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Enabling weakened hedges in linguistic multi-criteria decision making

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Abstract: A semantics-based model is proposed to enable weakened hedges, such as “more or less” and “roughly” in the context of linguistic multi-criteria decision making. First, the resemblance relations are defined based on the semantics of terms on the domain. Then, the hedges can be represented after the upper and loose upper approximations of a linguistic term are derived. Accordingly, some compact formulae can be derived for the semantics of linguistic expressions with hedges. Parameters in these formulae are objectively determined according to the semantics of original terms. The proposed model presents a more natural way to express the decision information under uncertainties and its semantics is clear. The proposed model is clarified by solving the problem of evaluation and selection of sustainable innovative energy technologies. Computational results demonstrate that the model can deal with various uncertainties of the problem. Finally, the model is compared with existing techniques and extended to the case when the semantics of terms are represented by trapezoidal fuzzy numbers.

Key words: decision making; multi-criteria decision making; linguistic term sets; linguistic hedges; similarity relation

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Multi-criteria decision making (MCDM) problems, which refer to evaluating, prioritizing or selecting among some available alternatives with respect to multiple criteria, are very common in practice[1]. The general challenges in collecting decision information are the complexities of problems and the way to express preferences of decision makers (DMs) or experts “accurately”[2]. Therefore, linguistic terms are commonly used due to the fact that DMs can present more “accurate” information with higher confidence. Serving as techniques of computing with words[3], linguistic representation models including the semantic models and symbolic linguistic computing models[4] are vital for MCDM.

A linguistic variable is a variable whose values are words or sentences in natural or artificial languages[3]. A linguistic term, defined by a linguistic descriptor and its semantics, is less precise than a number but closer to human cognitive processes[4]. Generally, a linguistic term set (LTS) is considered to make all the terms to be distributed in a predefined ordered scale. The semantics of terms can be represented by fuzzy numbers defined in [0, 1], described by membership functions. The triangular membership functions are usually used to represent uniformly distributed ordered sets. To enrich the linguistic model, the uncertain linguistic terms and hesitant fuzzy linguistic term sets (HFLTS)[5-6] have been proposed to consider more than one term at once.

However, the existing extensions are still limited. For example, a decision organization is authorized to evaluate an energy technology with respect to its contribution to regional development. Due to the uncertainties and risks of both the current status and the future, the organization may be not very clear about how to express the opinion by one certain term. The most natural linguistic expression experts used may be “more or less high” if “high” is supported by most of the evidence. However, there is not such a technique in MCDM. Furthermore, once a term is selected, some linguistic hedges may be used to express one’s preference more accurately, or to raise the confidence level of the expressed opinion. For example, we may modify the term “high” by “roughly”, “more or less” or “definitely” (The final one can be omitted.). Generally, linguistic hedges are special linguistic expressions by which linguistic terms are modified. It is useful to develop a technique to conduct this kind of linguistic expressions in MCDM.

Therefore, we focus on modeling linguistic hedges, such as “more or less” and “roughly”, in MCDM based on the approximation of fuzzy sets in this paper. We introduce the linguistic hedges with non-inclusive interpretation into the framework of MCDM. The original terms in a linguistic term set are referred to as linguistic terms (or terms for short). Moreover, linguistic terms and linguistic terms modified by hedges are called linguistic expressions. In addition, sustainable development is one of the most important issues for governments. However, it is complex and time consuming to assess these technologies with respect to a set of criteria[7]. Therefore, we en-

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able the experts to use linguistic expressions with hedges during evaluation, and solve the problem by the proposed approach.

1 Approximations of Fuzzy Sets and Linguistic Hedges

Given a nonempty domain X, a fuzzy set F on X is characterized by its membership function F: $X \rightarrow [0, 1]$. The class of all fuzzy sets on X is denoted by $F(X)$. A LTS can be denoted by $S = \{s_0, s_1, \dots, s_\tau\}$, where τ is a positive integer, and s_i represents a possible value of a linguistic variable, the semantics of which is illustrated by fuzzy numbers. We set $X = [0, 1]$ in this paper. De Cock et al. [8] introduced tight and loose approximations for fuzzy rough sets to fit the case that one element belongs to some degree to several fuzzy similarity classes at the same time. The concept is recalled as follows:

Definition 1[8] Given a nonempty domain X, a fuzzy relation R and a fuzzy set F on X, then the loose and (usual) upper approximations of F are defined as

$$R \uparrow\uparrow F(y) = \sup_{z \in X} \mathcal{T}(R(y, z), \sup_{x \in X} \mathcal{T}(R(x, z), F(x)))$$
$$R \uparrow F(y) = \sup_{x \in X} \mathcal{T}(R(x, y), F(x))$$

respectively, for all $y \in X$, where $\mathcal{T}$ is a t-norm.

