Journal of Southeast University (English Edition) Vol. 32, No. 2, pp. 240 - 249 June 2016 ISSN 1003—7985

A group decision making method based on double hesitant linguistic preference relations

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Abstract: A simple decision method is proposed to solve the group decision making problems in which the weights of decision organizations are unknown and the preferences for alternatives are provided by double hesitant linguistic preference relations. First, double hesitant linguistic elements are defined as representing the uncertain assessment information in the process of group decision making accurately and comprehensively, and the double hesitant linguistic weighted averaging operator is developed based on the defined operational laws for double hesitant linguistic elements. Then, double hesitant linguistic preference relations are defined and a means to objectively determine the weights of decision organizations is put forward using the standard deviation of scores of preferences provided by the individual decision organization for alternatives. Finally the correlation coefficient between the scores of preferences and the scores of preferences are provided by the other decision organizations. Accordingly, a group decision method based on double hesitant linguistic preference relations is proposed, and a practical example of the Jiudianxia reservoir operation alternative selection is used to illustrate the practicability and validity of the method. Finally, the proposed method is compared with the existing methods. Comparative results show that the proposed method can deal with the double hesitant linguistic preference information directly, does not need any information transformation, and can thus reduce the loss of original decision information.

Key words: group decision making; double hesitant linguistic elements; double hesitant linguistic preference relations; double hesitant linguistic weighted averaging operator

doi: 10.3969/j.issn.1003 -7985.2016.02.018

In reality, people are used to using natural languages to assess the qualitative aspects of problems. For example, when assessing the environmental quality of a city, experts prefer to use natural languages, such as "very good", "good" and "poor", etc. The fuzzy linguistic approach is a technique to deal with the qualitative information[1]. So far, many linguistic models have been developed to extend and improve the fuzzy linguistic approaches in information modeling and computing processes, such as the semantic model[2], the symbolic model[3] and the 2-tuple linguistic model[4], etc. Among them, the symbolic model implements direct computations on linguistic labels, and thus possesses the merits of simple computational processes and high interpretability, which has been widely applied to many fields, such as decision making[5], information retrieval[6], supply chain management[7], marking[8], sustainable energy management[9], etc.

In the above-mentioned linguistic models, an expert can only use a single linguistic term to express his/her assessment for an alternative under a criterion, which indicates that the degree of the alternative to the linguistic term under the criterion being 1. Nevertheless, sometimes a single linguistic term is inadequate to exactly express the expert's assessment for the alternative under the criterion because there may be some ambiguities when he/she provides the linguistic term as the assessment of the alternative under the criterion. For coping with such cases, Wang and Li[10] proposed the concept of intuitionistic linguistic sets, by which an alternative can be assessed by a linguistic term with a membership degree and a non-membership degree, both of which are values in $[0, 1]$. Liu et al. [11] defined hesitant intuitionistic fuzzy linguistic sets, in which the membership and non-membership degrees of an element to a linguistic term are denoted by a set of intuitionistic fuzzy numbers. It is worthwhile mentioning that in the process of group decision making or anonymous assessment, different decision makers or evaluators may give different linguistic terms with different membership and non-membership degrees, which are subintervals of $[0, 1]$. For example, a decision organization composed of several experts is invited to assess the environmental quality of a city in terms of the linguistic term set: $S = \{s_{-3}$: extremely poor, s_{-2}: very poor, s_{-1}: poor, s_0: fair, s_1: good, s_2: very good, s_3: extremely good$\}$. Suppose two experts think that the environmental quality of the city is surely not "good". One of them deems the degree of the environmental quality of the city

Received 2015-09-10.

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Foundation items: The National Natural Science Foundation of China (No. 61273209, 71571123), the Scientific Research Foundation of Graduate School of Southeast University (No. YBJJ1527), the Scientific Innovation Research of College Graduates in Jiangsu Province (No. KYLX_0207).

Citation: Zhao Na, Xu Zeshui. A group decision making method based on double hesitant linguistic preference relations[J]. Journal of Southeast University (English Edition), 2016, 32(2): 240 - 249. doi: 10.3969/j.issn.1003 -7985.2016.02.018.

to be "good" and the degree of it not belonging to "good" as 0.6 and 0.3, respectively, and the other deems it as [0.7,0.8] and [0.1,0.2], respectively. The rest of the experts think that the environmental quality of the city is doubtlessly "very good". Assume that they all insist on their own viewpoints and cannot persuade each other. To represent such assessments accurately, we will develop a new linguistic presentation model: double hesitant linguistic elements. By our model, we can use $\{\langle s_1,(0.6,0.3),([0.7,0.8],[0.1,0.2])\rangle,\langle s_2,(1,0)\rangle\}$ to express the assessment information of the decision organization. Double hesitant linguistic elements consider people's ambiguities when providing assessments, and encompass much more decision information; which is a very useful means to collect the decision makers' assessment information in large group decision making or anonymous assessment problems accurately and comprehensively.

In the process of decision making, the decision makers are often required to provide their preferences by comparing each pair of alternatives and construct preference relations. Up to now, there have been many different kinds of preference relations, such as fuzzy preference relations[12], intuitionistic preference relations[13], interval-valued intuitionistic preference relations[14], and linguistic preference relations[15]. It is worth noting that none of the existing preference relations permit the decision makers to provide all possible linguistic terms with the membership and non-membership degrees, which are the subintervals of $[0,1]$, as their preferences for a pair of alternatives. So another aim of this paper is to define double hesitant linguistic preference relations to overcome this limitation, and investigate their applications in group decision making.

