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Comparison of signal reconstruction under different transforms

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Abstract: A new algorithm, called MagnitudeCut, to recover a signal from its phase in the transform domain, is proposed. First, the recovery problem is converted to an equivalent convex optimization problem, and then it is solved by the block coordinate descent (BCD) algorithm and the interior point algorithm. Finally, the one-dimensional and two-dimensional signal reconstructions are implemented and the reconstruction results under the Fourier transform with a Gaussian random mask (FTGM), the Cauchy wavelets transform (CWT), the Fourier transform with a binary random mask (FTBM) and the Gaussian random transform (GRT) are also comparatively analyzed. The analysis results reveal that the MagnitudeCut method can reconstruct the original signal with the phase information of different transforms; and it needs less phase information to recover the signal from the phase of the FTGM or GRT than that of FTBM or CWT under the same reconstruction error.

Key words: MagnitudeCut algorithm; signal reconstruction; different transforms; convex optimization; phase information
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The problem of restoring a signal from its phase in complex transform has gained more and more attention. Generally, the phase and the magnitude of the signal in the transform domain are mutually independent, so the signal cannot be recovered only from the partial knowledge of either one. However, Hayes et al. [1] pointed out that it is possible to recover a signal from the phase-only information under certain concrete conditions. For example, exact or approximate prior information (positivity, asymmetry, sparsity, etc.) on the original signal. Many methods have been proposed to solve the above problem, including the iterative method, the statistical method, the alternating projections method[2], and the partial phase information approach[3]. Hua and Orchard[4] proposed a new image reconstruction algorithm with the simple geometrical model. Loveimi and Ahadi[5] reconstructed the speech signal via the least square error estimation and the overlap add methods. Recently, Boufounos[6] explored compressive sensing to recover sparse signals from phase information, where both the theoretical and experimental results suggest that the exact reconstruction is possible. These methods have been used in acoustical and optical hologram, electron microscopy, and X-ray crystallography. In this paper, we propose a novel approach to reconstruct signals with no assumption on the signals. Moreover, many experiments have been simulated with the phase of different matrix transforms, which are the Fourier transform with a Gaussian random mask (FTGM), the Cauchy wavelets transform (CWT), the Fourier transform with a binary random mask (FTBM), and the Gaussian random transform (GRT).

We consider the signal reconstruction problem using only the phase information because most of the signal information contained in the phase is more important than that incorporated in the magnitude with the same number of signals[1]. Inspired by the two methods reported in Refs. [7 – 8], the authors propose a novel magnitude recovery method called MagnitudeCut. The authors cast the original problem as a new convex optimization[9] problem, and solve it by the block coordinate descent (BCD) algorithm and the interior point algorithm. It is well known that the phase information has been utilized in many applications such as image retrieval[10] and object recognition[11]. Hence, the authors expect to recover the signal by using a small amount of phase information rather than the magnitude information.

1 Phase-Only Signal Reconstruction

The original problem is formulated as

$$\begin{array}{ll} \text{find} & \boldsymbol{x} \\ \text{such that} & \mathrm{e}^{j\angle A\boldsymbol{x}} = \boldsymbol{u} \end{array} \tag{1}$$

where $\boldsymbol{x} \in \mathbf{C}^p$ is the original signal and $\mathbf{C}^p$ is a p-dimensional complex vector space; $\boldsymbol{u} \in \boldsymbol{C}^n$ is the Fourier phase information; $|u_i| = 1$ for $i = 1, 2, \dots, n$ and n is the size

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of $\boldsymbol{u}$; $\boldsymbol{A} \in \mathbf{C}^{n \times p}$ is the Fourier matrix and $n \times p$ is the size of $\boldsymbol{A}$; $\angle \boldsymbol{Ax}$ denotes the phase angle of $\boldsymbol{Ax}$. Our objective is to recover the original signal with an appropriate number of samplings n.

