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Distribution algorithm of entangled particles for wireless quantum communication mesh networks

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Abstract: With ensured network connectivity in quantum channels, the issue of distributing entangled particles in wireless quantum communication mesh networks can be equivalently regarded as a problem of quantum backbone nodes selection in order to save cost and reduce complexity. A minimum spanning tree (MST)-based quantum distribution algorithm (QDMST) is presented to construct the mesh backbone network. First, the articulation points are found, and for each connected block uncovered by the articulation points, the general centers are solved. Then, both articulation points and general centers are classified as backbone nodes and an MST is formed. The quantum path between every two neighbor nodes on the MST is calculated. The nodes on these paths are also classified as backbone nodes. Simulation results validate the advantages of QDMST in the average backbone nodes number and average quantum channel distance compared to the existing random selection algorithm under multiple network scenarios.

Key words: wireless quantum communication networks; entangled particles distribution; wireless mesh networks; minimum spanning tree

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In the traditional research of quantum physics, the study of entangled particle distribution focuses on how to produce high quality and high intensity entangled particles, and to distribute them onto two nodes with maximum distance between them[1-4]. During the research of wireless quantum networks[5-10], it is necessary to study the entangled particle distribution in terms of the entire network. Future wireless quantum networks based on entangled states may be large scale, which makes it impossible to distribute high quality entangled particles directly between the source and destination because of the long distance. Instead, the quantum path needs to be constructed with several quantum channels hop by hop via intermediate nodes. Therefore, in order to determine the nodes' capability of producing entangled particles, investigation on transmission speed and fidelity of entangled particles in the network becomes necessary, which is usually referred to as an entangled particles distribution problem.

For wireless quantum networks, one of the essential requirements for quantum information transmission between two nodes is the existence of the quantum path. The path can be either a direct quantum channel between them or a quantum path between them via entanglement swapping. Generally in the network, the entangled particles can be produced by nodes involved in the communication or by dedicated devices.

In a quantum communication network based on the backbone mesh structure[11-13], each node has the capability of quantum teleportation. They are able to exchange routing information and quantum measurement information with traditional radio communication transceivers. The backbone of the quantum network is composed of backbone nodes with extra capability of producing and distributing entangled particles. While each non-backbone node is connected to one or more backbone nodes, and it has no capability of producing and distributing entangled particles for saving cost and reducing complexity. Thus, the entangled particles distribution issue in large-scale quantum communication networks with the mesh structure can be regarded as a quantum backbone node selection problem while ensuring the network connectivity of quantum channels.

1 The Model of QDMST Algorithm

There are two kinds of channels, i. e. quantum channels and radio channels in a wireless quantum communication network. While in a traditional radio communication network, there may be no direct quantum channel between two neighbor nodes, and vice versa, as shown in Fig. 1. Due to the variations with entangled particles in

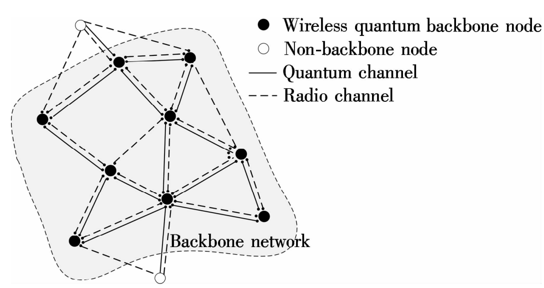

Fig. 1 Wireless quantum communication network based on mesh structure

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quantum channels, the topology of a quantum network varies much more significantly than that of a traditional radio network. Therefore, it is necessary to develop a dedicated algorithm to construct the backbone network for producing and distributing entangled particles. This paper presents a quantum distribution algorithm based on a minimum spanning tree (QDMST) to reconstruct a backbone network when network outage occurs due to topology change.

Assume that N quantum nodes are randomly distributed within a square region, which includes M backbone nodes and $N-M$ non-backbone nodes. Each non-backbone network node acts as a source and/or a destination node, which is connected to at least one backbone node via a quantum channel and a radio channel. In the case of a fixed number of nodes, fewer nodes in the backbone network means fewer nodes capable of generating and distributing entangled particles. Thus, the algorithm focuses on minimizing the number of backbone nodes in order to reduce network cost and complexity. Also, to directly ensure high-quality, low-cost teleportation[14-16] between the two quantum nodes, the distance between them cannot be too long[16-17] where a threshold is assumed to be R. In order to ensure network connectivity and that each terminal node can transfer quantum information through the backbone network, the distance between each terminal node and at least one backbone node must be less than the effective teleportation distance threshold. The objective of the QDMST algorithm is to minimize the number of backbone nodes:

$$\min M \tag{1}$$

$$\text{s. t.} \quad M < N, \ r_i < R \tag{2}$$

where r_i is the shortest distance between the terminal node i and backbone nodes, and $1 \leqslant i \leqslant N-M$.

