Journal of Southeast University (English Edition) Vol. 32, No. 2, pp. 183 - 186 June 2016 ISSN 1003—7985

An adaptive switching control approach for trajectory tracking of robotic manipulators

Yang Zhen[1,2] Fei Shumin[1] Wang Fang[2] Bao Anping[1] Liu Guquan[1]

([1]Key Laboratory of Measurement and Control of Complex Systems of Engineering of Ministry of Education, Southeast University, Nanjing 210096, China)

([2]School of Information Science and Engineering, Zaozhuang University, Zaozhuang 277100, China)

Abstract: In order to design a suitable controller which can achieve accurate trajectory tracking and a good control performance, and guarantee the stability and robustness of a robot system due to external disturbances error and internal parameter variations, an adaptive switching control strategy is proposed. The proposed scheme is designed under the condition of bounded distances and consists of an adaptive switching law and a PD controller. Based on the Lyapunov stability theory, it is proved that the proposed scheme can guarantee the tracking performance of the robotic manipulator and is adapted to varying unknown loads. Simulations are carried out on a two-link robotic manipulator, which illustrate the feasibility and validity of the proposed control scheme and the robustness for variational payloads.

Key words: adaptive control; switch control; robotic manipulator; trajectory tracking

doi: 10.3969/j. issn. 1003 - 7985. 2016. 02. 009

The trajectory tracking of robot control is an important issue in robotics because of its nonlinear, powerful coupling and time-varying systems. Many control schemes have been presented in the past decades[1-4]. Generally, in the process of operating robotic manipulators, there are many uncertainties and disturbances, such as the nonlinear friction and variational payloads, which can cause the robot system to be unstable and deteriorate the system performance. In order to cope with the parameter uncertainty of the robotic system, the adaptive method for robot control is proposed[5-6]. However, for the traditional adaptive control, it is usually with the assumption that the uncertain parameter must be constant. Practically, robots must pick up or lay down some objects and the load, due to the manipulator, is not constant. Therefore, parameter jumping is present in this system. So, it is difficult for the traditional adaptive control to solve the above problem. It is well known that a system with a jumping parameter can be viewed as a switched system whose subsystems differ from each other only by parameters. In order to deal with the above problem, there are a few works that combine the adaptive control with the switched system[7-9]. Linear PD control is one of the most simple and effective control methods, which has been widely used in the field of industrial robots. However, it is shown that a very large initial output requirement for the driving mechanism is a limitation for extending the application of linear PD control[10].

In this paper, an adaptive switching controller with PD parameters for a serial n-joint robotic manipulator is discussed. By the common Lyapunov function method, the adaptive updated laws and the switching signals have been developed to guarantee that the closed-loop system is asymptotically Lyapunov stable and the position of the manipulator's joint can follow any given desired signal. Finally, a simulation example of the robotic manipulator is given to illustrate the proposed method.

1 Problem Formulation

Considering an n-link robotic manipulator:

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})=\boldsymbol{\tau}+\boldsymbol{\omega} \tag{1}$$

where $\boldsymbol{q}\in\mathbf{R}^n$, $\dot{\boldsymbol{q}}\in\mathbf{R}^n$, $\ddot{\boldsymbol{q}}\in\mathbf{R}^n$ are the vectors of joint angles, velocity and acceleration, respectively; $\boldsymbol{\tau}\in\mathbf{R}^n$ is the torque input vector; $\boldsymbol{\omega}\in\mathbf{R}^n$ is the disturbance input and errors; $\boldsymbol{D}(\boldsymbol{q})\in\mathbf{R}^{n\times n}$ is the symmetric positive definite inertial matrix; $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\in\mathbf{R}^n$ is the vector of centripetal and Coriolis torques; and $\boldsymbol{G}(\boldsymbol{q})\in\mathbf{R}^n$ stands for the vector of gravitational forces.

Some properties[6] and assumptions of the robot (1) are listed, which is useful in the following stability analysis.