We solve (1) by separating the magnitude and phase variables. Letting $\boldsymbol{Ax} = \mathrm{diag}(\boldsymbol{u})\boldsymbol{b}$, where $\boldsymbol{b} \in \mathbf{R}^n$ denotes the magnitude vector, and $\mathbf{R}^n$ is an n-dimensional real vector space. The original problem (1) can thus be written as

$$\min_{\boldsymbol{x} \in \mathbf{C}^p, \boldsymbol{b} \in \mathbf{R}^n} \| \boldsymbol{Ax} - \mathrm{diag}(\boldsymbol{u})\boldsymbol{b} \|_2^2 \tag{2}$$

The minimization problem of formula (2) with respect to $\boldsymbol{x}$ is a standard least square problem and can be solved by setting $\boldsymbol{x} = \boldsymbol{A}^{\dagger}\mathrm{diag}(\boldsymbol{u})\boldsymbol{b}$, where $(\cdot)^{\dagger}$ is the pseudo-inverse operator. Therefore, formula (2) can be transformed equivalently to

$$\min_{\boldsymbol{b} \in \mathbf{R}^n} \| \boldsymbol{AA}^{\dagger}\mathrm{diag}(\boldsymbol{u})\boldsymbol{b} - \mathrm{diag}(\boldsymbol{u})\boldsymbol{b} \|_2^2 \tag{3}$$

The above formula can be rewritten as

$$\left\| \boldsymbol{AA}^{\dagger}\mathrm{diag}(\boldsymbol{u})\boldsymbol{b} - \mathrm{diag}(\boldsymbol{u})\boldsymbol{b} \right\|_2^2 = \boldsymbol{b}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{u}^{\mathrm{H}})\widetilde{\boldsymbol{M}}\mathrm{diag}(\boldsymbol{u})\boldsymbol{b}$$

where $\widetilde{\boldsymbol{M}} = (\boldsymbol{AA}^{\dagger} - \boldsymbol{I})^{\mathrm{H}}(\boldsymbol{AA}^{\dagger} - \boldsymbol{I}) = \boldsymbol{I} - \boldsymbol{AA}^{\dagger}$. Let $\boldsymbol{M}$ be the positive semi-definite matrix given by $\boldsymbol{M} = \mathrm{diag}(\boldsymbol{u}^{\mathrm{H}}) \cdot (\boldsymbol{I} - \boldsymbol{AA}^{\dagger})\mathrm{diag}(\boldsymbol{u})$. Finally, formula (3) becomes

$$\min \boldsymbol{b}^{\mathrm{T}}\boldsymbol{Mb} \quad \text{s.t.} \quad \boldsymbol{b} \in \mathbf{R}^n \tag{4}$$

Next, we denote $\boldsymbol{B} = \boldsymbol{bb}^{\mathrm{T}} \in \mathbf{S}_n^+$, where $\mathbf{S}_n^+$ denotes the set of positive symmetric matrices, and then problem (4) is

$$\begin{gathered} \min \mathrm{Tr}(\boldsymbol{BM}) \\ \text{s.t.} \quad \boldsymbol{B} \geqslant 0, \boldsymbol{B} \in \mathbf{S}_n^+, \mathrm{rank}(\boldsymbol{B}) = 1 \end{gathered} \tag{5}$$

After dropping the non-convex rank constraint, we obtain the following convex relaxation:

$$\min \mathrm{Tr}(\boldsymbol{BM}) \quad \text{s.t.} \quad \boldsymbol{B} \geqslant 0 \tag{6}$$

In order to solve the convex optimization problem (6), we use the BCD [12] to make formula (6) more conveniently solved. The proposed MagnitudeCut method is applied as the barrier version of MaxCut [13] to relax matrix $\boldsymbol{B}$, so formula (6) becomes

$$\min \mathrm{Tr}(\boldsymbol{BM}) - \mu \log\det(\boldsymbol{B}) \quad \mu > 0 \tag{7}$$

