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A note on ribbon elements of Hopf group-coalgebras

Zhao Xiaofan Wang Shuanhong

(Department of Mathematics, Southeast University, Nanjing 211189, China)

Abstract: Let G be a discrete group with a neutral element and H be a quasitriangular Hopf G-coalgebra over a field k. Then the relationship between G-grouplike elements and ribbon elements of H is considered. First, a list of useful properties of a quasitriangular Hopf G-coalgebra and its Drinfeld elements are proved. Secondly, motivated by the relationship between the grouplike and ribbon elements of a quasitriangular Hopf algebra, a special kind of G-grouplike elements of H is defined. Finally, using the Drinfeld elements, a one-to-one correspondence between the special G-grouplike elements defined above and ribbon elements is obtained.

Key words: quasitriangular Hopf G-coalgebra; G-grouplike element; ribbon element; Drinfeld element

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In the theory of the classical Hopf algebras[1−2], one of the celebrated results is the theory of ribbon Hopf algebras, which plays an important role in constructing invariants of framed links embedded in 3-dimensional space[3]. One important aspect of ribbon Hopf algebras is the relationship between grouplike elements and ribbon elements[4].

As a generalization of ordinary Hopf algebras, Hopf group-coalgebras related to homotopy quantum field theories were introduced by Turaev in Ref. [5]. A purely algebraic study of Hopf group-coalgebras, such as the main properties of quasitriangular and ribbon Hopf group-coalgebras, can be found in Refs. [6 − 9].

In this paper, we consider the following question: for a group G, how to use a special kind of G-grouplike elements to describe the ribbon elements of a Hopf G-coalgebra.

Throughout this paper, we let G be a discrete group with a neutral element 1 and k be a field. Assume that H is a Hopf G-coalgebra over k. Denote the set of all G-grouplike elements of H by $G(H)$.

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Biographies: Zhao Xiaofan (1986—), female, graduate; Wang shuanhong (corresponding author), male, doctor, professor, shuanhwang@seu.edu.cn.

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1 Preliminaries

Definition 1 *A* Hopf G-coalgebra $H=(\{H_\alpha\},\Delta,\varepsilon,S)$ is said to be crossed provided it is endowed with a family $\phi=\{\phi_\beta: H_\alpha\to H_{\beta\alpha\beta^{-1}}\}_{\alpha,\beta\in G}$ of k-linear maps (the crossing) such that for all $\alpha,\beta,\gamma\in G$, 1) ϕ_β is an algebra isomorphism; 2) $(\phi_\beta\otimes\phi_\beta)\Delta_{\alpha,\gamma}=\Delta_{\beta\alpha\beta^{-1},\beta\gamma\beta^{-1}}\phi_\beta$; 3) $\varepsilon\phi_\beta=\varepsilon$; 4) $\phi_{\alpha\beta}=\phi_\alpha\phi_\beta$.

Definition 2 A quasitriangular Hopf G-coalgebra is a crossed Hopf G-coalgebra $H=(\{H_\alpha\},\Delta,\varepsilon,S,\phi)$ endowed with a family $R=\{R_{\alpha,\beta}\in H_\alpha\otimes H_\beta\}_{\alpha,\beta\in G}$ of invertible elements (the R-matrix) such that for all $\alpha,\beta,\gamma\in G$, and $x\in H_{\alpha\beta}$,

$$R_{\alpha,\beta}\Delta_{\alpha,\beta}(x)=\sigma_{\beta,\alpha}(\phi_{\alpha^{-1}}\otimes\mathrm{id}_{H_\alpha})\Delta_{\alpha\beta\alpha^{-1},\alpha}(x)R_{\alpha,\beta}$$
$$(\mathrm{id}_{H_\alpha}\otimes\Delta_{\beta,\gamma})(R_{\alpha,\beta\gamma})=(R_{\alpha,\gamma})_{1\beta3}(R_{\alpha,\beta})_{12\gamma}$$
$$(\Delta_{\alpha,\beta}\otimes\mathrm{id}_{H_\gamma})(R_{\alpha\beta,\gamma})=[(\mathrm{id}_{H_\alpha}\otimes\phi_{\beta^{-1}})(R_{\alpha,\beta\gamma\beta^{-1}})]_{1\beta3}(R_{\beta,\gamma})_{\alpha23}$$
$$(\phi_\beta\otimes\phi_\beta)(R_{\alpha,\gamma})=R_{\beta\alpha\beta^{-1},\beta\gamma\beta^{-1}}$$