1 Preliminaries

In this paper, we consider a subscript symmetric additive linguistic term set $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$, where τ is a positive integer. In general, S is required to satisfy the following conditions[15]:

1) There is a negation operator $\operatorname{neg}(s_i)=s_{-i}$, especially, $\operatorname{neg}(s_0)=s_0$;

2) The set is ordered, $s_i\leqslant s_j \Leftrightarrow i\leqslant j$.

To facilitate the calculation of linguistic terms, Xu[15] extended the discrete linguistic term set S to a continuous set $\bar{S}$, and defined two operational laws as follows:

Definition 1[15] Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set, then $\bar{S}=\{s_\alpha \mid \alpha\in[-q,q]\}$ is called the extended linguistic term set of S, where $q(q>\tau)$ is a sufficiently large natural number.

Usually, we call s_α an original linguistic term if $s_\alpha\in S$; otherwise, we call s_α a virtual linguistic term.

Definition 2[15] Let $s_\alpha, s_\beta\in\bar{S}$, $\lambda\in[0,1]$, then

$$s_\alpha\oplus s_\beta=s_{\alpha+\beta},\ \lambda s_\alpha=s_{\lambda\alpha}$$

In order to express the membership and non-membership degrees of an element to a linguistic term, Wang and Li[10] defined intuitionistic linguistic sets.

Definition 3[10] Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set, $\bar{S}$ be its extended linguistic term set, and X be a given domain. Then an intuitionistic linguistic set (ILS) in X is defined as

$$A=\{\langle x[s_{\theta(x)},(u_A(x),v_A(x))]\rangle \mid x\in X\}$$

where $s_{\theta(x)}\in\bar{S}$, and $u_A(x)$ and $v_A(x)$ denote the membership and non-membership degrees of the element $x\in X$ to the linguistic term $s_{\theta(x)}$, respectively, with the condition $0\leqslant u_A(x)\leqslant 1$, $0\leqslant v_A(x)\leqslant 1$, $0\leqslant u_A(x)+v_A(x)\leqslant 1$.

Afterwards, Liu et al.[11] defined hesitant intuitionistic fuzzy linguistic sets, in which the membership and non-membership degrees of an element to a linguistic term are denoted by a set of intuitionistic fuzzy numbers.

Definition 4[11] Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set, $\bar{S}$ be its extended linguistic term set, and X be a given domain. Then a hesitant intuitionistic fuzzy linguistic set (HIFLS) in X is defined by

$$B=\{\langle x[s_{\theta(x)},h_B(x)]\rangle \mid x\in X\}$$

where $s_{\theta(x)}\in\bar{S}$, and $h_B(x)=\cup_{(u_B(x),v_B(x))\in h_B(x)}\{(u_B(x),v_B(x))\}$. For any $x\in X$, $0\leqslant u_B(x)\leqslant 1$, $0\leqslant v_B(x)\leqslant 1$, $0\leqslant u_B(x)+v_B(x)\leqslant 1$, where $u_B(x)$ and $v_B(x)$ represent the possible membership and non-membership degrees of the element $x\in X$ to the linguistic term $s_{\theta(x)}$, respectively.

2 Double Hesitant Linguistic Elements and Basic Operations

In this section, we will define double hesitant linguistic elements and some basic operational laws.

Definition 5 Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1\cdots,\tau\}$ be a linguistic term set, $\bar{S}$ be its extended linguistic term set, and X be a given domain. Then a double hesitant linguistic set (DHLS) in X is defined as

$$D=\{\langle x\{[s_{\theta_i(x)},M_i(x)] \mid i=1,2,\cdots,t_x\}\rangle \mid x\in X\}$$

where $s_{\theta_i(x)}\in\bar{S}$; t_x is a positive integer and for $i=1,2,\cdots,t_x$, $M_i(x)=\{([h_r^{i-}(x),h_r^{i+}(x)],[g_r^{i-}(x),g_r^{i+}(x)]) \mid 0\leqslant h_r^{i-}(x)\leqslant h_r^{i+}(x)\leqslant 1, 0\leqslant g_r^{i-}(x)\leqslant g_r^{i+}(x)\leqslant 1, 0\leqslant h_r^{i+}(x)+g_r^{i+}(x)\leqslant 1, r=1,2,\cdots,l_i^x\}$, in which l_i^x is a positive integer. Here for $r=1,2,\cdots,l_i^x$, the intervals $[h_r^{i-}(x),h_r^{i+}(x)]$ and $[g_r^{i-}(x),g_r^{i+}(x)]$ represent the possible membership and non-membership degrees of the element $x\in X$ to the linguistic term $s_{\theta_i(x)}$, respectively.

For convenience, we call $d(x)=\{\langle s_{\theta_i(x)},M_i(x)\rangle \mid i=1,2,\cdots,t_x\}$ a double hesitant linguistic element (DHLE). Clearly, the DHLS D can be written as $D=$

$\{\{\langle s_{\theta_i(x)}, M_i(x)\rangle \mid i=1,2,\cdots,t_x\} \mid x\in X\}$. Thus, DHLEs are the basic unites of a DHLS.

From Definition 5, we can derive some special results: if for all $x\in X$, $t_x=1$ and $M_1(x)=([h_1(x),h_1(x)],[g_1(x),g_1(x)])$, where $0\leqslant h_1(x)$, $g_1(x)\leqslant 1$ and $0\leqslant h_1(x)+g_1(x)\leqslant 1$, then the DHLS D reduces to an ILS, which means that ILS is a particular case of DHLS; if for all $x\in X$, $t_x=1$ and $M_1(x)=\{([h_r(x),h_r(x)],[g_r(x),g_r(x)]) \mid 0\leqslant h_r(x)\leqslant 1, 0\leqslant g_r(x)\leqslant 1, 0\leqslant h_r(x)+g_r(x)\leqslant 1, r=1,2,\cdots,l^x\}$, then the DHLS D reduces to an HIFLS, which illustrates that HIFLS is a particular case of DHLS.

