

Detection optimization for resonance region radar with dense multi-carrier waveform

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Abstract: Unlike the existing resonance region radar systems (RRRS) that transmit the orthogonal frequency division multiplexing (OFDM) multi-carrier waveform, the dense multi-carrier (DMC) radar waveform which has a narrower frequency interval than the traditional OFDM waveform is proposed. Therefore, in the same frequency bandwidth, the DMC waveform contains more sub-carriers and provides more frequency diversity. Additionally, to further improve detection performance, a novel optimal weight accumulation target detection (OWATD) method is proposed, where the echo electromagnetic waves at different frequencies are accumulated with the optimal weight coefficients. Then, with the signal-to-noise ratio (SNR) of echo waveform approaching infinity, the asymptotic detection performance is analyzed, and the condition that the OWATD method with the DMC outperforms the matched filter with the OFDM is presented. Simulation results show that the DMC outperforms the OFDM in the target detection performance, and the OWATD method can further improve the detection performance of the traditional methods with both the OFDM and DMC radar waveform.

Key words: constant false alarm rate (CFAR); dense multi-carrier waveform; detection optimization; resonance region radar system

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In the study of modern anti-stealth radar systems^[1], the electromagnetic waves resonate with the target when the target feature size is comparable with the wavelength of the transmitted waveform^[2-3]. Hence, this resonance can be utilized to significantly improve target detection performance in the resonance region radar systems (RRRS)^[4-6].

As wideband waveforms can provide better range and velocity resolution, they are widely utilized in traditional radar systems^[7-8]. Furthermore, the wideband waveform can cover the target resonating frequency to improve the anti-stealth ability. Therefore, the multi-carrier phase-co-

ded (MCPC) waveform^[9-10] as a realization of the wide-band signal is applied to the radar systems by Levanon et al.^[7,11-13], and the MCPC waveform can provide larger bandwidth-time (BT) product than the traditional wide-band ones^[8]. The orthogonal frequency division multiplexing (OFDM) signal can take advantage of the spectral efficiency and can be generated conveniently, so it is widely adopted to the conventional MCPC waveform^[14]. In addition, the target detection performance can be improved by the OFDM waveform, where frequency diversity is attained by the target scattering coefficients (TSC) at different frequencies^[15].

At the target detection part of the conventional OFDM radar systems, the echo waveform directly passes through the corresponding matched filter to obtain the maximal signal-to-noise ratio (SNR) while maintaining the performance of the range resolution^[16-18], which is known as the pulse compress process. However, when we consider the frequency diversity provided by the TSC, the target detection performance can be further improved.

In this work, a novel MCPC waveform based on the dense multi-carrier (DMC) is proposed for the RRRS, where the frequency interval is narrower than that of the OFDM signal and more sub-carriers can be provided within the same bandwidth. Therefore, the performance of the target detection can be further improved by the frequency diversity. Moreover, a novel target detection method, namely optimal weight accumulation target detection (OWATD), is proposed by accumulating the echo electromagnetic waves with the optimally weight coefficients at different frequencies. Then, the asymptotic detection performance is illustrated as the SNR of the echo waveform tends to infinity, and the condition that the OWATD method with DMC outperforms the matched filter with OFDM is proposed.

1 Model of RRRS with Multi-Carrier Waveform

The model of the RRRS considered in this work is shown in Fig. 1. As the extended binary phase shift keying (EBPSK) modulation can provide higher bandwidth and power efficiency with the tighter spectrum than the traditional binary phase shift keying (BPSK) modulation^[19], the MCPC waveform based on the EBPSK modulation is utilized as the transmitted waveform. The matrix $S \triangleq (s_1, s_2, \dots, s_N) \in \mathbf{R}^{L \times N}$ denotes the collection of the