where $\sigma_{\beta,\alpha}$ denotes the flip map $H_\beta\otimes H_\alpha\to H_\alpha\otimes H_\beta$, for two k-spaces P, Q and $r=\sum_j p_j\otimes q_j\in P\otimes Q$, we set $r_{12\gamma}=r\otimes1_\gamma\in P\otimes Q\otimes H_\gamma$, $r_{\alpha23}=1_\alpha\otimes r\in H_\alpha\otimes P\otimes Q$ and $r_{1\beta3}=\sum_j p_j\otimes1_\beta\otimes q_j\in P\otimes H_\beta\otimes Q$.

Remark 1 Let $H=(\{H_\alpha,m_\alpha,1_\alpha\},\Delta,\varepsilon,S,\phi,R)$ be a quasitriangular Hopf G-coalgebra. The generalized Drinfeld elements of H are defined by $\mu_\alpha=m_\alpha(S_{\alpha^{-1}}\phi_\alpha\otimes\mathrm{id}_{H_\alpha})\sigma_{\alpha,\alpha^{-1}}(R_{\alpha,\alpha^{-1}})\in H_\alpha$, for any $\alpha\in G$.

Definition 3 A quasitriangular Hopf G-coalgebra $H=(\{H_\alpha\},\Delta,\varepsilon,S,\phi,R)$ is said to be a ribbon if it is endowed with a family $\theta=\{\theta_\alpha\in H_\alpha\}_{\alpha\in G}$ of invertible elements (the twist) such that for all $\alpha,\beta\in G$ and $x\in H_\alpha$, 1) $\phi_\alpha(x)=\theta_\alpha^{-1}x\theta_\alpha$; 2) $S_\alpha(\theta_\alpha)=\theta_{\alpha^{-1}}$; 3) $\phi_\beta(\theta_\alpha)=\theta_{\beta\alpha\beta^{-1}}$; 4) $\Delta_{\alpha,\beta}(\theta_{\alpha\beta})=(\theta_\alpha\otimes\theta_\beta)\sigma_{\beta,\alpha}((\phi_{\alpha^{-1}}\otimes\mathrm{id}_{H_\alpha})(R_{\alpha\beta\alpha^{-1},\alpha}))R_{\alpha,\beta}$.

In the following proofs, for all $\alpha,\beta\in G$ and $x\in H_{\alpha\beta}$, we write $\Delta_{\alpha,\beta}(x)=\sum_{(x)}x_{(1,\alpha)}\otimes x_{(2,\beta)}\in H_\alpha\otimes H_\beta$, or shortly $\Delta_{\alpha,\beta}(x)=x_{(1,\alpha)}\otimes x_{(2,\beta)}$. When we write a compontent $R_{\alpha,\beta}$ of an R-matrix as $R_{\alpha,\beta}=a_\alpha\otimes b_\beta$, it is to signify that $R_{\alpha,\beta}=\sum_j a_j\otimes b_j$ for some $a_j\in H_\alpha$ and $b_j\in H_\beta$, where j runs over a finite set of indices.

2 A New Description of Ribbon Hopf G-Coalgebras

Lemma 1 Let $H=(\{H_\alpha\},\Delta,\varepsilon,S,\phi,R)$ be a quasitriangular Hopf G-coalgebra. Then for any $\alpha,\beta,\gamma\in G$, and