Given a LTS S defined on X, the terms in S act, actually, as a fuzzy partition of X. The approximations of a term can be illustrated by the following example.

Example 1 Given a linguistic term s_3, represented by the triangular fuzzy number $(0.333, 0.5, 0.667)$, the fuzzy relation $R(x, y) = 1 - 6|x - y|$, $\mathcal{T}(x, y) = \mathcal{T}_M(x, y) = \min\{x, y\}$, then the membership functions of s_3, $R \uparrow s_3$, $R \uparrow\uparrow s_3$ are shown in Fig. 1.

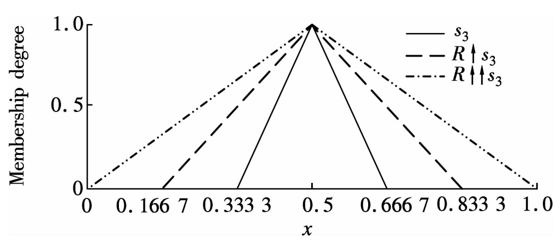

Fig. 1 An example of the upper and loose upper approximations of a term

Since the powering model was introduced by Zadeh[9], linguistic hedges have been widely investigated. Furthermore, the applications come down to several areas, such as fuzzy classifiers[10], database queries[11] and fuzzy modal logic. De Cock et al. [12] proposed the following general definitions of linguistic hedges by fuzzy sets, which were referred to as fuzzy hedges or fuzzy modifiers.

Definition 2[12] Given a nonempty domain X, a fuzzy hedge h on X is a $F(X) \rightarrow F(X)$ mapping.

There are two interpretations of linguistic hedges: the inclusive interpretation and the non-inclusive interpretation[13]. The non-inclusive interpretation was demonstrated by psycholinguistic research. Fig. 2(a) illustrates the possible membership functions for "old", "more or less old" and "very old" in the non-inclusive interpretation in the universe of ages depicted in years. We can see that a hedge modifies a base term to another different term rather than a subset or a superset of it. This kind of hedges has been widely used in linguistic MCDM[5, 14-15]. In the inclusive interpretation, the semantic entailment is always assumed to be held. That is, for $A \in F(X)$ and $x \in X$,

$$x \text{ is very } A \Rightarrow x \text{ is } A \Rightarrow x \text{ is more or less } A \quad (1)$$

Using denotations of fuzzy sets, Eq. (1) can be rewritten as

$$\text{very } A \subseteq A \subseteq \text{more or less } A \quad (2)$$

Fig. 2(b) illustrates the possible membership functions for "old", "more or less old" and "very old" in the inclusive interpretation.

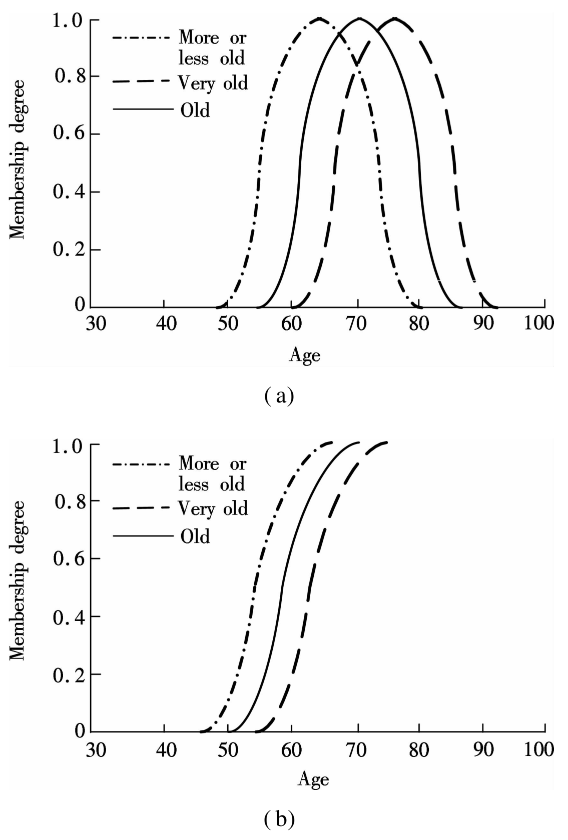

Fig. 2 An example of linguistic hedges with different interpretations. (a) Non-inclusive interpretation; (b) Inclusive interpretation