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transmitted waveform, where \mathbf{s}_i denotes the sample vector of the i -th sub-carrier signal; L denotes the length of the signal vector; and N denotes the number of the sub-carriers. The waveform is controlled by the coding matrix $\mathbf{A} \in \mathbf{R}^{N \times M}$, where M denotes the code length of each sub-carrier. When the target feature size is comparable with the wavelength of the transmitted waveform, the electromagnetic waves resonate with the target, and the echo signal \mathbf{r} can be modeled as^[20]

$$\mathbf{r} = \mathbf{S}\boldsymbol{\alpha} + \mathbf{n} \quad (1)$$

where $\boldsymbol{\alpha} \triangleq \{\alpha_1, \alpha_2, \dots, \alpha_N\}^T$ denotes the TSC vector at different frequencies; $\mathbf{n} \sim N(0, \sigma_n^2 \mathbf{I})$ denotes the additive white Gaussian noise (AWGN) with the mean and covariance being 0 and $\sigma_n^2 \mathbf{I}$, respectively; \mathbf{I} denotes an identity matrix; σ_n^2 denotes the variance of noise. To facilitate the analysis, we assume that the TSC follows the independent Gaussian distribution with the same variance but different means, i. e., $\boldsymbol{\alpha} \sim N(\mathbf{m}, \sigma_\alpha^2 \mathbf{I})$, where $\mathbf{m} \triangleq \{m_1, m_2, \dots, m_N\}^T$ denotes the mean of the TSC, and σ_α^2 denotes the variance of the TSC. After receiving the echo waveform \mathbf{r} , the detection method based on the constant false alarm rate (CFAR) is adopted to detect the presence of the target. In the following section, the traditional detection method based on the matched filter is presented.

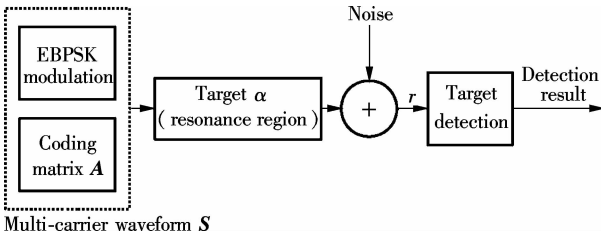


Fig. 1 RRRS model with multi-carrier waveform

2 Target Detection Based on OMMF Method

The target detection method based on the matched filter and OFDM waveform is named the OMMF method in this paper. With the knowledge of the TSC, the compressed waveform can be obtained after passing the matched filter,

$$\lambda \triangleq \mathbf{r}^T \mathbf{h}_m \quad (2)$$

where $\mathbf{h}_m \triangleq \sum_{n=1}^N \mathbf{s}_n$ denotes the matched filter for the multi-carrier waveform. Suppose that H_1 and H_0 represent the presence and absence of the target, respectively, then the target detection based on the CFAR theory is

$$\lambda \underset{H_0}{\overset{H_1}{\geq}} v \quad (3)$$

where v is the target detection threshold. When the target is absent, only noise can be received. Then the compressed waveform from Eq. (2) is

$$\lambda_0 \triangleq \lambda \mid H_0 = \mathbf{n}^T \mathbf{h}_m \quad (4)$$

As the noise follows the Gaussian distribution, the linear transformation of \mathbf{n} , denoted as λ_0 , also follows the Gaussian distribution, which can be determined by the mean and variance parameters as

$$\begin{aligned} \varepsilon\{\lambda_0\} &= \varepsilon\{\mathbf{n}^T\} \mathbf{h}_m = 0 \\ \varepsilon\{(\lambda_0 - \varepsilon\{\lambda_0\})^2\} &= \varepsilon\{(\mathbf{n}^T \mathbf{h}_m)^2\} = P_s \sigma_n^2 \end{aligned} \quad (5)$$

where P_s denotes the transmitted power and $\varepsilon\{\cdot\}$ denotes the expectation operation. Hence, the probability of false alarm based on the detection method (3) can be expressed as

$$P_{FA} = P(\lambda_0 \geq v) = \int_v^\infty G(\lambda, 0, P_s \sigma_n^2) d\lambda = Q\left(\frac{v}{\sqrt{P_s \sigma_n^2}}\right) \quad (6)$$

where $G(x, \mu, \sigma^2) \triangleq \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ and

$$Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt.$$

From the probability of the false alarm P_{FA} , the threshold of the CFAR detection can be set as

$$v = Q^{-1}(P_{FA}) \sqrt{P_s \sigma_n^2} \quad (7)$$

Therefore, the target detection process represented in formula (3) is

$$\mathbf{r}^T \mathbf{h}_m \underset{H_0}{\overset{H_1}{\geq}} Q^{-1}(P_{FA}) \sqrt{P_s \sigma_n^2} \quad (8)$$

After obtaining the threshold of the CFAR detection, the probability of target detection can be obtained. With the presence of target, the compressed waveform from Eq. (2) is

$$\lambda_1 \triangleq \lambda \mid H_1 = (\mathbf{S}\boldsymbol{\alpha} + \mathbf{n})^T \mathbf{h}_m \quad (9)$$