Property 1 $\boldsymbol{D}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$, $\boldsymbol{G}(\boldsymbol{q})$ of the dynamic model (1) can be linearly parameterized as

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})=\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta} \tag{2}$$

where $\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\in\mathbf{R}^{n\times m}$ is the regression matrix on the vector of joints, and $\boldsymbol{\theta}$ is the unknown constant vector on the load of the robotic manipulator.

Assumption 1 The desired vector of joint position, joint speed and joint acceleration are denoted by $\boldsymbol{q}_{\mathrm{d}}(t)$,

Received 2015-09-11.

Biographies: Yang Zhen(1975—), male, graduate; Fei Shumin(corresponding author), male, doctor, professor, 101005171@seu.edu.cn.

Foundation item: The National Natural Science Foundation of China (No. 61273119, 61374038, 61473079).

Citation: Yang Zhen, Fei Shumin, Wang Fang, et al. An adaptive switching control approach for trajectory tracking of robotic manipulators [J]. Journal of Southeast University (English Edition), 2016, 32(2): 183 - 186. doi: 10.3969/j. issn. 1003 - 7985. 2016. 02. 009.

$\dot{\boldsymbol{q}}_d(t)$, $\ddot{\boldsymbol{q}}_d(t) \in \mathbf{R}^n$, which are bounded.

Assumption 2 The disturbance input and errors $\boldsymbol{\omega}$ satisfy

$$\|\boldsymbol{\omega}\| \leqslant d_1 + d_2 \|\boldsymbol{e}\| + d_3 \|\dot{\boldsymbol{e}}\| \tag{3}$$

where d_1, d_2, d_3 are all positive constants; and $\boldsymbol{e} = \boldsymbol{q} - \boldsymbol{q}_d$ is the tracking error.

Considering the payload variation, we use the following switched model of the robotic manipulator for subsystems:

$$\boldsymbol{D}_\sigma(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_\sigma(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}_\sigma(\boldsymbol{q}) = \boldsymbol{\tau} + \boldsymbol{\omega} = \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta}_\sigma \tag{4}$$

where $\sigma(t): [0, +\infty) \to \Lambda = \{1, 2, \ldots, N\}$ is the switching signal dominated by the load.

2 Design of Adaptive Switching Controller

This section introduces the adaptive switching controller applied to the robotic manipulator. Our purpose is to design a robustly stable controller to ensure system stability and improve tracking performance. For the dynamic model of robot (4), the proposed controller is as follows:

$$\boldsymbol{\tau} = -\boldsymbol{K}_v\dot{\boldsymbol{e}} - \boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\hat{\boldsymbol{\theta}}_\sigma + \boldsymbol{u} \tag{5}$$

where $\boldsymbol{u} = \{u_1, u_2, \ldots, u_n\}^T$; $u_i = -(d_1 + d_2\|\boldsymbol{e}\| + d_3\|\dot{\boldsymbol{e}}\|) \cdot \operatorname{sgn}(\boldsymbol{e}_i)$; the positive-definite matrices $\boldsymbol{K}_p$ and $\boldsymbol{K}_v$ are the proportional and derivative gain matrices, respectively; $\hat{\theta}_i$ is the estimation of θ_i. Only when the i-th subsystem is active, will $\hat{\theta}_i$ work on it. The presented adaptive law is

$$\left.\begin{array}{ll} \dot{\tilde{\boldsymbol{\theta}}}_i^T = -\boldsymbol{e}^T\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\Gamma}_i^{-1} & \sigma = i \\ \dot{\tilde{\boldsymbol{\theta}}}_i^T = \boldsymbol{0} & \sigma \neq i \end{array}\right\} \tag{6}$$

where $\tilde{\boldsymbol{\theta}}_\sigma = \hat{\boldsymbol{\theta}}_\sigma - \boldsymbol{\theta}_\sigma$.