The existing models of representing linguistic hedges can be roughly divided into two strategies. The first strategy considers only an artificial operator that transforms the membership function of A into an acceptable version. The second strategy is endowed with a clear inherent semantics by taking the mutual relationships in the universe into account[12]. The basic intuition is that a man can be called more or less old if he resembles someone who is actually old. In this strategy, some fuzzy relations are usually defined to express hedges by modeling resemblance.

From the perspective of linguistic MCDM, we can draw the following conclusions: 1) Linguistic hedges with non-inclusive interpretation have been investigated and applied to MCDM problems. 2) However, linguistic hedges with the other interpretation have not emerged in this area although there have been several techniques to model them. Most of this techniques focus on other applications such as fuzzy control[16], algorithm refinements, approximate reasoning[17], fuzzy relation equations[18] and so on. Therefore, we intend to introduce some linguistic hedges to MCDM to enable the expressions of DMs' uncertain opinions in a more natural way. Only weakened hedges, such as "more or less" and "roughly", are possible to be used in uncertain setting.

2 Modeling Weakened Hedges in MCDM

In order to introduce some linguistic hedges with the inclusive interpretation to linguistic MCDM, we will study the mathematical representation of linguistic expressions built up by the following scheme:

<hedge> : = roughly | more or less

<linguistic expression> : = <base term> |

<weakened hedge> <base term>

where <base term> is an original term of a given LTS. We focus on two weakened hedges, i. e., "roughly" and "more or less", which may be used to represent evaluations in linguistic MCDM problems. An intuition is that "roughly" has more weakening force than "more or less". Similar to Ref. [19], we assume the following semantic entailment holds: for $s_\alpha \in S$ and $x \in X$,

$$x \text{ is } s_\alpha \Rightarrow x \text{ is more or less } s_\alpha \Rightarrow x \text{ is roughly } s_\alpha \quad (3)$$

By means of fuzzy sets, Eq. (3) corresponds to

$$s_\alpha \subseteq \text{more or less } s_\alpha \subseteq \text{roughly } s_\alpha \quad (4)$$

2.1 Distance measures and resemblance relations

The key task is to seek a proper fuzzy relation so that the approximations of a term can be derived. It is important that the fuzzy relation should be determined objectively. A rational way is to mine the fuzzy relation from the semantics of the terms defined on X. Let us begin with the distance measure of objects in X.

Definition 3 Given $x, y \in X$, the distance between x and y is defined by

$$d(x, y) = |\phi(x) - \phi(y)| \quad (5)$$

where $\phi: X \to X$ is a monotonic increasing function.

Generally, the original term s_α is defined by a content-free grammar. The distance between each two terms may not be equal. The function ϕ is used to conduct a distance transformation so that the linguistic term defined on the domain is uniformly distributed. It is easy to prove that the distance measure defined by Definition 3 satisfies the following theorem.

Theorem 1 Let d be the distance measure defined by Definition 3, $\mathcal{S}$ be the t-conorm, then for all $x, y, z \in X$,

1) $d(x, x) = 0$;
2) $d(x, y) = d(y, x)$;
3) $0 \leqslant d(x, y) \leqslant 1$;
4) $\mathcal{S}(d(x, y), d(y, z)) \geqslant d(x, z)$.

In fact, (X, d) forms a pseudometric space if d is defined by Definition 3. As we will only focus on the uniformly distributed domain, the function ϕ can be specified by

$$\phi(x) = kx \quad (6)$$

where $k > 0$ is a constant value. Then, we define the following fuzzy relation, namely resemblance relation, to model the similarity of any $x, y \in X$.

Definition 4 Given $x, y \in X$, the resemblance relation R is defined by

$$R(x, y) = 1 - d(x, y) \quad (7)$$

Similarly, we have the following theorem.

Theorem 2 Let R be the resemblance relation defined by Definition 4, then for all $x, y, z, u \in X$,

1) $R(x, x) = 1$;
2) $R(x, y) = R(y, x)$;
3) $0 \leqslant R(x, y) \leqslant 1$;
4) $d(x, y) \leqslant d(z, u)$ implies $R(x, y) \geqslant R(z, u)$.