Set $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{P} & \boldsymbol{y} \\ \boldsymbol{y}^{\mathrm{T}} & b^2 \end{bmatrix}$, where $\boldsymbol{y} = \begin{bmatrix} b_1 b_n \\ \vdots \\ b_{n-1} b_n \end{bmatrix}$; $\boldsymbol{P} = \begin{bmatrix} b_1^2 & \cdots & b_1 b_{n-1} \\ \vdots & \ddots & \vdots \\ b_{n-1} b_1 & \cdots & b_{n-1}^2 \end{bmatrix}$; $b^2 = b_n^2$. According to Ref. [14], we know that

$$\begin{bmatrix} \boldsymbol{P} & \boldsymbol{y} \\ \boldsymbol{y}^{\mathrm{T}} & b^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{P}^{-1}\boldsymbol{y} \\ 0 & \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P} & 0 \\ \boldsymbol{y}^{\mathrm{T}} & b^2 - \boldsymbol{y}^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y} \end{bmatrix}$$

So, we obtain $\det(\boldsymbol{B}) = \det(\boldsymbol{P})\det(b^2 - \boldsymbol{y}^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y})$. Since both $\boldsymbol{M}$ and $\boldsymbol{B}$ belong to $\boldsymbol{H}_n$, where $\boldsymbol{H}_n$ is the Hermitian matrices of dimension n, we can formulate the complex program in $\boldsymbol{H}_n$ as the real programs [12], and obtain the following equation:

$$\mathrm{Tr}(\Gamma(\boldsymbol{B})\Gamma(\boldsymbol{M})) = 2\mathrm{Tr}(\boldsymbol{BM}) \tag{8}$$

where $\Gamma(\cdot) = \begin{bmatrix} \mathrm{Re}(\cdot) & -\mathrm{Im}(\cdot) \\ \mathrm{Im}(\cdot) & \mathrm{Re}(\cdot) \end{bmatrix}$. Applying $\Gamma(\cdot)$ to Eq. (8), we have

$$\mathrm{Tr}(\Gamma(\boldsymbol{B})\Gamma(\boldsymbol{M})) = \mathrm{Tr}(2(\boldsymbol{B}\mathrm{Re}(\boldsymbol{M}))) \tag{9}$$

Hence, Eq. (7) becomes

$$\min \mathrm{Tr}(\boldsymbol{B}\mathrm{Re}(\boldsymbol{M})) - \mu \log\{\det(\boldsymbol{P})\det(b^2 - \boldsymbol{y}^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y})\} \tag{10}$$

By using the BCD and setting $\mathrm{Re}(\boldsymbol{M}) = \boldsymbol{R}$, formula (10) can be rewritten as

$$\min_{\boldsymbol{y}_{ji} \in R^{n-1}} \boldsymbol{R}_{jj}^{\mathrm{T}}\boldsymbol{y}_{ji} + R_{ii}b_i^2 - \mu\log(b_i^2 - \boldsymbol{y}_{ji}^{\mathrm{T}}\boldsymbol{P}_{jj}^{-1}\boldsymbol{y}_{ji}) \tag{11}$$

where $j \in \{1, \cdots, i-1, i+1, \cdots, n\}$, $i = \{1, \cdots, n\}$. Letting $\boldsymbol{y}_{ji} = \boldsymbol{y}'_{ji}b_i$, we can express the objective function in (11) as

$$F(\boldsymbol{y}'_{ji}, b_i) = \boldsymbol{R}_{ji}^{\mathrm{T}}\boldsymbol{y}'_{ji}b_i + R_{ii}b_i^2 - \mu\log(1 - \boldsymbol{y}'^{\mathrm{T}}_{jj}\boldsymbol{P}_{jj}^{-1}\boldsymbol{y}'_{ji}) - \mu\log b_i^2 \tag{12}$$

In order to obtain the minimum value of Eq. (12), we need to find a set of solutions $(\boldsymbol{y}'_{ji}, b_i)$. In the following, we use a log-barrier algorithm [9] to solve the convex optimization problem. For simplicity, we set $\boldsymbol{y}' = \boldsymbol{y}'_{ji}$, $\boldsymbol{Q} = \boldsymbol{R}_{ji}$ and $\boldsymbol{P} = \boldsymbol{P}_{jj}$, and then differentiate (12) with respect to $\boldsymbol{y}'$ and b_n.