Theorem 1 For robotic system (4), the adaptive switching controllers (5) and (6) can guarantee global convergence of the tracking error. That is $\lim_{t\to\infty}\boldsymbol{e}(t) = \boldsymbol{0}$.

Proof Combining Eq. (4) with Eq. (5), we can obtain

$$-\boldsymbol{K}_v\dot{\boldsymbol{e}} - \boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\hat{\boldsymbol{\theta}}_\sigma + \boldsymbol{u} + \boldsymbol{\omega} = \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta}_\sigma$$

$$\boldsymbol{K}_v\dot{\boldsymbol{e}} = -\boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\tilde{\boldsymbol{\theta}}_\sigma + \boldsymbol{u} + \boldsymbol{\omega} \tag{7}$$

Choose a Lyapunov function candidate:

$$V(\boldsymbol{e},\boldsymbol{H}) = \frac{1}{2}[\boldsymbol{e}^T\boldsymbol{K}_v\boldsymbol{e} + \operatorname{tr}(\boldsymbol{H}^T\boldsymbol{\Gamma}\boldsymbol{H})]$$

where $\boldsymbol{H}^T = [\tilde{\boldsymbol{\theta}}_1 \quad \tilde{\boldsymbol{\theta}}_2 \quad \cdots \quad \tilde{\boldsymbol{\theta}}_N]$ and $\boldsymbol{\Gamma} = \operatorname{diag}(\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \cdots, \boldsymbol{\Gamma}_N)$ with $\boldsymbol{\Gamma}_i > 0$, $i = 1, 2, \cdots, N$.

Taking the time of derivative of $V(\boldsymbol{e},\boldsymbol{H})$ and using Eqs. (6) and (7), we have

$$\dot{V}(\boldsymbol{e},\boldsymbol{H}) = \boldsymbol{e}^T\boldsymbol{K}_v\dot{\boldsymbol{e}} + \dot{\tilde{\boldsymbol{\theta}}}_\sigma^T\boldsymbol{\Gamma}_\sigma\tilde{\boldsymbol{\theta}}_\sigma =$$

$$\boldsymbol{e}^T(-\boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\tilde{\boldsymbol{\theta}}_\sigma + \boldsymbol{u} + \boldsymbol{\omega}) + \dot{\tilde{\boldsymbol{\theta}}}_\sigma^T\boldsymbol{\Gamma}_\sigma\tilde{\boldsymbol{\theta}}_\sigma =$$

$$-\boldsymbol{e}^T\boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{e}^T\boldsymbol{u} + \boldsymbol{e}^T\boldsymbol{\omega} + (\boldsymbol{e}^T\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) + \dot{\tilde{\boldsymbol{\theta}}}_\sigma^T\boldsymbol{\Gamma}_\sigma)\tilde{\boldsymbol{\theta}}_\sigma =$$

$$-\boldsymbol{e}^T\boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{e}^T\boldsymbol{u} + \boldsymbol{e}^T\boldsymbol{\omega}$$

Based on Assumption 2, we have

$$\boldsymbol{e}^T\boldsymbol{u} = \sum_{i=1}^n e_i[-(d_1 + d_2\|\boldsymbol{e}\| + d_3\|\dot{\boldsymbol{e}}\|)\operatorname{sgn}(e_i)] =$$

$$\sum_{i=1}^n (-d_1 - d_2\|\boldsymbol{e}\| - d_3\|\dot{\boldsymbol{e}}\|)|e_i| \leqslant$$

$$\sum_{i=1}^n (-\|\boldsymbol{\omega}\| \cdot |e_i|) = -\|\boldsymbol{\omega}\| \cdot \|\boldsymbol{e}\|$$

Since $\boldsymbol{e}^T\boldsymbol{\omega} \leqslant \|\boldsymbol{e}^T\| \cdot \|\boldsymbol{\omega}\|$, $\boldsymbol{e}^T\boldsymbol{u} + \boldsymbol{e}^T\boldsymbol{\omega} \leqslant 0$,

$$\dot{V}(\boldsymbol{e},\boldsymbol{H}) \leqslant -\boldsymbol{e}^T\boldsymbol{K}_p\boldsymbol{e} < 0 \qquad \forall\, \boldsymbol{e} \neq \boldsymbol{0} \tag{8}$$