2 Implementation of QDMST Algorithm

In this paper, the mesh network architecture model is constructed based on the graph theory, and the QDMST algorithm is implemented. Assume that the square area of the model is denoted as $G(V, E)$, where $V = \{v_1, v_2, \dots v_N\}$ is the quantum node set. Only if the distance between the pair of nodes is less than R, there is an edge between them, namely the existence of a quantum channel. The edge set is defined as $E = \{e_1, e_2, \dots, e_n\}$ $(n < N)$, the distance of which is set as the weight of the edge denoted by $l(e_i)$ because the distribution quality of entangled particles is associated with the distance. Now, we present the description of the proposed algorithm QDMST as follows:

1) First a connected graph $G(V, E)$ is randomly generated.

2) The articulation points (if they exist) of the connected graph are labelled as the initial nodes of the backbone network.

3) If all nodes are covered by backbone nodes, directly go to Step 6).

4) Connected components composed by nodes not belonging to the backbone network are found.

5) The general center of each connected block is obtained and classified as the backbone nodes. Go to Step 3).

6) The minimum spanning tree (MST) of the backbone network is formed according to the distance weights.

7) If there is a quantum channel between any two adjacent nodes on the MST, the algorithm stops; otherwise go to Step 8).

8) The shortest path in G between the two nodes is found, and the corresponding nodes on the path are classified as the backbone nodes. Go to Step 6).

The steps concerned in the algorithm include four sub-algorithms: the shortest distance algorithm, the articulation point algorithm, the general center algorithm and the MST algorithm.

First, the algorithm needs to obtain the shortest path between any two nodes in the network. The success probability of single-hop teleportation will decrease exponentially as distance increases due to the loss or distortion of entangled particles caused by environmental noise in free space. Therefore, the shortest distance algorithm is the basis of the QDMST algorithm, and it is used to calculate the articulation points and general centers. The Warshall-Floyd algorithm[18] is a classic shortest distance algorithm which takes advantage of dynamic programming by using an adjacency matrix to describe the topology. Combined with the structure characteristics of the quantum network, the shortest distance algorithm used in this paper is described as follows:

Algorithm 1 Shortest distance algorithm

Input: $\boldsymbol{W} = (w_{ij})_{N \times N}$ is the adjacency matrix of graph G; w_{ij} is the weight of e_{ij}; k_1 and k_2 are the two nodes.

Output: P is the shortest path between k_1 and k_2, and nodes on the path are sorted by sequence order; min $\boldsymbol{D}$ is the distance of the shortest path.

```
D = (d_ij);   // d_ij is the shortest distance from v_i to v_j.
for each I, j    // Initialize d_ij.
        d_ij = w_ij, k = 1;
for each i, j   // Update d_ij.
    if   d_ik + d_kj < d_ij then
        d_ij = d_ik + d_kj;
    if   k = N then
        stop;
min D = D(k_1, k_2);
k_x = k_2, 0 = 1, P(s) = k_2;
for each i
    if   D(k_1, i) = D(k_1, k_x) - W(i, k_x) then
        k_x = i, P(s + 1) = i, s = s + 1;
    If   k_x = k_1 then
        stop;
```

A connected graph without articulation points[19] is called a biconnected graph. Deleting any node of the biconnected graph will not damage the network connectivity. If articulation points exist, they are classified as backbone nodes to reduce the number of them and meanwhile ensure network connectivity. According to their characteristics, the steps of the algorithm to determine articulation points are presented in Algorithm 2.

Algorithm 2 Articulation point determining algorithm

Input: $\boldsymbol{W} = (w_{ij})_{N\times N}$, the adjacency matrix of graph G;

Output: Articulation point v_i.

for each i

//Find the adjacency matrix $\boldsymbol{A}_i$ of the subgraph $G - v_i$.

Delete the i-th row and the j-th column from $\boldsymbol{W} \to \boldsymbol{A}_i$;

do Algorithm 1//Find the shortest distance matrix $\boldsymbol{D}_i$ of subgraph $G - v_i$.

input $\boldsymbol{A}_i$;

output $\boldsymbol{D}_i$;

//Determine whether v_i is the articulation point by $\boldsymbol{D}_i$.