$x \in H_\alpha$, the following identities hold: 1) $\phi_1 \mid H_\alpha = \mathrm{id}_{H_\alpha}$; 2) $\phi_\beta S_\alpha = S_{\beta\alpha\beta^{-1}} \phi_\beta$; 3) $(S_\alpha \otimes S_\beta)(R_{\alpha,\beta}) = (\phi_\alpha \otimes \mathrm{id}_{H_{\beta^{-1}}})(R_{\alpha^{-1},\beta^{-1}})$; 4) $(R_{\beta,\gamma})_{\alpha 23}(R_{\alpha,\gamma})_{1\beta 3}(R_{\alpha,\beta})_{12\gamma} = (R_{\alpha,\beta})_{12\gamma} \cdot [(\mathrm{id}_{H_\alpha} \otimes \phi_{\beta^{-1}})(R_{\alpha,\beta\gamma\beta^{-1}})]_{1\beta 3}(R_{\beta,\gamma})_{\alpha 23}$; 5) $\mu_\alpha^{-1} = m_\alpha(\mathrm{id}_{H_\alpha} \otimes S_{\alpha^{-1}} S_\alpha)\sigma_{\alpha,\alpha}(R_{\alpha,\alpha})$; 6) $S_{\alpha^{-1}} S_\alpha \phi_\alpha(x) = \mu_\alpha x \mu_\alpha^{-1}$.

Lemma 2 Let $H = (\{H_\alpha\}, \Delta, \varepsilon, S, \phi, R)$ be a quasi-triangular Hopf G-coalgebra. Then for any $\alpha, \beta \in G$,

$$S_{\alpha^{-1}} S_\alpha(a_\alpha) \otimes S_{\beta^{-1}} S_\beta(b_\beta) = a_\alpha \otimes b_\beta \qquad (1)$$

$$S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) \otimes S_{\beta^{-1}} S_\beta(a_\beta) = \phi_\alpha(b_\alpha) \otimes a_\beta \qquad (2)$$

Proof We first check the identity (1). For any $\alpha, \beta \in G$

$$S_{\alpha^{-1}} S_\alpha(a_\alpha) \otimes S_{\beta^{-1}} S_\beta(b_\beta) = S_{\alpha^{-1}} \phi_\alpha(a_{\alpha^{-1}}) \otimes S_{\beta^{-1}}(b_{\beta^{-1}}) = \phi_\alpha S_{\alpha^{-1}}(a_{\alpha^{-1}}) \otimes S_{\beta^{-1}}(b_{\beta^{-1}}) = \phi_\alpha \phi_{\alpha^{-1}}(a_\alpha) \otimes b_\beta = a_\alpha \otimes b_\beta$$

Next we show the proof of the identity (2). For any $\alpha, \beta \in G$, we have the following computations:

$$S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) \otimes S_{\beta^{-1}} S_\beta(a_\beta) = S_{\alpha^{-1}} \phi_\alpha S_\alpha(b_\alpha) \otimes S_{\beta^{-1}} S_\beta(a_\beta) = S_{\alpha^{-1}} \phi_\alpha(b_{\alpha^{-1}}) \otimes S_{\beta^{-1}} \phi_\beta(a_{\beta^{-1}}) = \phi_\alpha S_{\alpha^{-1}}(b_{\alpha^{-1}}) \otimes \phi_\beta S_{\beta^{-1}}(a_{\beta^{-1}}) = \phi_\alpha(b_\alpha) \otimes \phi_\beta \phi_{\beta^{-1}}(a_\beta) = \phi_\alpha(b_\alpha) \otimes a_\beta$$

Lemma 3 Let $H = (\{H_\alpha\}, \Delta, \varepsilon, S, \phi, R)$ be a quasi-triangular Hopf G-coalgebra. Then for any $\alpha, \beta \in G$,

$$\Delta_{\alpha,\beta}(\mu_{\alpha\beta}^{-1}) = (\mu_\alpha^{-1} \otimes \mu_\beta^{-1})(\phi_\alpha \otimes \mathrm{id}_{H_\beta})\sigma_{\beta,\alpha}(R_{\beta,\alpha})R_{\alpha,\beta}$$