For comparing DHLEs, the following concepts are proposed.

Definition 6 Let $d(x)=\{\langle s_{\theta_i(x)}, M_i(x)\rangle \mid i=1,2,\cdots,t_x\}$ be a DHLE, where for $i=1,2,\cdots,t_x$, $M_i(x)=\{([h_r^{i-}(x),h_r^{i+}(x)],[g_r^{i-}(x),g_r^{i+}(x)]) \mid 0\leqslant h_r^{i-}(x)\leqslant h_r^{i+}(x)\leqslant 1, 0\leqslant g_r^{i-}(x)\leqslant g_r^{i+}(x)\leqslant 1, 0\leqslant h_r^{i+}(x)+g_r^{i+}(x)\leqslant 1, r=1,2,\cdots,l_i^x\}$. Then we call $\bar{h}_i(x)=\left[\sum_{r=1}^{l_i^x} h_r^{i-}(x)/l_i^x, \sum_{r=1}^{l_i^x} h_r^{i+}(x)/l_i^x\right]$ the average membership degree of the element x to the linguistic term $s_{\theta_i(x)}$, denoted by $[\bar{h}_i^-(x), \bar{h}_i^+(x)]$, $\bar{g}_i(x)=\left[\sum_{r=1}^{l_i^x} g_r^{i-}(x)/l_i^x, \sum_{r=1}^{l_i^x} g_r^{i+}(x)/l_i^x\right]$ the average non-membership degree of the element x to the linguistic term $s_{\theta_i(x)}$, denoted by $[\bar{g}_i^-(x), \bar{g}_i^+(x)]$.

Definition 7 Let $d=\{\langle s_{\theta_i(d)}, M_i(d)\rangle \mid i=1,2,\cdots,t_x\}$ be a DHLE, where for $i=1,2,\cdots,t_x$, $M_i(x)=\{([h_r^{i-}(x),h_r^{i+}(x)],[g_r^{i-}(x),g_r^{i+}(x)]) \mid 0\leqslant h_r^{i-}(x)\leqslant h_r^{i+}(x)\leqslant 1, 0\leqslant g_r^{i-}(x)\leqslant g_r^{i+}(x)\leqslant 1, 0\leqslant h_r^{i+}(x)+g_r^{i+}(x)\leqslant 1, r=1,2,\cdots,l_i^x\}$. Then the score function of d is defined as

$$F(d)=\sum_{i=1}^{t_x}\frac{1}{2t_x}\left(\frac{\bar{h}_i^-(x)+\bar{h}_i^+(x)}{2}+1-\frac{\bar{g}_i^-(x)+\bar{g}_i^+(x)}{2}\right)Q(s_{\theta_i(d)}) \qquad (1)$$

and the accuracy function of d is defined by

$$H(d)=\sum_{i=1}^{t_x}\frac{1}{2t_x}\left(\frac{\bar{h}_i^-(x)+\bar{h}_i^+(x)}{2}+\frac{\bar{g}_i^-(x)+\bar{g}_i^+(x)}{2}\right)\cdot Q(s_{\theta_i(d)}) \qquad (2)$$

where $Q(s_i)=i$; $\bar{h}_i^-(x)$, $\bar{h}_i^+(x)$, $\bar{g}_i^-(x)$ and $\bar{g}_i^+(x)$ are defined by Definition 6.

Then, based on Definitions 6 and 7, the following comparison rules are introduced.

Definition 8 Let $d_1=\{\langle s_{\theta_i(d_1)}, M_i(d_1)\rangle \mid i=1,2,\cdots,t_1\}$ and $d_2=\{\langle s_{\theta_j(d_2)}, M_j(d_2)\rangle \mid j=1,2,\cdots,t_2\}$ be any two DHLEs, then

1) If $F(d_1)>F(d_2)$, then d_1 is superior to d_2, denoted by $d_1 \succ d_2$;

2) If $F(d_1)=F(d_2)$, then

If $H(d_1)>H(d_2)$, then d_1 is superior to d_2, denoted by $d_1 \succ d_2$;

If $H(d_1)=H(d_2)$, then d_1 is equivalent to d_2, denoted by $d_1 \sim d_2$.

In what follows, an example is provided to illustrate the comparison method shown in Definition 8.

Example 1 Let $S=\{s_{-3}$: extremely poor, s_{-2}: very poor, s_{-1}: poor, s_0: fair, s_1: good, s_2: very good, s_3: extremely good$\}$ be a linguistic term set, $d_1=\{\langle s_{-3}, (0.6,0.3), ([0.8,0.9],[0,0.1])\rangle, \langle s_2, (0.8,0.2)\rangle\}$ and $d_2=\{\langle s_0, ([0.6,0.7],[0.1,0.2])\rangle, \langle s_{-2}, (0.15,0.8), ([0.2,0.4],[0.5,0.6])\rangle\}$ be two DHLVs, then by Eq. (1), we obtain

$$F(d_1)=-0.362\,5, \quad F(d_2)=-0.275$$

Since $F(d_1)<F(d_2)$, we have $d_1 \prec d_2$.

After giving the comparison rules of DHLEs, the next thing we need to do is to introduce their operational laws.