If there is non-zero element in $\boldsymbol{D}_i$ in addition to the main diagonal element then

$v_i \notin$ Articulation points;

else $v_i \in$ Articulation points;

The initially selected backbone nodes may not be able to cover all nodes in the network. For the uncovered nodes, they are divided into various connected components and their centers are then nominated as backbone nodes for completing the coverage. The object of the QDMST is to construct a mesh quantum network with the minimum number of backbone nodes, each of which can cover more other nodes. The general center is the node that has a minimum distance to the most remote node among all nodes in the network[20]. Therefore, it is appropriate to select general centers as backbone nodes. The general center algorithm is presented as follows:

Algorithm 3 General center determining algorithm

Input: $\boldsymbol{W} = (w_{ij})_{N\times N}$, the adjacency matrix of graph G, where w_{ij} is the weight of e_{ij};

Output: The general center i_0 of the graph.

//Find the shortest distance matrix of each node $\boldsymbol{D}(d_{i,k})$, where $i, k = 1, 2, \ldots, N$;

do Algorithm 1

input $\boldsymbol{W}$

output $\boldsymbol{D}(d_{i,k})$;

//Calculate the farthest distance for each node to each edge, where k_1 and k_2 are two nodes on e_j;

for each I, j

$d'_{ij} = 0.5\ [d_{i,k1} + l(e_j) + d_{i,k2}]$;

//Find the node whose distance to the farthest node is the minimum

for each i, j

$d_0 = \min\{\min\{d'_{ij}\}\}$;

i_0 ⟵ the SN of this node.

After selecting necessary backbone nodes, a connected graph is formed based on these nodes by an MST algorithm. The MST is the tree that the sum of all edge weights is minimized among all of the spanning trees of the connected graph. The Prim algorithm and Kruskal algorithm[21] are the most popular MST algorithms. The Kruskal MST algorithm used in this paper is presented as follows:

Algorithm 4 Minimum spanning tree

Input: $\boldsymbol{W} = (w_{ij})_{N\times N}$, adjacency matrix of graph G, where w_{ij} is the weight of e_{ij};

Output: Adjacency matrix of MST b.

$T = \phi$ //Set the tree empty

for each i //Join all nodes in T

$T = T \cup \{v_i\}$ //T includes all nodes without edge

for each i, j

do $e_{i,j}(\in E)$ sorting by the weights w_{ij} ascending

for each $e_{i,j}(\in E)$

if v_i and v_j are not in the same connected component

$T = T \cup \{e_{ij}\}$; //Join e_{ij} in T

Combine the two connected component;

b ⟵ adjacency matrix of T

For the MST obtained by Algorithm 4, the constraint in (2) that $r_i < R$ may not be guaranteed. Therefore, we need to use the shortest distance algorithm to reconnect some pair of vertices on the tree whose edge weights are greater than R.

Based on the above algorithms, the proposed algorithm QDMST can be summarized as follows:

Algorithm 5 QDMST algorithm

Randomly generate a connected graph $G(V, E)$;

$F = \phi$ //Set backbone nodes set empty

do Algorithm 2 //Set articulation nodes as initial backbone

input $\boldsymbol{W}(G)$

output articulation nodes of G

$F = F \cup \{$articulation nodes of $G\}$

Step 1 for each $v_i(\in V)$

if v_i is not covered by backbone nodes then

goto Step 2;

goto Step 3; //All nodes covered by backbone

Step 2 do find $\{T_n\}$, the connected components composed by the nodes uncovered by backbone;

for each T_n //get general center of connected components into backbone

do Algorithm 3

input $\boldsymbol{W}(T_n)$

output general center

$F = F \cup \{$general center$\}$

goto Step 1;

Step 3 do Algorithm 4//get the MST of backbone network

input $\boldsymbol{W}(F)$

output MST $B(V_B, E_B)$

for each $e_{i,j}(\in E_B)$ //determine whether a quantum channel exists between any two adjacent nodes on the MST

if the quantum channel does not exist between v_i and v_j then

goto Step 4;

end //If quantum channels exist for all edge, the algorithm ends

Step 4 do Algorithm 1 //Find the shortest path

input $\boldsymbol{W}(G)$

output nodes on the shortest path

$F = F \cup \{$nodes on the shortest path$\}$ //Join nodes on shortest path in the backbone

goto Step 3;

3 Performance Analysis and Simulation

To demonstrate algorithm procedures and verify their effectiveness, the steps of an exemplified QDMST algorithm are shown in Fig. 2 to Fig. 5. A connected graph is randomly generated within a square area of 1 000 m × 1 000 m, as shown in Fig. 2. The quantum teleportation distance threshold R is 250 m, and the total number of nodes N is 30. Each black line indicates the existence of a reliable quantum channel and a radio channel between two relevant quantum nodes.

