0.00	0.00	0.00
0.9	1.00	52.61	28.89	27.40	0.52	0.95	0.00	0.00	0.00
	0.72	54.76	29.69	27.97	0.51	0.94	0.00	0.00	0.00
	0.49	62.60	33.72	31.34	0.50	0.93	0.00	0.00	0.00
	0.25	81.82	43.62	39.96	0.49	0.92	0.00	0.00	0.01

vey or each time and retrieving two outgoing unit-loads within a single DR operation. Experimental results illustrate that the DR machine needs almost half the retrieval time to retrieve the same outgoing unit-loads as compared with the normal HM even if the DR operations are generated by a simple greedy heuristic. The RSHM is an effective and efficient method, which returns better DR operations than those obtained by the simple greedy heuristic.

4 Conclusion

In this paper, the DR sequencing problem in flow-rack AS/RSs is analyzed. The RSHM, modified from the Hungarian method, is proposed for addressing this problem. Symmetry marking and recursion are introduced to ensure feasible DR operations and to break the endless loop caused by the symmetry marking, respectively. Simulation experiments prove that a DR machine needs almost half of the total retrieval time when compared against a normal HM for retrieving the same outgoing unit-loads; and the RSHM is an efficient method, which returns better solutions than those obtained by a greedy heuristic.

In the future, more attention should be paid to the establishment of a travel time model for a dual-shuttle flow-rack AS/RSs. A storage assignment for flow-rack AS/RSs is also necessary to reduce the number of blocking unit-loads.

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一种用于双负载重力货架自动存取系统的排序方法

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摘要:重力货架自动存取系统的双提取操作排序问题等价于在一组提取货物中进行两两配对以生成最小执行成本的双提取操作序列,因此双提取操作排序问题被建模为一个匹配问题.提出一个基于匈牙利方法的启发式方法 RSHM 来生成具有最小总行驶时间的双提取命令.为了保证生成可行的双提取操作序列,对称标记法被引入 RSHM;为了打破对称标记法引起的无限循环,RSHM 被设计成一个递归过程.仿真实验对双负载装卸设备的成本效益和 RSHM 的性能进行了评价和分析.仿真实验结果显示:与单负载设备相比,采用双负载装卸设备可以节省超过 40% 的提取操作总行驶时间;与贪婪规则相比,RSHM 能够减少 5% ~10% 的总行驶时间.

关键词:双负载; 排序; 重力货架; 自动存取系统(AS/RS)
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