Definition 9 Let $d_1=\{\langle s_{\theta_i(d_1)}, M_i(d_1)\rangle \mid i=1,2,\cdots,t_1\}$ and $d_2=\{\langle s_{\theta_j(d_2)}, M_j(d_2)\rangle \mid j=1,2,\cdots,t_2\}$ be any two DHLEs, where for $i=1,2,\cdots,t_1$, $M_i(d_1)=\{([h_t^{i-}(d_1),h_t^{i+}(d_1)],[g_t^{i-}(d_1),g_t^{i+}(d_1)]) \mid t=1,2,\cdots,l_i^1\}$ and for $j=1,2,\cdots,t_2$, $M_j(d_2)=\{([h_r^{j-}(d_2),h_r^{j+}(d_2)],[g_r^{j-}(d_2),g_r^{j+}(d_2)]) \mid r=1,2,\cdots,l_j^2\}$, and $\lambda\geqslant 0$. Then the following operations are valid:

1) $\mathrm{neg}(d_1)=\{\langle s_{-\theta_i(d_1)}, M_i(d_1)\rangle \mid i=1,2,\cdots,t_1\}$.

2) $d_1\oplus d_2=\{\langle s_{\theta_i(d_1)+\theta_j(d_2)}, M_{ij}(d_1\oplus d_2)\rangle \mid i=1,2,\cdots,t_1; j=1,2,\cdots,t_2\}$, where for $i=1,2,\cdots,t_1$, $j=1,2,\cdots,t_2$, $M_{ij}(d_1\oplus d_2)=\{([h_t^{i-}(d_1)+h_r^{j-}(d_2)-h_t^{i-}(d_1)\cdot h_r^{j-}(d_2), h_t^{i+}(d_1)+h_r^{j+}(d_2)-h_t^{i+}(d_1)h_r^{j+}(d_2)], [g_t^{i-}(d_1)g_r^{j-}(d_2), g_t^{i+}(d_1)g_r^{j+}(d_2)]) \mid t=1,2,\cdots,l_i^1; r=1,2,\cdots,l_j^2\}$.

3) $\lambda d_1=\{\langle s_{\lambda\theta_i(d_1)}, M_i(\lambda d_1)\rangle \mid i=1,2,\cdots,t_1\}$, where for $i=1,2,\cdots,t_1$, $M_i(\lambda d_1)=\{([1-(1-h_t^{i-}(d_1))^\lambda, 1-(1-h_t^{i+}(d_1))^\lambda], [(g_t^{i-}(d_1))^\lambda, (g_t^{i+}(d_1))^\lambda]) \mid t=1,2,\cdots,l_i^1\}$.

Example 2 Let $S=\{s_{-3}$: extremely poor, s_{-2}: very poor, s_{-1}: poor, s_0: fair, s_1: good, s_2: very good, s_3: extremely good$\}$ be a linguistic term set, $d_1=\{\langle s_1, (0.6,0.2), ([0.7,0.9],[0,0.1])\rangle, \langle s_3, (0.7,0.1)\rangle\}$ and $d_2=\{\langle s_0, ([0.6,0.8],[0.1,0.2])\rangle, \langle s_2, (0.8,0.2)\rangle, \langle s_3, (0.5,0.4), ([0.6,0.7],[0.2,0.3])\rangle\}$ be two DHLEs. Then according to Definition 9, we obtain

1) $\mathrm{neg}(d_1)=\{\langle s_{-3}, (0.7,0.1)\rangle, \langle s_{-1}, (0.6,0.2), ([0.7,0.9],[0,0.1])\rangle\}$;

2) $0.4d_1=\{\langle s_{0.4}, (0.306\,9, 0.525\,3), ([0.382\,2, 0.601\,9],[0,0.398\,1])\rangle, \langle s_{1.2}, (0.382\,2, 0.398\,1)\rangle\}$;

3) $d_1\oplus d_2=\{\langle s_1, ([0.84,0.92],[0.02,0.04]), ([0.88,0.98],[0,0.02])\rangle, \langle s_3, (0.92,0.04), ([0.94,0.98],[0,0.02]), ([0.88,0.94],[0.01,$

$0.02]])\rangle, \langle s_4, (0.8, 0.08), ([0.84, 0.88], [0.04, 0.06]), ([0.85, 0.95], [0, 0.04]), ([0.88, 0.97], [0, 0.03])\rangle, \langle s_5, (0.94, 0.02)\rangle, \langle s_6, (0.85, 0.04), ([0.88, 0.91], [0.02, 0.03])\rangle\}$.

For the operations of DHLEs in Definition 9, the following desirable properties are satisfied.

Property Let $d_1 = \{\langle s_{\theta_i(d_1)}, M_i(d_1)\rangle \mid i = 1, 2, \cdots, t_1\}$, $d_2 = \{\langle s_{\theta_j(d_2)}, M_j(d_2)\rangle \mid j = 1, 2, \cdots, t_2\}$ and $d_3 = \{\langle s_{\theta_k(d_3)}, M_k(d_3)\rangle \mid k = 1, 2, \cdots, t_3\}$ be any three DHLEs, and λ, λ_1, $\lambda_2 \geqslant 0$. Then we obtain

1) $\lambda \operatorname{neg}(d_1) = \operatorname{neg}(\lambda d_1)$;

2) $\operatorname{neg}(d_1) \oplus \operatorname{neg}(d_2) = \operatorname{neg}(d_1 \oplus d_2)$;

3) $d_1 \oplus d_2 = d_2 \oplus d_1$;

4) $(d_1 \oplus d_2) \oplus d_3 = d_1 \oplus (d_2 \oplus d_3)$;

5) $\lambda(d_1 \oplus d_2) = \lambda d_1 \oplus \lambda d_2$.