$$\frac{\partial F(\boldsymbol{y}', b_i)}{\partial \boldsymbol{y}'} = \boldsymbol{Q}b_i + \frac{2\mu\boldsymbol{P}^{-1}\boldsymbol{y}'}{(1 - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y}')} = I(\boldsymbol{y}') \tag{13}$$

$$\frac{\partial F(\boldsymbol{y}', b_i)}{\partial b_i} = \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{y}' + 2R_{ii}b_i - \frac{2\mu}{b_i} = J(b_i) \tag{14}$$

Next, we use the second-order Tayler series expansion to express Eq. (13) and Eq. (14),

$$I(\boldsymbol{y}' + \Delta\boldsymbol{y}') = I(\boldsymbol{y}') + \frac{2\mu(\boldsymbol{P}^{-1})^{\mathrm{T}}}{(1 - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y}')\Delta\boldsymbol{y}'} + \frac{4\mu\boldsymbol{P}^{-1}\boldsymbol{y}'(\boldsymbol{P}^{-1}\boldsymbol{y}')^{\mathrm{T}}}{(1 - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y}')^2\Delta\boldsymbol{y}'} \tag{15}$$

$$J(b_i + \Delta b_i) = J(b_i) + 2R_{ii}\Delta b_i + \frac{2\mu}{b_i^2}\Delta b_i \tag{16}$$

Then, we define

$$\begin{gathered} \boldsymbol{G} = -\mu\log(1 - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y}')\boldsymbol{g}_{y'} = \frac{\mathrm{d}\boldsymbol{G}}{\mathrm{d}\boldsymbol{y}'} = \frac{2\mu\boldsymbol{P}^{-1}\boldsymbol{y}'}{1 - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y}'} \\ \boldsymbol{H}_{y'} = \frac{\mathrm{d}^2\boldsymbol{G}}{\mathrm{d}\boldsymbol{y}'^2} = \frac{2\mu(\boldsymbol{P}^{-1})^{\mathrm{T}}}{1 - \boldsymbol{y}'^{\mathrm{T}}P^{-1}\boldsymbol{y}'} + \frac{4\mu\boldsymbol{P}^{-1}\boldsymbol{y}'(\boldsymbol{P}^{-1}\boldsymbol{y}')^{\mathrm{T}}}{(1 - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{y}')^2} \end{gathered} \tag{17}$$

$$K = R_{nn}b_n^2 - \mu\log(b_n^2), k_{b_n} = 2R_{nn}b_n - \frac{2\mu}{b_n}, L_{b_n} = 2R_{nn} + \frac{2\mu}{b_n^2} \tag{18}$$

Hence, Eq. (15) and Eq. (16) are simplified as

$$\boldsymbol{Q}b_i + \boldsymbol{g}_{y'} + \boldsymbol{H}_{y'}\Delta\boldsymbol{y}' = 0, \ \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{y}' + k_{b_i} + L_{b_i}\Delta b_i = 0 \tag{19}$$

Solving Eq. (19), we have

$$\Delta \boldsymbol{y}' = -(\boldsymbol{H}_{y'})^{-1}(\boldsymbol{Q}b_i + \boldsymbol{g}_{y'}),\ \Delta b_i = \frac{-(\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{y}' + k_{b_i})}{\boldsymbol{L}_{b_i}} \quad (20)$$

By updating Eq. (20), we obtain a better solution $\boldsymbol{B}$ as

$$\boldsymbol{y}'_{ji} = \boldsymbol{y}'_{ji} + s\Delta\boldsymbol{y}'_{ji},\ b_i = b_i + s\Delta b_i \quad (21)$$

where s is the step size. If the rank of the solution $\boldsymbol{B}$ is one, the relaxation is tight and the vector $\boldsymbol{b}$ satisfying $\boldsymbol{B} = \boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}$ is the optimal solution of Eq. (4). When the rank of the solution $\boldsymbol{B}$ is larger than one, a leading eigenvector $\boldsymbol{b}$ of $\boldsymbol{B}$ is used as an approximate solution. Finally, we can obtain the reconstruction signal by $\hat{\boldsymbol{x}} = \boldsymbol{A}^{\dagger}\mathrm{diag}(\boldsymbol{u})\boldsymbol{b}$.