As the parameters $\boldsymbol{\alpha}$ and \mathbf{n} follow the Gaussian distribution, the linear transformation also follows the Gaussian distribution, which can be determined by the mean and the variance,

$$\begin{aligned} m &= \varepsilon\{\lambda_1\} = \varepsilon\{\boldsymbol{\alpha}^T \mathbf{S}^T \mathbf{h}_m\} = (\mathbf{S}\mathbf{m})^T \mathbf{h}_m \\ \sigma^2 &= \varepsilon\{(\lambda_1 - \varepsilon\{\lambda_1\})^2\} = \varepsilon\{[\mathbf{h}_m^T (\mathbf{S}\boldsymbol{\alpha} + \mathbf{n}) - \\ &\quad (\mathbf{S}\mathbf{m})^T \mathbf{h}_m]^2\} = \sigma_\alpha^2 \frac{P_s^2}{N} + P_s \sigma_n^2 \end{aligned} \quad (10)$$

Eq. (10) indicates that the variance of λ_1 is increased by the TSC, so the performance of target detection is decreased. Finally, we can obtain the probability of target detection based on the OMMF method as

$$P_D(\mathbf{m}) = \int_v^\infty G(\lambda, m, \sigma^2) d\lambda = Q\left(\frac{Q^{-1}(P_{FA}) \sqrt{P_s \sigma_n^2} - m}{\sqrt{\sigma_\alpha^2 \frac{P_s^2}{N} + P_s \sigma_n^2}}\right) \quad (11)$$

The expectation of the probability of target detection with

different means of the TSC is

$$\bar{P}_D = \mathcal{E}_m \{P_D(\mathbf{m})\} \quad (12)$$

3 Target Detection Based on the OWATD Method

3.1 The OWATD method

In this subsection, we propose a OWATD method, which can maximize the probability of the target detection with the mean knowledge of the TSC. In this method, a vector $\mathbf{x} \triangleq \{x_1, x_2, \dots, x_N\}^T$ is utilized to control the allocation of the weight coefficients. Therefore, the target detection based on the CFAR can be expressed as

$$\lambda(\mathbf{x}, \alpha) \stackrel{H_1}{\geq} \stackrel{H_0}{v}(\mathbf{x}) \quad (13)$$

where $v(\mathbf{x})$ is the detection threshold, and

$$\lambda(\mathbf{x}, \alpha) \triangleq (\mathbf{S}\mathbf{x})^T \mathbf{r} \quad (14)$$

The vector of the weight coefficients \mathbf{x} is optimized to maximize the probability of target detection.

First, we give the threshold of the CFAR detection with the vector of the weight coefficients. When the target is absent, the probability of the false alarm is

$$P_{FA}(\mathbf{x}, \alpha) \triangleq P(\lambda_0(\mathbf{x}, \alpha) \geq v(\mathbf{x})) \quad (15)$$

where $\lambda_0(\mathbf{x}, \alpha) \triangleq (\mathbf{S}\mathbf{x})^T \mathbf{n}$ follows the Gaussian distribution. The mean and variance respectively are

$$\begin{aligned} m_{H_0} &\triangleq \mathcal{E}\{\lambda_0(\mathbf{x}, \alpha)\} = 0 \\ \sigma_{H_0}^2 &\triangleq \mathcal{E}\{\lambda_0^2(\mathbf{x}, \alpha)\} = \sigma_n^2 (\mathbf{S}\mathbf{x})^T (\mathbf{S}\mathbf{x}) \end{aligned} \quad (16)$$

Then, the probability of false alarm is

$$P_{FA}(\mathbf{x}, \alpha) = \int_{v(\mathbf{x})}^{\infty} G(t, 0, \sigma_{H_0}^2) dt = Q\left(\frac{v(\mathbf{x})}{\sigma_{H_0}}\right) \quad (17)$$