Proof Here we assume for $i = 1, 2, \cdots, t_1$, $M_i(d_1) = \{([h_t^{i-}(d_1), h_t^{i+}(d_1)], [g_t^{i-}(d_1), g_t^{i+}(d_1)]) \mid t = 1, 2, \cdots, l_i^1\}$, for $j = 1, 2, \cdots, t_2$, $M_j(d_2) = \{([h_r^{j-}(d_2), h_r^{j+}(d_2)], [g_r^{j-}(d_2), g_r^{j+}(d_2)]) \mid r = 1, 2, \cdots, l_j^2\}$, and for $k = 1, 2, \cdots, t_3$, $M_k(d_3) = \{([h_p^{k-}(d_3), h_p^{k+}(d_3)], [g_p^{k-}(d_3), g_p^{k+}(d_3)]) \mid p = 1, 2, \cdots, l_k^3\}$. Then by Definition 9, we obtain

1) $\lambda \operatorname{neg}(d_1) = \lambda\{\langle s_{-\theta_i(d_1)}, M_i(d_1)\rangle \mid i = 1, 2, \cdots, t_1\} = \{\langle s_{-\lambda\theta_i(d_1)}, M_i(\lambda d_1)\rangle \mid i = 1, 2, \cdots, t_1\} = \operatorname{neg}(\{\langle s_{\lambda\theta_i(d_1)}, M_i(\lambda d_1)\rangle \mid i = 1, 2, \cdots, t_1\}) = \operatorname{neg}(\lambda d_1)$.

2) $\operatorname{neg}(d_1) \oplus \operatorname{neg}(d_2) = \{\langle s_{-\theta_i(d_1)}, M_i(d_1)\rangle \mid i = 1, 2, \cdots, t_1\} \oplus \{\langle s_{-\theta_j(d_2)}, M_j(d_2)\rangle \mid j = 1, 2, \cdots, t_2\} = \{\langle s_{-(\theta_i(d_1)+\theta_j(d_2))}, M_{ij}(d_1 \oplus d_2)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\} = \operatorname{neg}(\{\langle s_{\theta_i(d_1)+\theta_j(d_2)}, M_{ij}(d_1 \oplus d_2)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\}) = \operatorname{neg}(d_1 \oplus d_2)$.

3) The result is easily obtained by 2) in Definition 9.

4) $(d_1 \oplus d_2) \oplus d_3 = \{\langle s_{\theta_i(d_1)+\theta_j(d_2)}, M_{ij}(d_1 \oplus d_2)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\} \oplus \{\langle s_{\theta_k(d_3)}, M_k(d_3)\rangle \mid k = 1, 2, \cdots, t_3\} = \{\langle s_{\theta_i(d_1)+\theta_j(d_2)+\theta_k(d_3)}, M_{ijk}((d_1 \oplus d_2) \oplus d_3)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2; k = 1, 2, \cdots, t_3\}$, where for $i = 1, 2, \cdots, t_1, j = 1, 2, \cdots, t_2, k = 1, 2, \cdots, t_3$, $M_{ijk}((d_1 \oplus d_2) \oplus d_3) = \{([h_{tr}^{ij-}(d_1 \oplus d_2) + h_p^{k-}(d_3) - h_{tr}^{ij-}(d_1 \oplus d_2) h_p^{k-}(d_3), h_{tr}^{ij+}(d_1 \oplus d_2) + h_p^{k+}(d_3) - h_{tr}^{ij+}(d_1 \oplus d_2) h_p^{k+}(d_3)], [g_{tr}^{ij-}(d_1 \oplus d_2) g_p^{k-}(d_3), g_{tr}^{ij+}(d_1 \oplus d_2) g_p^{k+}(d_3)]) \mid t = 1, 2, \cdots, l_i^1; r = 1, 2, \cdots, l_j^2; p = 1, 2, \cdots, l_k^3\}$. Since for $t = 1, 2, \cdots, l_i^1, r = 1, 2, \cdots, l_j^2$, $h_{tr}^{ij-}(d_1 \oplus d_2) = h_t^{i-}(d_1) + h_r^{j-}(d_2) - h_t^{i-}(d_1) h_r^{j-}(d_2)$, $h_{tr}^{ij}(d_1 \oplus d_2) = h_t^{i+}(d_1) + h_r^{j+}(d_2) - h_t^{i+}(d_1) h_r^{j+}(d_2)$, $g_{tr}^{ij-}(d_1 \oplus d_2) = g_t^{i-}(d_1) g_r^{j-}(d_2)$ and $g_{tr}^{ij+}(d_1 \oplus d_2) = g_t^{i+}(d_1) g_r^{j+}(d_2)$, then we obtain

$$M_{ijk}((d_1 \oplus d_2) \oplus d_3) = \{([1 - (1 - h_t^{i-}(d_1))(1 - h_r^{j-}(d_2))(1 - h_p^{k-}(d_3)), 1 - (1 - h_t^{i+}(d_1))(1 - h_r^{j+}(d_2))(1 - h_p^{k+}(d_3))], [g_t^{i-}(d_1) g_r^{j-}(d_2) g_p^{k-}(d_3), g_t^{i+}(d_1) g_r^{j+}(d_2) g_p^{k+}(d_3)]) \mid t = 1, 2, \cdots, l_i^1; r = 1, 2, \cdots, l_j^2; p = 1, 2, \cdots, l_k^3\}$$

Similarly, we obtain

$$d_1 \oplus (d_2 \oplus d_3) = \{\langle s_{\theta_i(d_1)+\theta_j(d_2)+\theta_k(d_3)}, M_{ijk}(d_1 \oplus (d_2 \oplus d_3))\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2; k = 1, 2, \cdots, t_3\}$$

where for $i = 1, 2, \cdots, t_1$, $j = 1, 2, \cdots, t_2$, $k = 1, 2, \cdots, t_3$,

$$M_{ijk}(d_1 \oplus (d_2 \oplus d_3)) = \{([1 - (1 - h_t^{i-}(d_1))(1 - h_r^{j-}(d_2))(1 - h_p^{k-}(d_3)), 1 - (1 - h_t^{i+}(d_1))(1 - h_r^{j+}(d_2))(1 - h_p^{k+}(d_3))], [g_t^{i-}(d_1) g_r^{j-}(d_2) g_p^{k-}(d_3), g_t^{i+}(d_1) g_r^{j+}(d_2) g_p^{k+}(d_3)]) \mid t = 1, 2, \cdots, l_i^1; r = 1, 2, \cdots, l_j^2; p = 1, 2, \cdots, l_k^3\} = M_{ijk}((d_1 \oplus d_2) \oplus d_3)$$

Therefore, we prove $(d_1 \oplus d_2) \oplus d_3 = d_1 \oplus (d_2 \oplus d_3)$.