In this subsection, we summarize the proposed MagnitudeCut method. After we obtain $\hat{\boldsymbol{x}}$ by Eqs. (17) to (21), we set $\boldsymbol{X}^0 = \boldsymbol{A}\hat{\boldsymbol{x}} \in F$, where set F satisfies Eq. (1), as the initial value of the modified Gerchberg-Saxton method[15] (MGS) and obtain a more accurate solution. From Eqs. (17) to (20), we know that the MagnitudeCut method only needs a real matrix vector product and real inner product in the iteration process. Although we obtain the result by the MGS finally, the MGS method has no significant contribution to computational complexity. The reason is that $\hat{\boldsymbol{x}}$ is sufficiently close to $\boldsymbol{x}$, the MGS method requires much fewer operations than the MagnitudeCut method. The core of the MagnitudeCut method is the interior point algorithm, so the convergence of the algorithm is guaranteed by the result in Ref. [9] and in fact the function logdet is strongly convex over compact subsets of the positive semi-definite cone [8].

2 Simulations

The simulations are implemented by Matlab. We implement the one-dimensional and two-dimensional signal reconstructions by the MagnitudeCut algorithm and compare the signal reconstruction results by four different kinds of transform matrices: FTGM, CWT, FTBM and GRT.

2.1 One-dimensional signal

The original signal $\boldsymbol{x} \in \mathbf{R}^{32}$ is shown in Fig. 1. Its phases in FTGM, CWT, FTBM and GRT are given. The sampling number is twice the length of the original signal and the reconstruction results $\hat{\boldsymbol{x}}$ are shown in Figs. 2(a) to (d), respectively. From this figure, we find that good signal reconstruction performance from the phase of FTGM, CWT, FTBM, and GRT can be achieved. For simplicity, we use the symbol C to denote the sampling number below.

To further illustrate the recovery results, we record the reconstruction error $E = \|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2^2 / \|\boldsymbol{x}\|_2^2$ with different C. For convenient observation, we set the vertical coordinates as the logarithmic function $-\log E$ of the reconstruction error in Fig. 3. The greater the value of the vertical coordinates, the better the reconstruction property. From these simulations, we can see that the original signal can be perfectly reconstructed by our MagnitudeCut algorithm when C is greater than or equal to the twice of the length of the original signal in FTGM and GRT. Simultaneously, the results also clearly show that when C is equal to the triple of the original signal in FTBM, our method can still restore signals with certain error budgets.