It means that $\dot{V}(\boldsymbol{e},\boldsymbol{H})$ is a non-increasing function over time t. Hence, $\forall\, t \geqslant 0, V(\boldsymbol{e}(t),\boldsymbol{H}(t)) \leqslant V(\boldsymbol{e}(0),\boldsymbol{H}(0))$, which implies that $V(\boldsymbol{e},\boldsymbol{H})$ is bound for the signals $\boldsymbol{e}$ and $\boldsymbol{H}$. Integrating both sides of (8) over $[0, +\infty)$ leads to

$$\int_0^\infty \boldsymbol{e}^T\boldsymbol{e}\,\mathrm{d}t \leqslant \frac{1}{\lambda_{\min}}\int_0^\infty \boldsymbol{e}^T\boldsymbol{K}_p\boldsymbol{e}\,\mathrm{d}t = \frac{1}{\lambda_{\min}}(V(0) - V(\infty)) \leqslant \frac{1}{\lambda_{\min}}V(0) \tag{9}$$

The above (9) implies $\boldsymbol{e} \in \boldsymbol{L}_2$. It can be seen that $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\ddot{\boldsymbol{q}}$ are all bounded by Assumption 1 and the boundedness of e. According to Property 1, the boundedness of $\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$ can be ensured. Therefore, from Eq. (7), $\dot{\boldsymbol{e}} \in \boldsymbol{L}_\infty$. Using Barbalat's lemma, we have $\lim_{t\to\infty}\boldsymbol{e}(t) = \boldsymbol{0}$. So far, the proof has been completed.

3 Simulation

In this section, the proposed adaptive switching strategy is applied to control the robotic manipulator to demonstrate feasibility and effectiveness. Without loss of generality, simulations are carried out for a two-link planar manipulator whose load is constantly changing. The linkage is composed of two rigid beams with actuators mounted at the joints. The load can be considered as the part of the second link. The parameters of the dynamic model (1) are as follows.

The length and mass of robot are: $r_1 = 1$, $r_2 = 0.8$; $m_1 = 0.5$, $m_2 = 0.5$ $(\sigma = 1)$, $m_2 = 1$ $(\sigma = 2)$.

The disturbance input and errors, given reference trajectory, and the initial state of system are

$$\omega = d_1 + d_2\|\boldsymbol{e}\| + d_3\|\dot{\boldsymbol{e}}\| \qquad d_1 = 2,\ d_2 = 3,\ d_3 = 6$$

$$q_{1d} = \sin(2\pi t),\ q_{2d} = \sin(2\pi t),\ \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \\ 0 \end{bmatrix}$$

The control parameters are

$$\boldsymbol{K}_p = \mathrm{diag}(50,50),\ \boldsymbol{K}_d = \mathrm{diag}(180,180),\ \boldsymbol{\Gamma} = \mathrm{diag}(5,5)$$

The switching signal is shown in Fig. 1. The curves of the θ_i estimate value are shown in Fig. 2. Fig. 3(a) denotes the tracking error performance of two links. In order to show the advantage of the adaptive switching controller, the tracking error using the PID controller is obtained in Fig. 3(b). Comparing Fig. 3(a) with Fig. 3(b), we can see that the proposed controller is superior to the PID controller apparently in terms of convergence speed and tracking accuracy. The bounded control input for the two-link of robotic manipulator is given in Figs. 4(a) and (b). From the above description, it can be seen that the proposed control scheme provides the better control performance. Similar to the claims of Theorem 1, Fig. 3(a) shows that the tracking errors converge to zero.