Proof Using Lemma 1 and Lemma 2, for any $\alpha, \beta \in G$, we compute

$$\begin{aligned}
&\Delta_{\alpha,\beta}(\mu_{\alpha\beta}^{-1}) = \Delta_{\alpha,\beta}(b_{\alpha\beta} S_{\beta^{-1}\alpha^{-1}} S_{\alpha\beta}(a_{\alpha\beta})) = \\
&\Delta_{\alpha,\beta}(b_{\alpha\beta})\Delta_{\alpha,\beta} S_{\beta^{-1}\alpha^{-1}} S_{\alpha\beta}(a_{\alpha\beta}) = \\
&(b_{\alpha\beta(1,\alpha)} \otimes b_{\alpha\beta(2,\beta)})(S_{\alpha^{-1}} \otimes S_{\beta^{-1}})(S_\alpha \otimes S_\beta)\Delta_{\alpha,\beta}(a_{\alpha\beta}) = \\
&(b_\alpha \otimes \bar{b}_\beta)(S_{\alpha^{-1}} \otimes S_{\beta^{-1}})(S_\alpha \otimes S_\beta)\Delta_{\alpha,\beta}(\bar{a}_{\alpha\beta} a_{\alpha\beta}) = \\
&(b_\alpha \otimes \bar{b}_\beta)(S_{\alpha^{-1}} \otimes S_{\beta^{-1}})(S_\alpha \otimes S_\beta)(\bar{a}_{\alpha\beta(1,\alpha)} a_{\alpha\beta(1,\alpha)} \otimes \\
&\bar{a}_{\alpha\beta(2,\beta)} a_{\alpha\beta(2,\beta)}) = [\phi_{\beta^{-1}}(b_{\beta\alpha\beta^{-1}})\bar{b}_\alpha \otimes \phi_{\beta^{-1}}(\tilde{b}_\beta)\hat{b}_\beta] \cdot \\
&(S_{\alpha^{-1}} \otimes S_{\beta^{-1}})(S_\alpha \otimes S_\beta)(\tilde{a}_\alpha a_\alpha \otimes \hat{a}_\beta \bar{a}_\beta) = \\
&\phi_{\beta^{-1}}(b_{\beta\alpha\beta^{-1}})\bar{b}_\alpha S_{\alpha^{-1}} S_\alpha(\tilde{a}_\alpha a_\alpha) \otimes \\
&\phi_{\beta^{-1}}(\tilde{b}_\beta)\hat{b}_\beta S_{\beta^{-1}} S_\beta(\hat{a}_\beta) S_{\beta^{-1}} S_\beta(\bar{a}_\beta) = \phi_{\beta^{-1}}(b_{\beta\alpha\beta^{-1}})\bar{b}_\alpha S_{\alpha^{-1}} S_\alpha \cdot \\
&(\tilde{a}_\alpha a_\alpha) \otimes \phi_{\beta^{-1}}(\tilde{b}_\beta)\mu_\beta^{-1} S_{\beta^{-1}} S_\beta(\bar{a}_\beta) = \phi_{\beta^{-1}}(b_{\beta\alpha\beta^{-1}})\bar{b}_\alpha S_{\alpha^{-1}} \cdot \\
&S_\alpha(\tilde{a}_\alpha a_\alpha) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta \phi_\beta \phi_{\beta^{-1}}(\tilde{b}_\beta) S_{\beta^{-1}} S_\beta(\bar{a}_\beta) = \phi_{\beta^{-1}}(b_{\beta\alpha\beta^{-1}}) \cdot \\
&\bar{b}_\alpha S_{\alpha^{-1}} S_\alpha(\tilde{a}_\alpha a_\alpha) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(\tilde{b}_\beta \bar{a}_\beta) = b_\alpha \bar{b}_\alpha S_{\alpha^{-1}} S_\alpha(\bar{a}_\alpha \tilde{a}_\alpha) \otimes \\
&\mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta \tilde{b}_\beta) = b_\alpha \mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha(\tilde{a}_\alpha) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta \tilde{b}_\beta) = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) S_{\alpha^{-1}} S_\alpha(\tilde{a}_\alpha) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta) S_{\beta^{-1}} S_\beta(\tilde{b}_\beta) = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) S_{\alpha^{-1}} \phi_\alpha(\tilde{a}_{\alpha^{-1}}) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta) S_{\beta^{-1}}(\tilde{b}_{\beta^{-1}}) = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) \phi_\alpha S_{\alpha^{-1}}(\tilde{a}_{\alpha^{-1}}) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta) S_{\beta^{-1}}(\tilde{b}_{\beta^{-1}}) = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) \phi_\alpha \phi_{\alpha^{-1}}(\tilde{a}_\alpha) \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta)\tilde{b}_\beta = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha)\tilde{a}_\alpha \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta)\tilde{b}_\beta = \\
&\mu_\alpha^{-1} \phi_\alpha(b_\alpha)\tilde{a}_\alpha \otimes \mu_\beta^{-1} a_\beta \tilde{b}_\beta = (\mu_\alpha^{-1} \otimes \mu_\beta^{-1})(\phi_\alpha \otimes \mathrm{id}_{H_\beta}) \cdot \\
&\sigma_{\beta,\alpha}(R_{\beta,\alpha})R_{\alpha,\beta}.
\end{aligned}$$