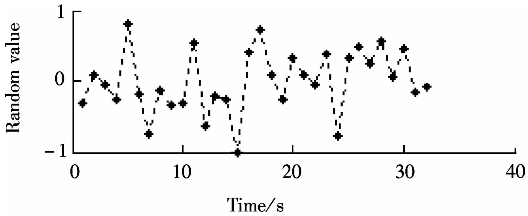

Fig. 1 The original signal

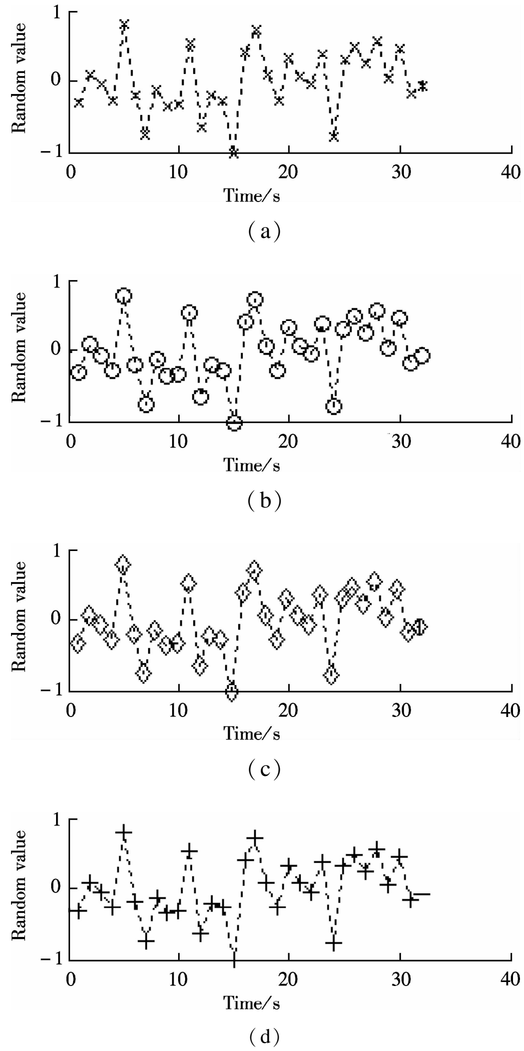

Fig. 2 Reconstruction results by four different transforms. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

So, on the one hand, in order to reconstruct the signal from a small amount of phase information under the same E, we need to choose FTGM or GRT. On the other hand, in the same transform domain, we can choose a set of phase information to describe a signal whose number is double the length of the original signal when a more accurate result is required. For example, in the field of info-

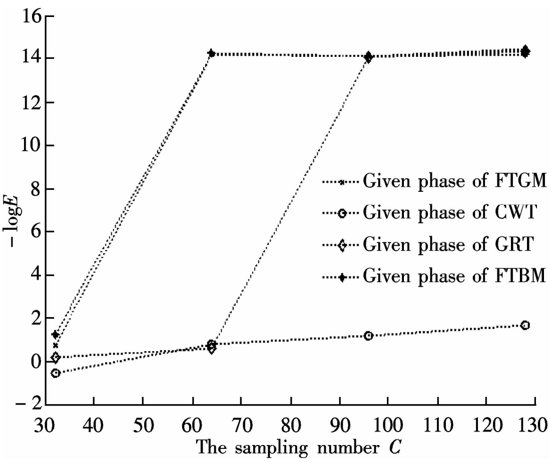

Fig. 3 The reconstruction error with different transforms

rmation encryption, we can use less phase information under FTGM to encrypt a signal.

2.2 Two-dimensional signal

In many cases, we need to deal with the two-dimensional signals. In this paper, three images are chosen. They are the moon's surface, a clock and Lena, as shown in Fig. 4. Supposing that the phase of each transform matrix is known, the reconstructed images with $C = 2$ are shown in Fig. 5 to Fig. 7. The results reconstructed by the phase of FTGM and GRT are better than those by the phase of CWT and FTBM. Similarly, the reconstructed results with $C = 4$ are shown in Fig. 8 to Fig. 10. It can be seen that with more sampling numbers, the results reconstructed by the phase of FTGM, CWT, FTBM and GRT are also better.

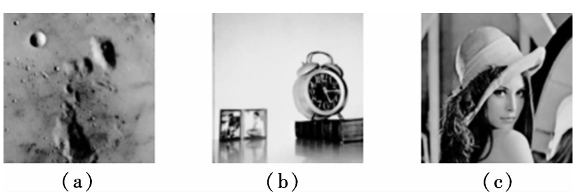

Fig. 4 Original images. (a) Moon's surface; (b) Clock; (c) Lena

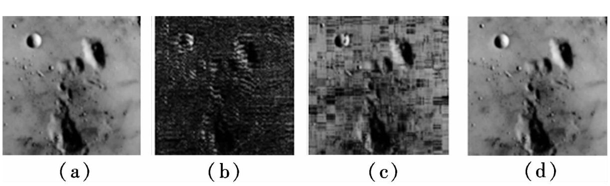

Fig. 5 Reconstructed results of the moon's surface by four different transforms with $C = 2$. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

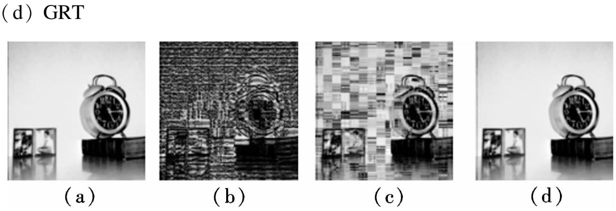

Fig. 6 Reconstructed results of the clock by four different transforms with $C = 2$. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