Theorems 1 and 2 ensure the intuitive rationality of the proposed resemblance relation. According to 4) of Theorem 2, the closer the two objects are to each other, the more they are approximately equal.

2.2 Derivation of semantics of linguistic terms with weakened hedges

Given an original linguistic term (a fuzzy set on X), Definition 1 can be used to compute its upper approximations based on the predefined resemblance relation. Then, linguistic hedges can be modeled by means of approximation. Let R be a resemblance relation on X, then (X, R) is a fuzzy approximation space because of the reflexive and symmetric properties of R. Moreover, for every $F \in F(X)$ $(x \in X)$, $R \uparrow F(x)$ is the degree to which the fuzzy set of objects resembling x overlaps F. Then, according to Definition 1 and Eq. (4), we present the following way of modeling two weakened hedges: given an original term s_α,

$$\text{more or less } s_\alpha: R \uparrow s_\alpha \quad (8)$$

$$\text{roughly } s_\alpha: R \uparrow\uparrow s_\alpha \quad (9)$$

As can be seen in literature, the symmetrically and uniformly distributed LTSs are frequently used in applications. Thus, we will discuss some specific issues of representing this kind of terms in this subsection.

Given a symmetrically and uniformly distributed LTS $S = \{s_\alpha \mid \alpha = 0, 1, \ldots, \tau\}$ and $x, y \in X$, $R(x, y) > 0$ iff $\exists \alpha \in \{0, 1, \ldots, \tau\}$ such that $s_\alpha(x) > 0$ and $s_\alpha(y) > 0$. For example, $R(0, 0.1)$ is greater than 0 and $R(0, 0.5)$ is equal to 0. Therefore, if $|x - y| \geqslant 1/\tau$, then $R(x, y) = 0$; if $|x - y| < 1/\tau$, then $R(x, y) > 0$. Thus, $1/\tau$ can be considered as the resemblance threshold. Based on the analysis, we can let $k = \tau$ in Eq. (6), then Eq. (7) is reduced to

$$R(x, y) = 1 - \tau |x - y| \tag{10}$$

For convenience, a fuzzy set F on X with triangular membership function is written as

$$F(x) = (a, b, c) = \begin{cases} \dfrac{(x-a)}{(b-a)} & \max(0, a) \leqslant x \leqslant b \\ \dfrac{(c-x)}{(c-b)} & b \leqslant x \leqslant \min(1, c) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Given an original term $s_\alpha = (a_\alpha, b_\alpha, c_\alpha) \in S$, we calculate $R \uparrow s_\alpha$ and $R \uparrow\uparrow s_\alpha$ to model "more or less s_α" and "roughly s_α", respectively. Since s_α is uniformly distributed in S, then $s_\alpha = \left(\frac{\alpha - 1}{\tau}, \frac{\alpha}{\tau}, \frac{\alpha + 1}{\tau}\right)$ according to Eq. (11), where $\alpha \in \{0, 1, \ldots, \tau\}$. Then, we have the following theorem.

Theorem 3 Let $s_\alpha = \left(\frac{\alpha - 1}{\tau}, \frac{\alpha}{\tau}, \frac{\alpha + 1}{\tau}\right) \in S(\alpha = 0, 1, \ldots, \tau)$ be a set of uniformly distributed original terms, R be the resemblance relation defined by Eq. (10), and $\mathcal{T} = \mathcal{T}_M$, then $\forall y \in [0, 1]$

$$R \uparrow s_\alpha(y) = \left(\frac{\alpha - 2}{\tau}, \frac{\alpha}{\tau}, \frac{\alpha + 2}{\tau}\right) \tag{12}$$

$$R \uparrow\uparrow s_\alpha(y) = \left(\frac{\alpha - 3}{\tau}, \frac{\alpha}{\tau}, \frac{\alpha + 3}{\tau}\right) \tag{13}$$

Proof 1) If $\max\left(0, \frac{\alpha - 2}{\tau}\right) \leqslant y \leqslant \max\left(0, \frac{\alpha - 1}{\tau}\right)$, then

$$\begin{aligned} R \uparrow s_\alpha(y) &= \sup_{x \in [0,1]} T_M(R(x, y), s_\alpha(x)) = \\ &\sup_{x \in [0,1]} \min(R(x, y), s_\alpha(x)) = \\ &\sup_{x \in [y - 1/\tau, y + 1/\tau]} \min(1 - \tau |x - y|, s_\alpha(x)) = \\ &\sup_{x \in [(\alpha - 1)/\tau, y + 1/\tau]} \min\left(1 - \tau(x - y), \frac{x - (\alpha - 1)/\tau}{1/\tau}\right) = \\ &\frac{y - (\alpha - 2)/\tau}{1/\tau} \end{aligned}$$