Therefore, the threshold of the CFAR target detection with the vector of the weight coefficients can be expressed as

$$v(\mathbf{x}) = Q^{-1}(P_{FA}) \sigma_{H_0} \quad (18)$$

Secondly, when the target is present, the probability of target detection is

$$P_D(\mathbf{x}, \alpha) \triangleq P(\lambda_1(\mathbf{x}, \alpha) \geq v(\mathbf{x})) \quad (19)$$

where $\lambda_1(\mathbf{x}, \alpha) \triangleq (\mathbf{S}\mathbf{x})^T (\mathbf{S}\alpha + \mathbf{n})$. When the mean TSC is known, $\lambda_1(\mathbf{x}, \alpha)$ in (19) follows the Gaussian distribution, and the mean and variance are

$$\begin{aligned} m_{H_1} &= \mathcal{E}\{\lambda_1(\mathbf{x}, \alpha)\} = (\mathbf{S}\mathbf{x})^T (\mathbf{S}\mathbf{m}) \\ \sigma_{H_1}^2 &= \mathcal{E}\{[\lambda_1(\mathbf{x}, \alpha) - m_{H_1}]^2\} = \\ &\sigma_n^2 (\mathbf{S}\mathbf{x})^T (\mathbf{S}\mathbf{x}) + \sigma_\alpha^2 (\mathbf{S}\mathbf{x})^T \mathbf{S} \mathbf{S}^T \mathbf{S}\mathbf{x} \end{aligned} \quad (20)$$

Then, the probability of target detection in (19) can be expressed as

$$P_D(\mathbf{x}, \alpha) = \int_{v(\mathbf{x})}^{\infty} G(t, m_{H_1}, \sigma_{H_1}^2) dt = Q\left(\frac{v(\mathbf{x}) - m_{H_1}}{\sigma_{H_1}}\right) \quad (21)$$

Since $Q(\cdot)$ is a monotonically decreasing function, the optimal weight coefficients which can maximize the probability of the target detection are equal to

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} \left\{ f(\mathbf{x}) = \frac{v(\mathbf{x}) - m_{H_1}}{\sigma_{H_1}} \right\} \quad (22)$$

The optimal weight coefficients \mathbf{x} can be obtained by letting the derivation of $f(\mathbf{x})$ be equal to 0, i. e. ,

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}^*} = \frac{g(\mathbf{x}^*)}{(\sigma_n^2 (\mathbf{S}\mathbf{x}^*)^T \mathbf{S}\mathbf{x}^* + \sigma_\alpha^2 \mathbf{x}^{*T} \mathbf{B}\mathbf{B}\mathbf{x}^*)^{3/2}} = 0 \quad (23)$$

where $\mathbf{B} = \mathbf{S}^T \mathbf{S}$ and

$$\begin{aligned} g(\mathbf{x}^*) &= \frac{Q^{-1}(P_{FA}) \sigma_n \sigma_\alpha^2}{\sqrt{\mathbf{x}^{*T} \mathbf{B}\mathbf{x}^*}} [(\mathbf{x}^{*T} \mathbf{B}\mathbf{B}\mathbf{x}^*) \mathbf{B}\mathbf{x}^* - \\ &(\mathbf{x}^{*T} \mathbf{B}\mathbf{x}^*) \mathbf{B}\mathbf{B}\mathbf{x}^*] + \sigma_n^2 [(\mathbf{x}^{*T} \mathbf{B}\mathbf{m}) \mathbf{B}\mathbf{x}^* - \\ &(\mathbf{x}^{*T} \mathbf{B}\mathbf{x}^*) \mathbf{B}\mathbf{m}] + \sigma_\alpha^2 [(\mathbf{x}^{*T} \mathbf{B}\mathbf{m}) \mathbf{B}\mathbf{B}\mathbf{x}^* - \\ &(\mathbf{x}^{*T} \mathbf{B}\mathbf{B}\mathbf{x}^*) \mathbf{B}\mathbf{m}] \end{aligned} \quad (24)$$

Letting $\mathbf{x}^* = \mathbf{m}$, we can obtain

$$g(\mathbf{x}^*) = \sigma_\alpha^2 \left(\frac{Q^{-1}(P_{FA}) \sigma_n}{\sqrt{\mathbf{m}^T \mathbf{B}\mathbf{m}}} - 1 \right) (\mathbf{m}^T \mathbf{B}\mathbf{B}\mathbf{m}\mathbf{m} - \mathbf{m}^T \mathbf{B}\mathbf{m}\mathbf{B}\mathbf{m}) \quad (25)$$