This completes the proof of the lemma.

Let $H = (\{H_\alpha\}, \Delta, \varepsilon, S, \phi, R)$ be a quasitriangular Hopf G-coalgebra. We define the set of a special kind of G-grouplike elements $(v = \{v_\alpha \in H_\alpha\}_{\alpha \in G})$ of H by $E = \{v \in G(H) \mid \phi_\beta(v_\alpha) = v_{\beta\alpha\beta^{-1}}, S_\alpha(\mu_\alpha) = v_{\alpha^{-1}}{}^{-1}\mu_{\alpha^{-1}} v_{\alpha^{-1}}{}^{-1}, S_{\alpha^{-1}} S_\alpha(x) = v_\alpha x v_\alpha^{-1}$, for any $\alpha, \beta \in G, x \in H_\alpha\}$. Denote the set of all ribbon elements of H by F.

Theorem 1 Suppose that $H = (\{H_\alpha\}, \Delta, \varepsilon, S, \phi, R)$ is a quasitriangular Hopf G-coalgebra. Then there is a one-to-one correspondence between E and F defined as above.

Proof Define a map $P: E \to F$ by $P(v) = \mu^{-1} v = \{\mu_\alpha^{-1} v_\alpha \in H_\alpha\}_{\alpha \in G}$, for any $v \in E$. First, we check that P is well defined, i. e., $\mu^{-1} v \in F$. From Definition 3, it suffices to verify the following five conditions: 1) $\mu^{-1} v$ is invertible; 2) $\phi_\alpha(x) = (\mu_\alpha^{-1} v_\alpha)^{-1} x \mu_\alpha^{-1} v_\alpha$; 3) $S_\alpha(\mu_\alpha^{-1} v_\alpha) = \mu_{\alpha^{-1}}^{-1} v_{\alpha^{-1}}$; 4) $\phi_\beta(\mu_\alpha^{-1} v_\alpha) = \mu_{\beta\alpha\beta^{-1}}^{-1} v_{\beta\alpha\beta^{-1}}$; 5) $\Delta_{\alpha,\beta}(\mu_{\alpha\beta}^{-1} v_{\alpha\beta}) = (\mu_\alpha^{-1} v_\alpha \otimes \mu_\beta^{-1} v_\beta)\sigma_{\beta,\alpha}((\phi_{\alpha^{-1}} \otimes \mathrm{id}_{H_\alpha})(R_{\alpha\beta\alpha^{-1},\alpha}))R_{\alpha,\beta}$, for any $\alpha, \beta \in G, x \in H_\alpha$. The first condition follows from the invertibility of μ and v. For the second condition, we have

$$\begin{aligned}
&(\mu_\alpha^{-1} v_\alpha)^{-1} x \mu_\alpha^{-1} v_\alpha = v_\alpha^{-1} \mu_\alpha x \mu_\alpha^{-1} v_\alpha = \\
&v_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(x) v_\alpha = v_\alpha^{-1} v_\alpha \phi_\alpha(x) v_\alpha^{-1} v_\alpha = \phi_\alpha(x)
\end{aligned}$$