Similarly, by some simple but trivial computations, we can prove that

$$R \uparrow s_\alpha(y) = \begin{cases} \dfrac{y - (\alpha - 2)/\tau}{1/\tau} & \max\left(0, \dfrac{\alpha - 2}{\tau}\right) \leqslant y \leqslant \dfrac{\alpha}{\tau} \\ \dfrac{(\alpha + 2)/\tau - y}{1/\tau} & \dfrac{\alpha}{\tau} \leqslant y \leqslant \min\left(1, \dfrac{\alpha + 2}{\tau}\right) \end{cases}$$

2) $R \uparrow\uparrow s_\alpha$ can be derived by Eq. (12) and Theorem 7 of Ref. [8].

Theorem 3 demonstrates a simple way to compute the approximation of linguistic terms with triangular membership functions. According to the theorem, "more or less s_α" and "roughly s_α" can be calculated by the 3-tuple of s_α. No parameter has to be determined subjectively. We further illustrate the procedure in the next example.

Example 2 If $\tau = 6$, then $s_3 = (0.333, 0.5, 0.667)$. Based on Eqs. (12) and (13), we have (see Fig. 1)

More or less s_3: $R \uparrow s_3 = (0.1667, 0.5, 0.8333)$

Roughly s_3: $R \uparrow\uparrow s_3 = (0, 0.5, 1)$

3 Enabling Hedges in MCDM

Based on the representational approach proposed in Section 2, the DMs can use linguistic expressions with hedges to conduct evaluations in qualitative environments. We will present an approach for such a class of MCDM problems in this section. The problem is described as follows: Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of m possible alternatives and $C = \{c_1, c_2, \ldots, c_n\}$ be a set of n criteria associated with its weighting vector $\boldsymbol{W} = \{w_1, w_2, \ldots, w_n\}^{\mathrm{T}}$, whose elements take the form of either numerical values or linguistic terms. A decision organization is authorized to evaluate the m alternatives with respect to the set of criteria. The organization provides the assessment information on A_i by a vector $\boldsymbol{V}_i = \{v_{i1}, v_{i2}, \ldots, v_{in}\}^{\mathrm{T}}$ according to a predefined LTS S, where v_{ij} is a linguistic variable denoting the assessment value of the alternative A_i with respect to the criterion c_j. The problem is to rank the alternatives of A based on the matrix.

3.1 The proposed approach

The resolution is presented step by step as follows:

Step 1 The choice of the LTS with its semantics. It is necessary to determine the granularity, labels and semantics of the LTS. Then, the domain of linguistic expressions can be established to evaluate the alternatives according to the criteria. The resemblance relation is fixed when the semantics of linguistic terms are prepared. In particular, if the LTS S is uniformly distributed, the original terms are represented by triangular membership functions, and then the resemblance relation defined by Eq.

(10) can be used.

Step 2 Evaluations. The decision organization is asked to evaluate alternatives with respect to each criterion and express the performance opinions by linguistic variables. The linguistic variable v_{ij} can be either original term $s_\alpha \in S$ or linguistic expressions constructed by hedges ("more or less" and "roughly") and original terms.

Step 3 The choice of the aggregation operator. The DMs have to establish or select an appropriate aggregation operator for fusing and combining the linguistic expressions provided in Step 2. The choice is mainly dependent on three aspects, i. e., the form of weighting vector, the percentage of known weights and the preference of the DM.

Step 4 The choice of the best alternatives. This step usually consists of two phases: the aggregation phase and the exploitation phase. The former combines the linguistic information provided in Step 2 by means of the aggregation operator chosen in Step 3. The latter establishes a priority among the alternatives and chooses the best alternative(s).