The condition that $g(\mathbf{x}) = 0$ in (25) is that the nonzero eigenvalues of \mathbf{B} are equal. Fig. 2 depicts the eigenvalues distribution of \mathbf{B} , when the frequency interval is 0.08 of the orthogonal one. Therefore, the nonzero eigenvalues of \mathbf{B} are approximately equal under the condition of the DMC waveform considered in this work. Besides, when sub-carriers are orthogonal, the nonzero eigenvalues are also equal. Therefore, $\mathbf{x}^* = \mathbf{m}$ is the optimal weight coefficients. This conclusion is similar to the maximum ratio combining method in the diversity theory, but the optimization objective function considered here is the probability of target detection. Then the process of the target detection based on CFAR is

$$(\mathbf{S}\mathbf{m})^T \mathbf{r} \stackrel{H_1}{\geq} \stackrel{H_0}{v}(\mathbf{m}) \quad (26)$$

where $v(\mathbf{m}) \triangleq Q^{-1}(P_{FA}) \sqrt{\sigma_n^2 (\mathbf{S}\mathbf{m})^T (\mathbf{S}\mathbf{m})}$.

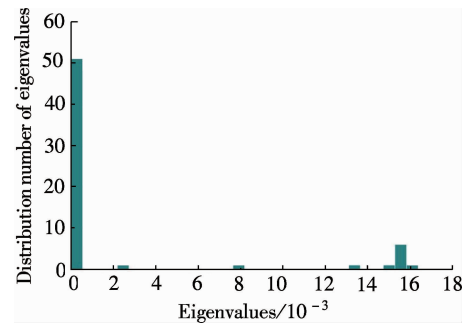


Fig. 2 RRRS model with multi-carrier waveform

Finally, we can obtain the probability of target detection with the optimal vector of the weight coefficients,

$$P_D(\mathbf{m}) = Q\left(\frac{v(\mathbf{m}) - (\mathbf{S}\mathbf{m})^T(\mathbf{S}\mathbf{m})}{\sqrt{\sigma_n^2(\mathbf{S}\mathbf{m})^T(\mathbf{S}\mathbf{m}) + \sigma_\alpha^2(\mathbf{S}\mathbf{m})^T\mathbf{S}\mathbf{S}^T\mathbf{S}\mathbf{m}}}\right) \quad (27)$$

The detection probability with the mean of the TSC is

$$\bar{P}_D = \varepsilon_m\{P_D(\mathbf{m})\} \quad (28)$$

3.2 Performance comparison between the DMC and OFDM waveform

This section presents the performance comparison between the dense and orthogonal multi-carrier waveform with the same signal bandwidth with the mean knowledge of the TSC. The number of the sub-carriers in the DMC is

$$N_d \triangleq \left\lfloor \frac{B}{\Delta f} \right\rfloor \quad (29)$$

where B is the signal bandwidth; $\Delta f \triangleq \zeta \Delta f_o$ denotes the frequency interval of the DMC waveform; Δf_o denotes the frequency interval of orthogonal multi-carrier waveform; and $\zeta \in (0, 1)$ denotes the dense parameter. Therefore, the number of dense sub-carriers N_d is $\frac{1}{\zeta}$ of the orthogonal one $N_o \triangleq \left\lfloor \frac{B}{\Delta f_o} \right\rfloor$.

The probability of the target detection for different ζ can be obtained from (21) that

$$P_D(\zeta) = Q\left(\frac{Q^{-1}(P_{FA})\sigma_{H_0} - m_{H_1}}{\sigma_{H_1}}\right) \quad (30)$$

As SNR tends to infinity, we can obtain the asymptotic probability of the target detection from (30) in the RRRS with the DMC waveform that

$$\bar{P}_D(\zeta) \triangleq \lim_{\text{SNR} \rightarrow +\infty} Q\left(\frac{Q^{-1}(P_{FA})\sigma_{H_0} - m_{H_1}}{\sigma_{H_1}}\right) = Q\left(-\frac{(\mathbf{S}\mathbf{m})^T(\mathbf{S}\mathbf{m})}{\sqrt{\sigma_\alpha^2(\mathbf{S}^T\mathbf{S}\mathbf{m})^T(\mathbf{S}^T\mathbf{S}\mathbf{m})}}\right) \quad (31)$$