Let us prove the third condition. We compute

$$\begin{aligned}
&S_\alpha(\mu_\alpha^{-1} v_\alpha) = S_\alpha(v_\alpha) S_\alpha(\mu_\alpha^{-1}) = \\
&v_{\alpha^{-1}}^{-1}(v_{\alpha^{-1}}^{-1} \mu_{\alpha^{-1}} v_{\alpha^{-1}}^{-1})^{-1} = v_{\alpha^{-1}}^{-1} v_{\alpha^{-1}} \mu_{\alpha^{-1}}^{-1} v_{\alpha^{-1}} = \\
&\mu_{\alpha^{-1}}^{-1} v_{\alpha^{-1}}
\end{aligned}$$

The fourth condition holds since $\phi_\beta(\mu_\alpha^{-1} v_\alpha) = \phi_\beta(\mu_\alpha^{-1})\phi_\beta(v_\alpha) = \mu_{\beta\alpha\beta^{-1}}^{-1} v_{\beta\alpha\beta^{-1}}$. To prove the fifth condition, it is enough to verify that

$$\begin{aligned}
&(\mu_\alpha^{-1} v_\alpha \otimes \mu_\beta^{-1} v_\beta)\sigma_{\beta,\alpha}((\phi_{\alpha^{-1}} \otimes \mathrm{id}_{H_\alpha})(R_{\alpha\beta\alpha^{-1},\alpha}))R_{\alpha,\beta} = \\
&\mu_\alpha^{-1} v_\alpha b_\alpha \tilde{a}_\alpha \otimes \mu_\beta^{-1} v_\beta \phi_{\alpha^{-1}}(a_{\alpha\beta\alpha^{-1}})\tilde{b}_\beta = \\
&\mu_\alpha^{-1} v_\alpha \phi_\alpha(b_\alpha)\tilde{a}_\alpha \otimes \mu_\beta^{-1} v_\beta \phi_{\alpha^{-1}} \phi_\alpha(a_\beta)\tilde{b}_\beta = \\
&\mu_\alpha^{-1} v_\alpha \phi_\alpha(b_\alpha)\tilde{a}_\alpha \otimes \mu_\beta^{-1} v_\beta a_\beta \tilde{b}_\beta = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) v_\alpha \tilde{a}_\alpha \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta) v_\beta \tilde{b}_\beta = \\
&\mu_\alpha^{-1} S_{\alpha^{-1}} S_\alpha \phi_\alpha(b_\alpha) S_{\alpha^{-1}} S_\alpha(\tilde{a}_\alpha) v_\alpha \otimes \mu_\beta^{-1} S_{\beta^{-1}} S_\beta(a_\beta) S_{\beta^{-1}} S_\beta(\tilde{b}_\beta) v_\beta = \\
&\mu_\alpha^{-1} \phi_\alpha(b_\alpha)\tilde{a}_\alpha v_\alpha \otimes \mu_\beta^{-1} a_\beta \tilde{b}_\beta v_\beta = \\
&\Delta_{\alpha,\beta}(\mu_{\alpha\beta}^{-1} v_{\alpha\beta})
\end{aligned}$$

Hence P is well defined. Secondly, we show that P has an inverse map. Define a map $Q: F \to E$ by $Q(\theta) = \mu\theta = \{\mu_\alpha \theta_\alpha \in H_\alpha\}_{\alpha \in G}$, for any $\theta \in F$. Clearly, $PQ = \mathrm{id}_F$, $QP = \mathrm{id}_E$. Following Ref. [6], we know that Q is well defined. This completes the proof of the theorem.

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关于 Hopf 群余代数 ribbon 元的注记

赵晓凡　王栓宏

（东南大学数学系，南京 211189）

摘要：设 G 是一个带有单位元的离散群，H 是域 k 上的拟三角 Hopf G-余代数. 考虑了 H 的 G-群像元和 ribbon 元之间的关系. 首先证明了拟三角 Hopf G-余代数以及它的 Drinfeld 元的一些重要性质. 受到 Hopf 代数中群像元和 ribbon 元之间关系的启发，定义了一类特殊的 G-群像元. 最后利用 Drinfeld 元得到了所定义的特殊的 G-群像元和 ribbon 元之间的一个一一对应关系.

关键词：拟三角 Hopf G-余代数；G-群像元；ribbon 元；Drinfeld 元
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