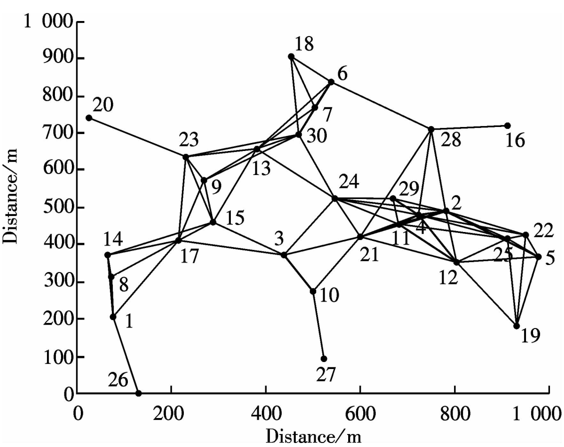

Fig. 2 Randomly generated connected graph

In the case of a connected graph in Fig. 2, the set of articulation points is first determined as {1, 10, 23, 28}, which does not cover all nodes, as shown in Fig. 3. The radius of the circle is R, and the small squares denote the backbone nodes. Uncovered nodes {{7, 18}, {5, 11, 12, 19, 22, 24, 25}} are separated into two connected components. Subsequently, the general centers {7, 12} of two uncovered connected components are added into the backbone nodes set as {1, 7, 10, 12, 23, 28}. There are still some uncovered nodes, which require the algorithm to further

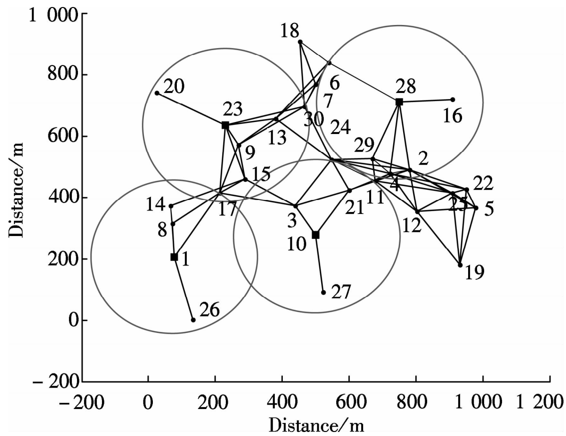

Fig. 3 Articulation points as initial backbone nodes

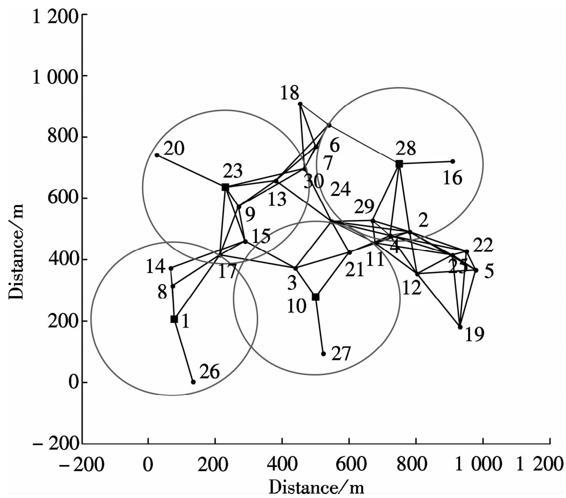

Fig. 4 MST formed by backbone nodes

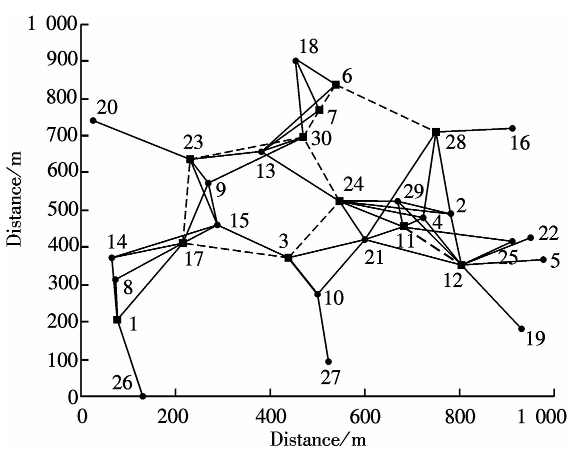

Fig. 5 Quantum mesh network obtained by QDMST

calculate the general centers of the uncovered connected components, and to expand the backbone network to {1, 7, 10, 12, 23, 24, 28}. Now all the nodes are covered by the backbone network. An MST is formed by these backbone nodes, as shown in Fig. 4.