The expectation of the asymptotic probability of target detection is

$$\bar{P}_D(\zeta) = \varepsilon_m(\bar{P}_D(\zeta)) \quad (32)$$

In order to analyze the performance of the target detection, the approximation result of (32) is given. If the TSC at different frequencies are the same, i. e., $\bar{m} = m_1 = \dots = m_N$, then (31) can be written as

$$P'_D(\zeta) = Q\left(-\frac{\bar{m}P_s}{\sqrt{\sigma_\alpha^2\mathbf{h}_m^T\mathbf{S}\mathbf{S}^T\mathbf{h}_m}}\right) \quad (33)$$

The expectation of (33) is

$$\bar{P}'_D(\zeta) \triangleq \varepsilon_m(P'_D(\zeta)) \quad (34)$$

We simplify (34) as follows. For arbitrary m , n , if the following holds

$$a \triangleq \sum_{i=1}^{N_d} \mathbf{s}_m^T \mathbf{s}_i = \sum_{i=1}^{N_d} \mathbf{s}_n^T \mathbf{s}_i \quad (35)$$

then

$$P_s = \sum_{m=1}^{N_d} \sum_{n=1}^{N_d} \mathbf{s}_m^T \mathbf{s}_n = N_d a \quad (36)$$

Then Eq. (33) can be simplified as

$$P''_D(\zeta) = Q\left(-\frac{m\sqrt{N_d}}{\sigma_\alpha}\right) = Q\left(-\frac{m}{\sigma_\alpha\sqrt{\zeta}}\right) \quad (37)$$

Eq. (37) shows that reducing the sub-carrier interval, which increases the number of sub-carriers N_d , can improve the target detection performance. The expectation of Eq. (37) is

$$\bar{P}''_D(\zeta) = \varepsilon_m\{P''_D(\zeta)\} \quad (38)$$

To analyze the performance of the target detection, the assumption that the TSC at different frequencies are the same in (34) and the expression of the target detection is further simplified by the condition in (35). The performance effect of these assumptions will be given in the next section.

3.3 The unknown mean knowledge of TSC

When the mean of the TSC is unknown, we can adopt the best estimation method, which is the least square (LS) estimation under the condition of this work, to estimate the mean of TSC $\hat{\mathbf{m}} = \mathbf{m} + \boldsymbol{\delta}$, where $\boldsymbol{\delta} \sim N(0, \boldsymbol{\Sigma})$ is the vector of estimation errors, and $\boldsymbol{\Sigma}$ is the covariance matrix.

The likelihood function $f(\mathbf{r}; \mathbf{m})$ of the mean of the TSC \mathbf{m} follows the Gaussian distribution as

$$f(\mathbf{r}; \mathbf{m}) \sim N(\mathbf{S}\mathbf{m}, \sigma_\alpha^2\mathbf{S}\mathbf{S}^T + \sigma^2\mathbf{I}) \quad (39)$$

The covariance matrix of the estimation error $\boldsymbol{\delta}$ can be obtained by the Fisher information, and it is

$$\mathbf{J} = -\varepsilon\left\{\frac{\partial^2 \ln f(\mathbf{r}; \mathbf{m})}{\partial \mathbf{m} \partial^T \mathbf{m}} \middle| \mathbf{m}\right\} \quad (40)$$

Substituting (39) into (40), we can obtain the entry of \mathbf{J} at the p -th row and the q -th column as

$$J_{pq} = \mathbf{s}_p^T (\sigma_\alpha^2\mathbf{S}\mathbf{S}^T + \sigma^2\mathbf{I})^{-1} \mathbf{s}_q \quad (41)$$

Then the covariance matrix is the inverse of the Fisher information.

$$\boldsymbol{\Sigma} = \mathbf{J}^{-1} \quad (42)$$

Therefore, the estimated mean of TSC $\hat{\mathbf{m}}$ is utilized during the process of the target detection method (26) proposed in this work.

$$(\hat{\mathbf{S}}\mathbf{m})^T \mathbf{r} \stackrel{H_1}{\underset{H_0}{\gtrless}} v(\hat{\mathbf{m}}) \quad (43)$$

where $v(\hat{\mathbf{m}}) \triangleq Q^{-1}(P_{FA}) \sqrt{\sigma_\alpha^2(\hat{\mathbf{S}}\mathbf{m})^T(\hat{\mathbf{S}}\mathbf{m})}$. Furthermore, the probability of target detection with the estimation error