5) $\lambda(d_1 \oplus d_2) = \lambda\{\langle s_{\theta_i(d_1)+\theta_j(d_2)}, M_{ij}(d_1 \oplus d_2)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\} = \{\langle s_{\lambda(\theta_i(d_1)+\theta_j(d_2))}, M_{ij}(\lambda(d_1 \oplus d_2))\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\}$, where for $i = 1, 2, \cdots, t_1$, $j = 1, 2, \cdots, t_2$, $M_{ij}(\lambda(d_1 \oplus d_2)) = \{([1 - (1 - h_{tr}^{ij-}(d_1 \oplus d_2))^{\lambda}, 1 - (1 - h_{tr}^{ij+}(d_1 \oplus d_2))^{\lambda}], [(g_{tr}^{ij-}(d_1 \oplus d_2))^{\lambda}, (g_{tr}^{ij+}(d_1 \oplus d_2))^{\lambda}]) \mid t = 1, 2, \cdots, l_i^1, r = 1, 2, \cdots, l_j^2\}$. Since for any $t = 1, 2, \cdots, l_i^1$; $r = 1, 2, \cdots, l_j^2$, $h_{tr}^{ij-}(d_1 \oplus d_2) = h_t^{i-}(d_1) + h_r^{j-}(d_2) - h_t^{i-}(d_1) h_r^{j-}(d_2)$, $h_{tr}^{ij+}(d_1 \oplus d_2) = h_t^{i+}(d_1) + h_r^{j+}(d_2) - h_t^{i+}(d_1) h_r^{j+}(d_2)$, $g_{tr}^{ij-}(d_1 \oplus d_2) = g_t^{i-}(d_1) g_r^{j-}(d_2)$ and $g_{tr}^{ij+}(d_1 \oplus d_2) = g_t^{i+}(d_1) g_r^{j+}(d_2)$, then we obtain

$$M_{ij}(\lambda(d_1 \oplus d_2)) = \{([1 - (1 - h_t^{i-}(d_1))^{\lambda}(1 - h_r^{j-}(d_2))^{\lambda}, 1 - (1 - h_t^{i+}(d_1))^{\lambda}(1 - h_r^{j+}(d_2))^{\lambda}], [(g_t^{i-}(d_1) g_r^{j-}(d_2))^{\lambda}, (g_t^{i+}(d_1) g_r^{j+}(d_2))^{\lambda}]) \mid t = 1, 2, \cdots, l_i^1; r = 1, 2, \cdots, l_j^2\}$$

Moreover, since

$$\lambda d_1 = \{\langle s_{\lambda\theta_i(d_1)}, M_i(\lambda d_1)\rangle \mid i = 1, 2, \cdots, t_1\}$$

where for $i = 1, 2, \cdots, t_1$,

$$M_i(\lambda d_1) = \{([1 - (1 - h_t^{i-}(d_1))^{\lambda}, 1 - (1 - h_t^{i+}(d_1))^{\lambda}], [(g_t^{i-}(d_1))^{\lambda}, (g_t^{i+}(d_1))^{\lambda}]) \mid t = 1, 2, \cdots, l_i^1\}$$

and

$$\lambda d_2 = \{\langle s_{\lambda\theta_j(d_2)}, M_j(\lambda d_2)\rangle \mid j = 1, 2, \cdots, t_2\}$$

where for $j = 1, 2, \cdots, t_2$,

$$M_j(\lambda d_2) = \{([1 - (1 - h_r^{j-}(d_2))^{\lambda}, 1 - (1 - h_r^{j+}(d_2))^{\lambda}], [(g_r^{j-}(d_2))^{\lambda}, (g_r^{j+}(d_2))^{\lambda}]) \mid r = 1, 2, \cdots, l_j^2\}$$

then we have

$$\lambda d_1 \oplus \lambda d_2 = \{\langle s_{\lambda\theta_i(d_1)+\lambda\theta_j(d_2)}, M_{ij}(\lambda d_1 \oplus \lambda d_2)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\} = \{\langle s_{\lambda(\theta_i(d_1)+\theta_j(d_2))}, M_{ij}(\lambda d_1 \oplus \lambda d_2)\rangle \mid i = 1, 2, \cdots, t_1; j = 1, 2, \cdots, t_2\}$$

where for $i = 1, 2, \cdots, t_1$, $j = 1, 2, \cdots, t_2$, $M_{ij}(\lambda d_1 \oplus \lambda d_2) = \{([h_{tr}^{ij-}(\lambda d_1 \oplus \lambda d_2), h_{tr}^{ij+}(\lambda d_1 \oplus \lambda d_2)], [g_{tr}^{ij-}(\lambda d_1 \oplus \lambda d_2), g_{tr}^{ij+}(\lambda d_1 \oplus \lambda d_2)]) \mid t = 1, 2, \cdots, l_i^1; r = 1, 2, \cdots,$

$l_j^2\}$.