Checking the topology in Fig. 2, it can be seen that on this tree there may be no quantum channel between two neighbor nodes, such as that between nodes 1 and 10.

The pair of neighbor nodes on the tree without direct quantum channel are connected using the shortest distance algorithm. The results are shown in Fig. 5. The dotted lines show the final connection of the backbone network. The final backbone nodes are {1, 3, 6, 7, 10, 11, 12, 17, 23, 24, 28, 30}, 12 in total. Each non-backbone node can be connected via one or more backbone nodes.

To evaluate the performance of the algorithm, the QDMST is compared with the random selection algorithm, by which a node is chosen to be a backbone node randomly, followed by determining whether the backbone structure mesh network can be composed in ensuring the connectivity currently. If not, one more node is randomly chosen to join the backbone node set until the backbone covers all nodes. One result of random selection is exemplified in Fig. 6, where the number of backbone nodes is larger than that of the QDMST.

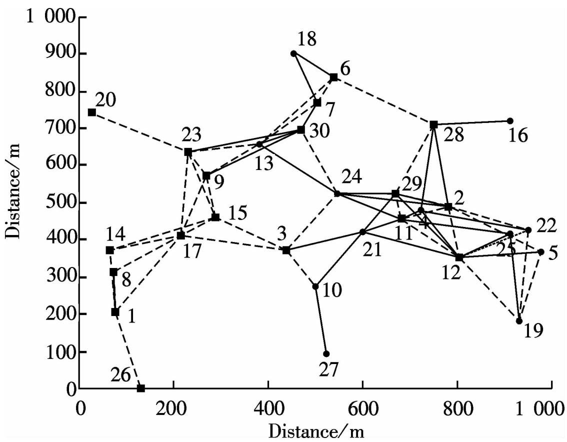

Fig. 6 Wireless quantum communication mesh network obtained by random selection algorithm

To evaluate the performance of the above algorithm more accurately, the average backbone nodes number A_{BN} and average quantum channel distance A_{QCD} are defined as two performance parameters.

$$A_{BN} = \sum_{i}^{n} \frac{M_i}{n} \tag{3}$$

$$A_{QCD} = \sum_{i}^{n} \frac{l(e_i)}{n} \tag{4}$$

The QDMST performance variations vs. node number N and communication radius threshold R are presented and compared with that of the random selection algorithm. 100 connected graphs are randomly generated, and the performance curves are shown in the following figures.

Fig. 7 shows that A_{BN} increases with the increasing total node number N and a fixed communication radius R. The QDMST obtains more obvious gain with a greater N. Fig. 8 shows that A_{BN} decreases while R increases with a fixed N. The QDMST obtains a more pronounced performance gain with a smaller R. A_{BN} tends to be at the same level when R is large. It can be seen from both the two figures that the QDMST always performs better than the random selection algorithm.