δ is

$$P_D(\delta, \mathbf{m}, \zeta) = P(\lambda > \nu | H_1) = \frac{P((\mathbf{S}\mathbf{m})^T(\mathbf{S}\mathbf{m} + \mathbf{S}\boldsymbol{\alpha}' + \mathbf{n} + \mathbf{S}\delta) + (\mathbf{S}\delta)^T\mathbf{S}\boldsymbol{\alpha}' + (\mathbf{S}\delta)^T\mathbf{n} > \nu(\hat{\mathbf{m}}))}{1} \quad (44)$$

where $\boldsymbol{\alpha}' \sim N(0, \sigma_a^2 \mathbf{I})$, and the expectation of the probability of target detection can be obtained under the estimation error and the mean of TSC,

$$\bar{P}_D(\zeta) = \mathcal{E}_{\delta, \mathbf{m}} \{P_D(\delta, \mathbf{m}, \zeta)\} \quad (45)$$

4 Simulation Results

In this section we conduct the simulation of the RRRS with the DMC waveform, and the simulation parameters are given in Tab. 1. As the practical radar data is difficult to obtain, the simulation data is utilized to realize the proposed RRRS with the DMC waveform and the OWATD method. The simulation and theoretical probability of the target detection with and without the mean knowledge of the TSC are, respectively, shown in Fig. 3 and Fig. 4, where the OWATD method proposed in this work is adopted. As shown in Fig. 3 and Fig. 4, the simulation results are consistent with the theoretical ones, which illustrates the accuracy of theoretical results obtained in (28) and (45).

Tab. 1 Simulation parameters

Parameter	Value
Number of orthogonal sub-carriers N_o	5
Dense parameter ζ	$\zeta \in \{0.1, 0.5, 0.9\}$
Number of dense sub-carriers N_d	$N_d \in \{50, 10, 5\}$
Transmitted power P_s	1
Variance of TSC σ_a^2	1
TSC α	Uniformly distributed within $[0, 1]$
Carrier frequency f_c /MHz	10
Pulse width T_p /s	10^{-5}

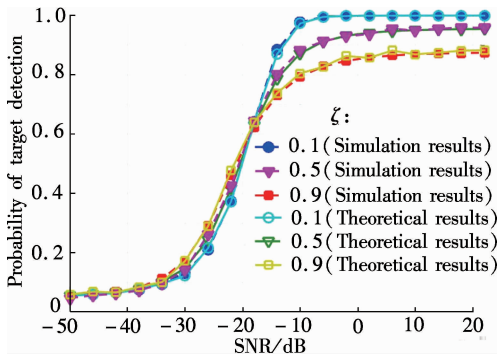


Fig. 3 Comparison between theoretical and simulation results using the OWATD method with mean knowledge of TSC

Fig. 5 depicts the comparison between the probability of target detection of the DMC waveform using the OWATD method and that of the OMMF method, where the mean of the TSC is known and the bandwidth is the

same. It can be seen from Fig. 5 that different frequency intervals of the sub-carriers have significant effects on the probability of target detection, which can be further improved by reducing the frequency interval at high SNR.

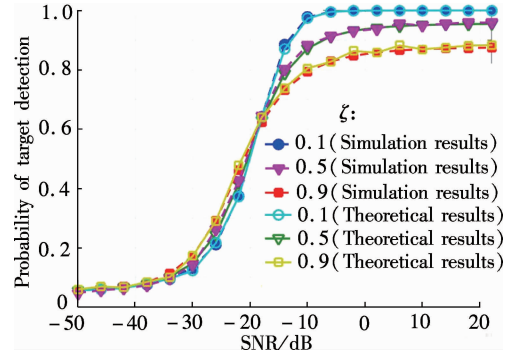


Fig. 4 Comparison between theoretical and simulation results using the OWATD method without mean knowledge of TSC

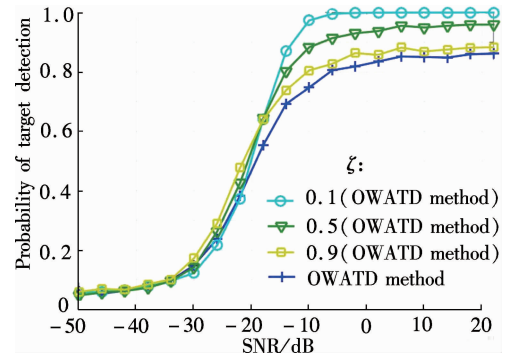


Fig. 5 Comparison between the OWATD method with DMC waveform and the OMMF method with mean knowledge of TSC

The comparison of the probability of target detection between the proposed OWATD method and the traditional OMMF method is shown in Fig. 6, where the mean of the TSC is known and the bandwidth are the same. As shown in Fig. 6, when we consider the effect of the frequency interval of sub-carriers, the probability of target detection is improved by enlarging the frequency interval at low SNR. However, the probability of target detection is reduced by enlarging the frequency interval at high SNR. Therefore, the DMC waveform has the advantages of the detection performance at a high SNR.