Since for $t=1,2,\cdots,l_i^1$, $r=1,2,\cdots,l_j^2$, $h_{tr}^{ij-}(\lambda d_1 \oplus \lambda d_2)=1-(1-h_t^{i-}(d_1))^{\lambda}(1-h_r^{j-}(d_2))^{\lambda}$, $h_{tr}^{ij+}(\lambda d_1 \oplus \lambda d_2)=1-(1-h_t^{i+}(d_1))^{\lambda}(1-h_r^{j+}(d_2))^{\lambda}$, $g_{tr}^{ij-}(\lambda d_1 \oplus \lambda d_2)=(g_t^{i-}(d_1))^{\lambda}(g_r^{j-}(d_2))^{\lambda}$ and $g_{tr}^{ij+}(\lambda d_1 \oplus \lambda d_2)=(g_t^{i+}(d_1))^{\lambda}(g_r^{j+}(d_2))^{\lambda}$, then we obtain $M_{ij}(\lambda d_1 \oplus \lambda d_2)=M_{ij}(\lambda(d_1 \oplus d_2))$. Thus, we prove $\lambda(d_1 \oplus d_2)=\lambda d_1 \oplus \lambda d_2$.

In the process of decision making, the aggregation operators are usually used to incorporate the individual decision information into the collective one. In order to fuse the double hesitant linguistic information, the following basic aggregation operator is developed based on the operational laws in Definition 9.

Definition 10 Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1\cdots,\tau\}$ be a linguistic term set, $d_i(i=1,2,\cdots,n)$ be a collection of DHLEs, and V be the set of all DHLEs. Then a double hesitant linguistic weighted averaging (DHLWA) operator is a mapping DHLWA: $V^n \to V$ such that

$$\mathrm{DHLWA}_w(d_1,d_2,\cdots,d_n)=\bigoplus_{i=1}^{n}(w_i d_i) \tag{3}$$

where $\boldsymbol{w}=\{w_1,w_2,\cdots,w_n\}^{\mathrm{T}}$ is the weight vector of $d_i(i=1,2,\cdots,n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i=1$. Particularly, if $\boldsymbol{w}=\{1/n,1/n,\cdots,1/n\}^{\mathrm{T}}$, then the DHLWA operator reduces to the double hesitant linguistic averaging (DHLA) operator:

$$\mathrm{DHLA}(d_1,d_2,\cdots,d_n)=\bigoplus_{i=1}^{n}\left(\frac{1}{n}d_i\right) \tag{4}$$

Theorem 1 Let $d_i=\{\langle s_{\theta_{p_i}(d_i)},M_{p_i}(d_i)\rangle \mid p_i=1,2,\cdots,t_i\}(i=1,2,\cdots,n)$ be a collection of DHLEs, where for $p_i=1,2,\cdots,t_i(i=1,2,\cdots,n)$, $M_{p_i}(d_i)=\{([h_{r_i}^{p_i-}(d_i),h_{r_i}^{p_i+}(d_i)],[g_{r_i}^{p_i-}(d_i),g_{r_i}^{p_i+}(d_i)]) \mid r_i=1,2,\cdots,l_{p_i}^{d_i}\}$. Then the aggregation result derived from Eq. (3) is still a DHLE, and

$$\mathrm{DHLWA}_w(d_1,d_2,\cdots,d_n)=\{\langle s_{\sum_{i=1}^{n} w\theta_{p_i}(d_i)},M_{p_1p_2\cdots p_n}(\bigoplus_{i=1}^{n} w_i d_i)\rangle \mid p_i=1,2,\cdots,t_i(i=1,2,\cdots,n)\} \tag{5}$$

where for $p_1=1,2,\cdots,t_1$, $p_2=1,2,\cdots,t_2,\cdots,p_n=1,2,\cdots,t_n$,

$$M_{p_1p_2\cdots p_n}(\bigoplus_{i=1}^{n} w_i d_i)=\left\{\left[1-\prod_{i=1}^{n}(1-h_{r_i}^{p_i-}(d_i))^{w_i},\ 1-\prod_{i=1}^{n}(1-h_{r_i}^{p_i+}(d_i))^{w_i}\right],\left[\prod_{i=1}^{n}(g_{r_i}^{p_i-}(d_i))^{w_i},\ \prod_{i=1}^{n}(g_{r_i}^{p_i+}(d_i))^{w_i}\right] \,\middle|\, r_i=1,2,\cdots,l_{p_i}^{d_i}(i=1,2,\cdots,n)\right\}$$

Proof According to Definitions 9 and 10, the theorem can be easily proven by mathematical induction.

3 A Method to Group Decision Making with Double Hesitant Linguistic Preference Relations

Definition 11 Let $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1\cdots,\tau\}$ be a linguistic term set and $X_1=\{x_1,x_2,\cdots,x_n\}$ be a fixed set. Then a double hesitant linguistic preference relation (DHLPR) on X_1 is represented by a matrix $\boldsymbol{R}=(d_{ij})_{n\times n}\subset X_1\times X_1$ with $d_{ij}=\{\langle s_{\theta_{p_{ij}}(d_{ij})},M_{p_{ij}}(d_{ij})\rangle \mid p_{ij}=1,2,\cdots,t_{ij}\}$ being a DHLE, where for all $i,j=1,2,\cdots,n$, $d_{ij}(i<j)$ satisfies the following requirements:

$$s_{\sigma(l)(d_{ij})} \oplus s_{\sigma(t_{ji}-l+1)(d_{ji})}=s_0,\ M_{\sigma(l)}(d_{ij})=M_{\sigma(t_{ji}-l+1)}(d_{ji})$$
$$d_{ii}=\{\langle s_0,(1,0)\rangle\},\ t_{ij}=t_{ji}$$