3.2 Application in evaluation of energy technologies

A sustainable energy system is crucial for any country and the key way is the implementation of new and innovative energy technologies[7]. However, the evaluation process is very complex because of a series of uncertainties and implications. A government formed a working organization with 25 experts from all the relevant energy "actors". To assess the technologies' impacts on the environmental, social, economical and technological aspects of sustainable development, a number of criteria are selected and shown in Tab. 1. Furthermore, the organization looked systematically into the longer-term future, sought the technologies which have not been used in any energy sector or been applied at the initial stage, but are likely to uphold sustainable development in the four aspects in Tab. 1. Finally, the technologies listed in Tab. 2 are pre-selected as alternatives.

Tab. 1 Selected criteria of appraising energy technologies

Aspect	Criterion
Economic	c_1: investment cost
	c_2: economic viability using payback period
Environmental	c_3: contribution to addressing the climate change phenomenon
	c_4: effects on natural environment
Technological	c_5: efficiency rate
	c_6: knowledge of the innovative technology
Social	c_7: contribution to employment opportunities creation
	c_8: contribution to regional development

Tab. 2 Pre-selected technologies

Category	Technology
Natural fossil fuels technologies	A_1: pressurized fluidized bed combustion
	A_2: pressurized pulverized coal combustion
	A_3: natural gas combined cycle
Hydrogen technologies	A_4: molten carbonate fuel cell
	A_5: fuel cell/turbine hybrids
Renewable energy technologies	A_6: biomass co-firing
	A_7: biomass gasification
	A_8: off-shore wind farms
	A_9: large-scale wind farms
	A_{10}: building integrated photovoltaics

We solve the problem by the approach proposed in Section 3.1.

Step 1 Considering the recommendation of the deciding organization, we use $S = \{s_0, s_1, \dots, s_6\}$. That is, $\tau = 6$, $s_\alpha = (a_\alpha, b_\alpha, c_\alpha) = \left(\frac{\alpha-1}{\tau}, \frac{\alpha}{\tau}, \frac{\alpha+1}{\tau}\right)$ for $\alpha = 1, 2, \dots, \tau$. The resemblance relation is defined by Eq. (10).

Step 2 During the process of evaluation, two weakened hedges (more or less and roughly) are used to represent uncertainties. Based on the domain of linguistic expressions, the performance of the 10 technologies is listed in Tab. 3 below.

Tab. 3 Performance of technologies per criterion

Alternative	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
A_1	VH	H	VH	mVH	H	H	H	H
A_2	H	H	H	P	L	M	H	H
A_3	VH	H	VH	VH	VH	mP	VH	H
A_4	VL	H	H	VH	H	r M	H	M
A_5	mN	M	H	VH	M	VL	L	L
A_6	H	P	rVL	H	H	H	VH	H
A_7	M	H	L	H	mVH	H	P	VH
A_8	H	M	L	L	M	H	mVH	VH
A_9	H	H	L	L	M	VH	VH	rP
A_{10}	VL	rN	H	VH	VH	VH	M	L

Note: N = Nothing, VL = very low, L = low, M = medium, H = high, VH = very high, P = perfect, m = more or less and r = roughly.

Step 3 The organization insists that the importance of eight criteria is equal. In this case, the TFOWA operator can be used to fuse the membership functions of a set of linguistic expressions. Yager[20] suggested an interesting way to derive the associated weighting vector of the TFOWA operator by means of fuzzy linguistic quantifiers. The weighting vectors for the three fuzzy quantifiers can be defined as follows[7]: "Most": $\boldsymbol{W} = \{0, 0, 0.15, 0.25, 0.25, 0.25, 0.1, 0\}^{\mathrm{T}}$; "At least half": $\boldsymbol{W} = \{0.25, 0.25, 0.25, 0.25, 0, 0, 0, 0\}^{\mathrm{T}}$; "As many as possible": $\boldsymbol{W} = \{0, 0, 0, 0, 0.25, 0.25, 0.25, 0.25\}^{\mathrm{T}}$.

Step 4 The aggregation results of 10 alternatives are derived by the TFOWA operator. Using the comparison law of triangular fuzzy numbers, the rankings of alternatives are shown in Tab. 4.

According to the message addressed in Tab. 4, the natural gas combined cycle (A_3) is the best technology de-serving special handling and support from the government. However, the technologies ranked at lower places, such as fuel cell/turbine hybrids (A_5), cannot be considered.