Fig. 7 depicts the asymptotic probability of target detection when SNR tends to infinity. In this paper, in order to analyze the performance of the target detection, we assume that the TSC at different frequencies are the same in (34), and further simplify the expression of the target detection with the condition in (35). The performance effect of these assumptions is highlighted in Fig. 7, and is compared with that without approximation assumption. These assumptions achieve a relatively small probability of the target detection, but have the same tendency as the frequency interval. Furthermore, the performance of the target detection with the DMC waveform via the

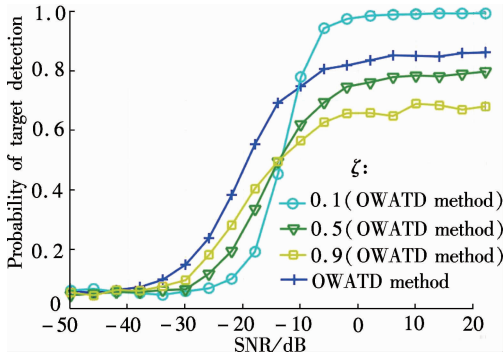


Fig. 6 Comparison between the OWATD method with DMC waveform and the OMMF method without mean knowledge of TSC

OWATD method is better than that via the OMMF method for all frequency intervals of the sub-carriers, and the asymptotic probability of the target detection without the mean knowledge of the TSC is lower than that using the OMMF method with the increase of the frequency interval. Therefore, for the asymptotic consideration, the DMC waveform via the OWATD method outperforms that via the traditional OMMF method.

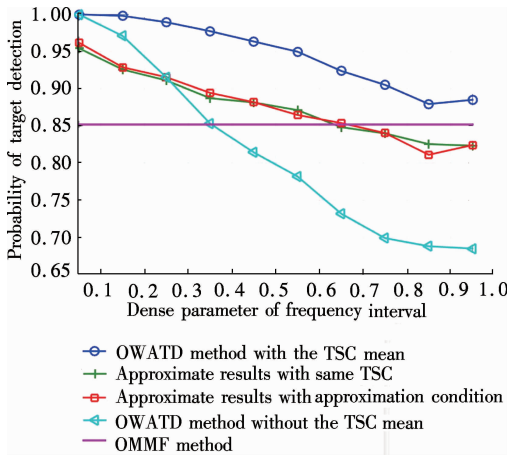


Fig. 7 Asymptotic probability of the target detection

5 Conclusion

The problem of the target detection in the RRRS is considered, and the DMC waveform is proposed as the transmitted waveform. By exploiting the frequency diversity, the performance of target detection with the DMC waveform outperforms the traditional OFDM signal. Furthermore, we propose the OWATD method to further improve the target detection performance. As the SNR of the echo signal tends to infinity, the condition that the asymptotic detection performance of the OWATD method using the DMC waveform outperforms the matched filter using the OFDM signal has also been illustrated. Finally, simulation results demonstrate the efficiency of the OWATD method with the DMC waveform in the RRRS. In the future work, the interference of clutter will be considered.

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基于密集多载波波形的谐振区雷达检测优化

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摘要:区别于现有发射正交频分复用(OFDM)多载波波形的谐振区雷达系统(RRRS),提出了拥有比传统OFDM波形更窄频率间隔的密集多载波(DMC)雷达波形.在相同带宽内,DMC波形包含了更多的子载波,从而能提供更高的频率分集.为了进一步提高检测性能,提出了一种新的最优权重累积目标检测(OWATD)方法.该方法采用最佳权重系数来累积不同频率的电磁回波,分析了当回波信噪比(SNR)趋于无穷大时的极限检测性能,并给出了采用DMC的OWATD方法优于采用OFDM的匹配滤波方法的条件.仿真结果表明,DMC的目标检测性能优于OFDM,而且OWATD方法可以进一步提高采用DMC和OFDM波形的传统方法的检测性能.

关键词:恒虚警概率;密集多载波波形;检测优化;谐振区雷达系统

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