Fig. 7 Reconstructed results of Lena by four different transforms with $C = 2$. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

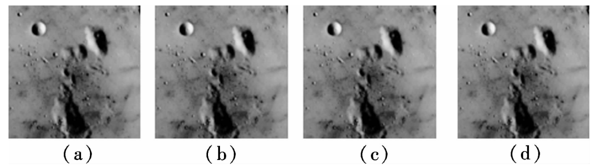

Fig. 8 Reconstructed results of the moon's surface by four different transforms with $C = 4$. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

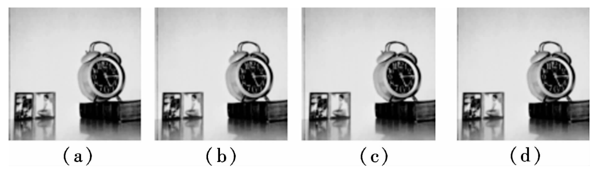

Fig. 9 Reconstructed results of the clock by four different transforms with $C = 4$. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

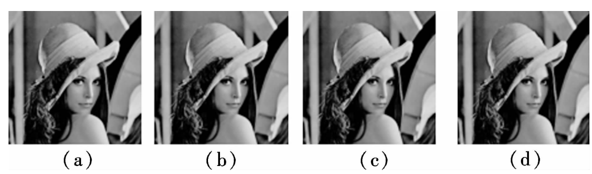

Fig. 10 Reconstructed results of Lena by four different transforms with $C = 4$. (a) FTGM; (b) CWT; (c) FTBM; (d) GRT

The experimental results indicate that the two-dimensional images can also be recovered from the phase under the corresponding four transforms by the MagnitudeCut method. As a comparison, if one wants to recover a signal of size p from the phase under FTGM or GRT, the sampling number should satisfy $C \geqslant 2p$. However, we find that good reconstruction performance can be realized with $C \geqslant 3p$ after observing a large number of simulation results when the transform is FTBM. In the case of CWT, we need the number of rows in the transform matrix to be equal to or larger than $4p$. This not only proves the importance of phase information in the signal reconstruction process, but also explains the significance of research of the MagnitudeCut algorithm.

3 Conclusion

We propose a new algorithm called MagnitudeCut to solve the problem of signal reconstruction from the phase-only information in different transform matrices, such as FTGM, CWT, FTBM and GRT. Experiments on the one-dimensional and two-dimensional signals are simulated to illustrate the feasibility of the algorithm. The merit of the proposed algorithm is that the original signal can be reconstructed with less amount of phase information than the PhaseCut algorithm. Furthermore, the results show that the phase with FTGM and GRT can obtain better results by the MagnitudeCut algorithm than the other two transforms. The phase information can preserve many more important features of a signal than the magnitude information. Therefore, if the phase information is used to describe the signal features, the requirements for the storage and the transmission bandwidth can be reduced. Since the PhaseCut algorithm is the basis of the scattering convolution networks, the proposed method shows that we can also construct a new convolution network by the phase.

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基于不同变换下的信号重建性能的比较

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摘要:提出了一种新的算法——MagnitudeCut 算法,用于从信号的变换域的相位来恢复信号. 首先将重建问题等价转换为一个凸优化问题,然后通过块坐标下降算法(BCD)和内点法解决原始信号重建问题. 最后,实现了一维和二维信号的重建,并对先通过高斯随机掩膜再进行傅里叶变换(简称 FTGM),柯西小波变换(CWT)相位,先通过二值随机掩膜再进行傅里叶变换(简称 FTBM),高斯随机变换(GRT)相位的信号重建结果做了比较分析. 分析结果表明,MagnitudeCut 算法可以完成已知信号不同变换域相位的信号重建,并且在相同的重建误差下,从 FTGM 和 GRT 相位信息重建信号比从 FTBM 和 CWT 需要的相位数目更少.

关键词: MagnitudeCut 算法; 信号重建; 不同变换; 凸优化; 相位信息

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