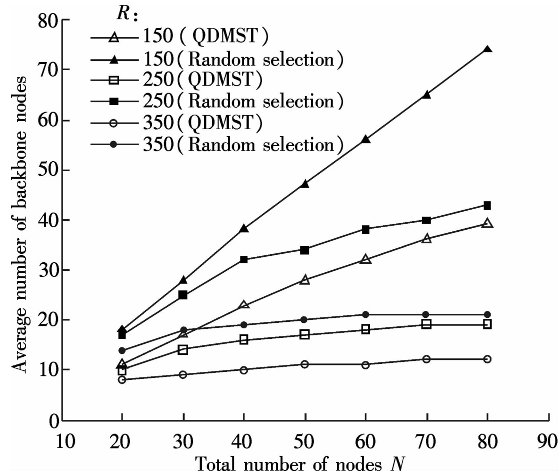

Fig. 7 Influences on A_{BN} with variation of N and fixed R

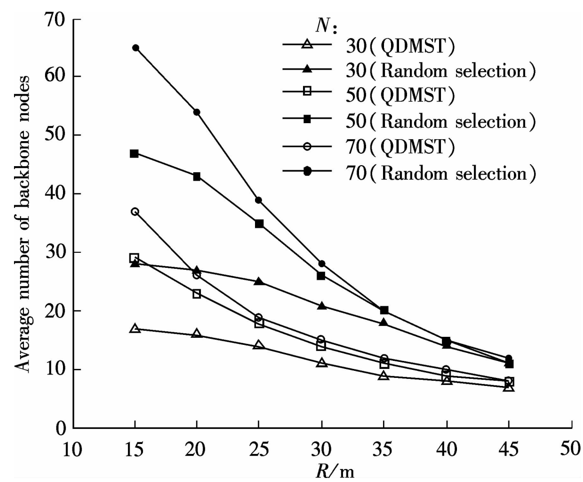

Fig. 8 Influences on A_{BN} with variation of R and fixed N

Fig. 9 shows that A_{QCD} decreases while N increases with a fixed R. Fig. 10 shows that A_{QCD} increases while R increases with a fixed N. By comparing the two configures, it can be seen that communication radius R is the dominant factor influencing the variation of A_{QCD}, and the performance of QDMST is superior to that of the random selection algorithm in both cases.

4 Conclusion

In summary, the QDMST algorithm can generate a topology for quantum communication networks based on the backbone mesh structure while ensuring the networks connectivity. It makes the distribution of entangled particles effective with cost saving and complexity reduction. Under the same network scenarios, the QDMST algorithm outperforms the random selection algorithm in terms of the backbone nodes numbers and quantum channel distance.

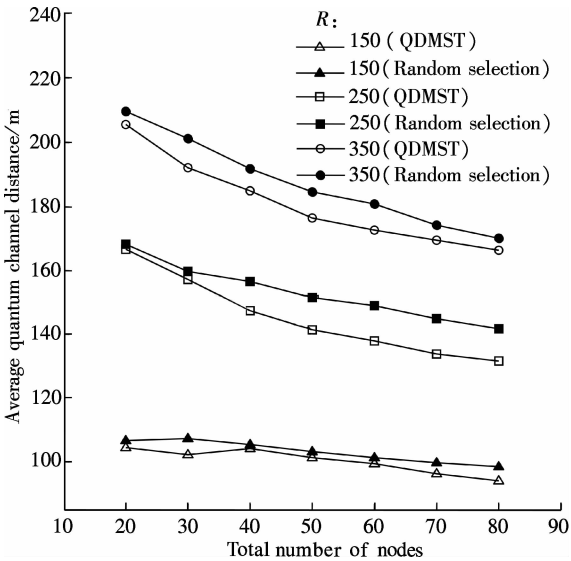

Fig. 9 Influences on A_{QCD} with variation of N and fixed R

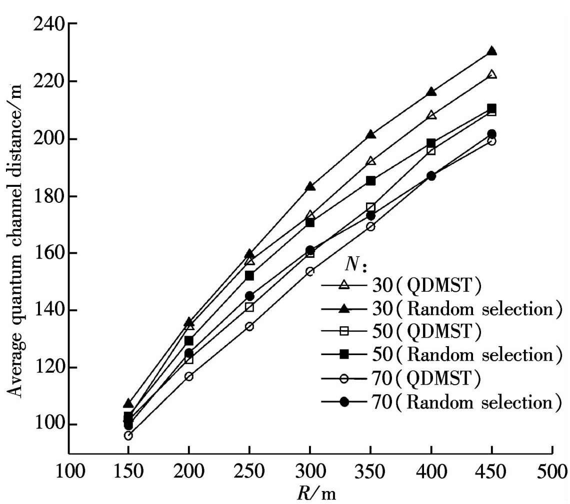

Fig. 10 Influences on A_{QCD} with variation of R and fixed N

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无线量子通信 mesh 网络的纠缠粒子分发算法

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摘要: 为了节约成本和降低复杂度,在保证量子信道意义上的网络连通性前提下,无线量子通信 mesh 网络中的纠缠粒子分发问题可被看作为量子骨干节点的选择问题. 提出了一种基于最小生成树的量子分发算法 QDMST,以构建 mesh 骨干网. 算法首先求解连通图的关节点,再求解未被关节点覆盖的各连通块的一般中心,将关节点和一般中心作为骨干网节点,并生成最小生成树,以最短径算法求得最小生成树上任意相邻节点间的量子通路,量子通路上的节点也加入骨干网. 对算法进行了分析和仿真,仿真结果表明在不同的网络场景下,QDMST 算法的平均骨干网节点数和平均量子信道距离均优于随机选择算法.

关键词: 无线量子通信网络;纠缠粒子分发;无线 mesh 网络; 最小生成树

中图分类号: TN91