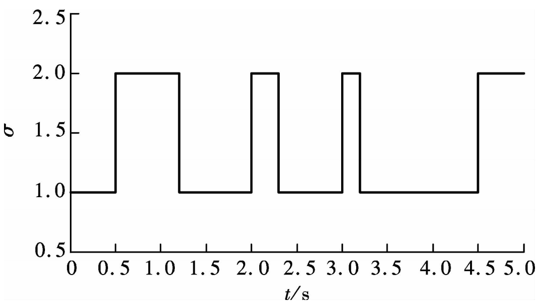

Fig. 1 The switching signal

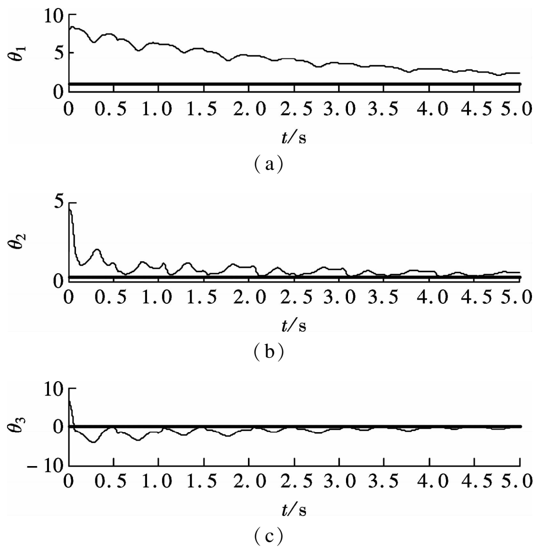

Fig. 2 Curves of θ_i estimate value. (a) θ_1; (b) θ_2; (c) θ_3

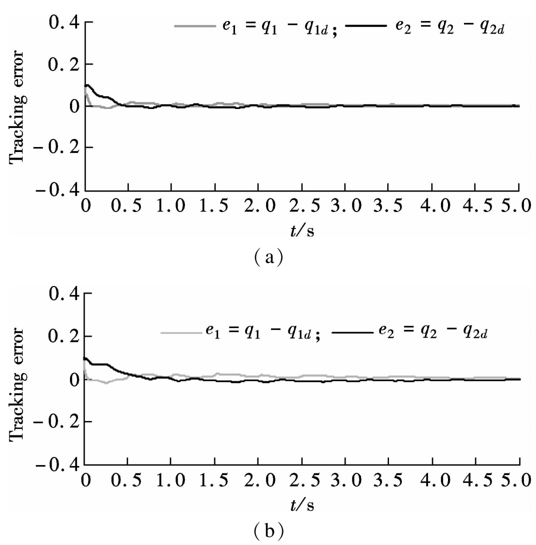

Fig. 3 Tracking error. (a) Adaptive switching control; (b) PID

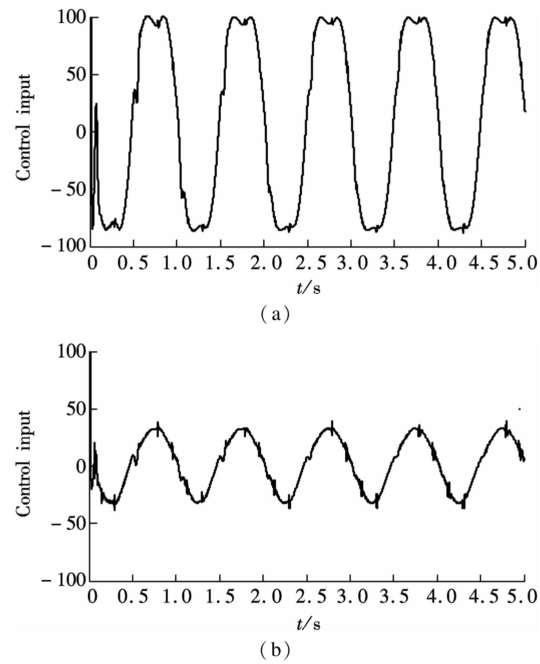

Fig. 4 The control input for two links. (a) Link 1; (b) Link 2

4 Conclusion

In this paper, an adaptive switching control approach is investigated for the robotic manipulator with changing loads. When the corresponding subsystem is activated, the proposed adaptive update law works. Based on the Lyapunov stability theory, it is shown that the proposed control scheme can guarantee the tracking performance of the robotic manipulator system. Simulation results show that the satisfactory tracking performance can be obtained. In the text, the proposed scheme is obtained under the condition of Assumption 2 which shows that the supremum of the bounded disturbance for robot manipulator is known. In our future work, we will consider the case without knowing the supremum.