Tab. 4 The overall ranks of 10 technologies with respect to different fuzzy quantifiers

Quantifier	Rank
Most	$A_3 > A_1 > A_7 > A_6 > A_2 > A_9 > A_4 > A_8 > A_{10} > A_5$
At least half	$A_3 > A_7 > A_9 > A_1 > A_6 > A_{10} > A_2 > A_8 > A_4 > A_5$
As many as possible	$A_3 > A_1 > A_2 > A_6 > A_7 > A_4 > A_9 > A_8 > A_{10} > A_5$

4 Discussion

There are mainly two linguistic computational models that extend the range of values of linguistic expressions in MCDM, i. e., the uncertain LTS and the HFLTS. The linguistic expression "between s_1 and s_3" can be represented by the uncertain linguistic term $[s_1, s_3]$ and the HFLTS $\{s_1, s_2, s_3\}$. Both of these two models can be classified in the symbolic computing model which can compute semantics with the terms directly. However, we enable linguistic hedges to modify the original terms. The proposed technique can be classified in the semantic model. The semantic of the linguistic variables is maintained by the membership function. Furthermore, we insist that it is more natural to express preference information by means of expressions with hedges. For example, if the information is not sufficient to prove that one object is s_2, we may say "more or less s_2" naturally. Only if no technique can be used to compute this expression "more or less s_2", we express our opinion by such as $[s_1, s_3]$ or $\{s_1, s_2, s_3\}$ as alternatives.

We use the triangular membership functions to represent linguistic terms in this paper because they are suitable for most cases. If the trapezoidal membership functions are adopted, the corresponding theories can be easily extended as follows: Suppose that the original term $s_\alpha \in S$ is represented by a trapezoidal fuzzy number. We consider the uniform and symmetrically distributed linguistic terms set as well for convenience and let $b_\alpha - a_\alpha = d_\alpha - c_\alpha$, $b_\alpha - a_\alpha$ and $c_\alpha - b_\alpha$ be the fixed numbers Δ_{ab} and Δ_{bc} for any $s_\alpha \in S$. Then, the following resemblance relation can be defined: for any $x, y \in X$,

$$R(x, y) = \max\left(0, \min\left(1, 1 - \frac{|x - y| - \Delta_{bc}}{\Delta_{ab}}\right)\right) \tag{14}$$

Similar to Theorem 3, we can easily obtain the corresponding results. Based on the above assumption, we specify the results as

$$R \uparrow s_\alpha = (a_\alpha - \Delta_{ab}, b_\alpha, c_\alpha, d_\alpha + \Delta_{ab}) \tag{15}$$

$$R \uparrow\uparrow s_\alpha = (a_\alpha - 2\Delta_{ab}, b_\alpha, c_\alpha, d_\alpha + 2\Delta_{ab}) \tag{16}$$

According to the analysis in Section 2. 2, the expressions "more or less s_α" and "roughly s_α" can be represented by $R \uparrow s_\alpha$ and $R \uparrow\uparrow s_\alpha$, respectively.

5 Conclusion

A semantic approach for hedges is presented to model linguistic expressions with weakened hedges. Based on the approach, the proposed MCDM approach can handle this kind of linguistic variables directly. Given a predefined LTS, the only parameter which indicates the threshold of similarity between two objects on the domain is objectively determined by the semantics of terms. The advantages of the proposed technique lie in some aspects. First, the linguistic expressions with hedges are more natural and close to human languages than uncertain linguistic terms and HFLTSs. Secondly, the fuzzy rough set-based approach is more objective than other methods which use fuzzy sets only.

As for future work, we consider the application of the proposed model, for instance, to support linguistic decision making in intelligent decision support systems, to act as a technique for the transformation of any two LTSs with distinct granularities, or to represent preference relations in a qualitative setting. The partial or total orders of linguistic expressions with hedges are also interesting.

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弱化语言修饰词在多属性决策中的应用

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摘要:为允许在语言多属性决策环境中使用“差不多”和“有点”等表示程度弱化的语言修饰词，提出了一种基于语义的表示模型. 首先，基于论域上语言标度的语义定义相似关系，并通过计算语言标度的上近似和松上近似得到语言修饰词的表示方法，进而推导出语言表达式的语义公式，其中参数由语言标度的语义客观确定. 该模型具有清晰的语义，且允许专家以更自然的方式表达不确定的决策信息. 将模型应用于可持续创新能源技术的评估和选择,计算结果表明,所提出的模型可以处理评估过程中存在的多种不确定性. 最后,讨论了该模型与现有技术的区别，并给出了该模型在语义为梯形模糊数情形下的拓展.

关键词:决策；多属性决策；语言标度集；语言修饰词；相似关系

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