References

[1] Van Cuong P, Wang Y N. Adaptive trajectory tracking neural network control with robust compensator for robot manipulators [J]. *Neural Computing and Applications*, 2016, **27**(2): 525 − 536. DOI: 10.1007/s00521-015-1873-4.

[2] Hernandez-Guzman V M, Santibanez V, Campa R. PID control of robot manipulators equipped with brushless DC motors [J]. *Robotica*, 2009, **27**(2): 225 − 233. DOI: 10.1017/s026357470800461x.

[3] Huh S H, Bien Z. Robust sliding mode-control of a robot manipulator based on variable structure-mode reference adaptive control approach [J]. *IET Control Theory and Applications*, 2007, **1**(5): 1355 – 1363. DOI:10.1049/iet-cta:20060440.

[4] Craig J J, Hsu P, Sastry S. S. Adaptive control of mechanical manipulators [J]. *International Journal of Robotic Research*, 1987, **6**(2): 16 – 28.

[5] Yu L, Fei S M, Sun L N, et al. An adaptive neural network switching control approach of robotic manipulators for trajectory tracking [J]. *International Journal of Computer Mathematics*, 2014, **91**(5): 983 – 995. DOI:10.1080/00207160.2013.813021.

[6] Wang X, Niu R S, Chen C, et al. H_∞ switched adaptive control for a class of robot manipulators [J]. *Transactions of the Institute of Measurement and Control*, 2014, **36**(3): 347 – 353. DOI:10.1177/0142331213503362.

[7] Yu L, Fei S M. Robustly stable switching neural control of robotic manipulators using average dwell-time approach [J]. *Transactions of the Institute of Measurement and Control*, 2014, **36** (6): 789 – 796. DOI: 10.1177/0142331213507593.

[8] Branicky M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems [J]. *IEEE Transactions on Automatic Control*, 1998, **43**(4): 475 – 482. DOI:10.1109/9.664150.

[9] Niu R S, Wang X, Zhao J, et al. Switched adaptive control for a class of robot manipulators [C]//*Proceedings of the 31st Chinese Control Conference*. Hefei, China, 2012: 4974 – 4978.

[10] Jiao X H, Li Y F, Fang Y M, et al. A method on robust adaptive control strategy of robot [J]. *Technology and Application of Robot*, 2002, **5**(3): 40 – 43. (in Chinese)

一种工业机器人轨迹跟踪的自适应切换控制方法

杨　振[1,2]　费树岷[1]　王　芳[2]　鲍安平[1]　刘顾全[1]

([1] 东南大学复杂工程系统测量与控制教育部重点实验室, 南京 210096)
([2] 枣庄学院信息科学与工程学院, 枣庄 277100)

摘要: 针对机器人系统易受外部干扰及内部参数变化影响的问题,以设计具有良好跟踪性能及优良控制品质、保证系统稳定性和鲁棒性的控制器为目标,提出了一种自适应切换控制方法. 该控制方法在系统干扰有界的条件下获得,包括 PD 控制器和自适应切换 2 部分. 应用李雅谱诺夫稳定性理论证明了所提控制方法既能够保证机器人的跟踪性能,也可以适应变化的未知负载. 以二连杆机器人为被控对象的仿真研究表明,所提出的控制方法有效可行,对系统负载的变化具有一定的鲁棒性.

关键词: 自适应控制;切换控制;工业机器人;轨迹跟踪
中图分类号: TP241