where $s_{\sigma(l)(d_{ij})}$ is the l-th smallest linguistic term of $s_{\theta_1(d_{ij})}$, $s_{\theta_2(d_{ij})},\cdots,s_{\theta_{t_{ij}}(d_{ij})}$, and $M_{\sigma(l)}(d_{ij})$ is the set of possible membership and non-membership degrees to $s_{\sigma(l)(d_{ij})}$. We call $d_{ij}=\{\langle s_{\theta_{p_{ij}}(d_{ij})},M_{p_{ij}}(d_{ij})\rangle \mid p_{ij}=1,2,\cdots,t_{ij}\}$ the double hesitant linguistic preference of x_i over x_j, in which the linguistic terms $s_{\theta_{p_{ij}}(d_{ij})},p_{ij}=1,2,\cdots,t_{ij}$ manifest all possible linguistic preferences of x_i over x_j. Assume that $M_{p_{ij}}(d_{ij})=\{([h_{r_{ij}}^{p_{ij}-}(d_{ij}),h_{r_{ij}}^{p_{ij}+}(d_{ij})],[g_{r_{ij}}^{p_{ij}-}(d_{ij}),g_{r_{ij}}^{p_{ij}+}(d_{ij})]) \mid 0\leqslant h_{r_{ij}}^{p_{ij}-}(d_{ij})\leqslant h_{r_{ij}}^{p_{ij}+}(d_{ij})\leqslant 1,0\leqslant g_{r_{ij}}^{p_{ij}-}(d_{ij})\leqslant g_{r_{ij}}^{p_{ij}+}(d_{ij})\leqslant 1,0\leqslant h_{r_{ij}}^{p_{ij}+}(d_{ij})+g_{r_{ij}}^{p_{ij}+}(d_{ij})\leqslant 1,r_{ij}=1,2,\cdots,l_{p_{ij}}^{d_{ij}}\}$, then for $r_{ij}=1,2,\cdots,l_{p_{ij}}^{d_{ij}}$, the intervals $[h_{r_{ij}}^{p_{ij}-}(d_{ij}),h_{r_{ij}}^{p_{ij}+}(d_{ij})]$ and $[g_{r_{ij}}^{p_{ij}-}(d_{ij}),g_{r_{ij}}^{p_{ij}+}(d_{ij})]$ indicate the possible membership and non-membership degrees to the linguistic term $s_{\theta_{p_{ij}}(d_{ij})}$, respectively.

Below we consider the group decision making problem. Let $G=\{G_1,G_2,\cdots,G_n\}$ be a set of alternatives, and $O=\{O_1,O_2,\cdots,O_m\}$ be a set of decision organizations. Suppose that each decision organization is composed of several experts and each expert provides his/her preferences for each pair of alternatives. To convey the preferences accurately, each expert independently offers his/her preferences by using a linguistic term in the linguistic term set $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$, the membership and non-membership degrees to the linguistic term. So the preference information given by each decision organization $O_k(k=1,2,\cdots,m)$ can construct a DHLPR $\boldsymbol{D}_k=(d_{ij}^k)_{n\times n}$ with $d_{ij}^k=\{\langle s_{\theta_{p_{ij}^k}(d_{ij}^k)},M_{p_{ij}^k}(d_{ij}^k)\rangle \mid p_{ij}^k=1,2,\cdots,t_{ij}^k\}$ being a DHLE, where $s_{\sigma(l)(d_{ij}^k)} \oplus s_{\sigma(t_{ji}^k-l+1)(d_{ji}^k)}=s_0$, $M_{\sigma(l)}(d_{ij}^k)=M_{\sigma(t_{ji}^k-l+1)}(d_{ji}^k)$, $d_{ii}^k=\{\langle s_0,(1,0)\rangle\}$, $t_{ij}^k=t_{ji}^k$ for all $i,j=1,2,\cdots,n$. Here $s_{\sigma(l)(d_{ij}^k)}$ is the l-th smallest linguistic term of $s_{\theta_1(d_{ij}^k)},s_{\theta_2(d_{ij}^k)},\cdots,s_{\theta_{t_{ij}}(d_{ij}^k)}$, and $M_{\sigma(l)}(d_{ij}^k)$ is the set of possible membership and non-membership degrees to $s_{\sigma(l)(d_{ij}^k)}$.

To solve the above group decision making problem with DHLPRs, the following steps are given.

Step 1 For the decision organization $O_k(k=1,2,\cdots,m)$ and the alternative $G_i(i=1,2,\cdots,n)$, we aggregate all $d_{ij}^k(j=1,2,\cdots,n,j\neq i)$ to obtain the average double hesitant linguistic preference of the alternative G_i over the others by using the DHLA operator:

$$d_i^k = \frac{1}{n-1} \bigoplus_{j=1, j\neq i}^{n} d_{ij}^k \qquad (6)$$

Step 2 Aggregate all $d_i^k, k=1,2,\cdots,m$ by the DHLWA operator to obtain the collective double hesitant linguistic preference of the alternative G_i over the others:

$$d_i = \bigoplus_{k=1}^{m} v_k d_i^k \qquad i=1,2,\cdots,n \qquad (7)$$

where $\boldsymbol{v} = \{v_1, v_2, \cdots, v_m\}^{\mathrm{T}}$ is the importance weight vector of decision organizations $O = \{O_1, O_2, \cdots, O_m\}$ with $v_k \geqslant 0$ and $\sum_{k=1}^{m} v_k = 1$.

Step 3 Compare d_i, $i=1,2,\cdots,n$ by Definition 8, and obtain the priority of alternatives G_i, $i=1,2,\cdots,n$ by ranking d_i, $i=1,2,\cdots,n$.

In Step 2, if the importance weights of decision organizations are unknown, the following method is used to determine the importance weight of each decision organization.

Assume that for the decision organization $O_k(k=1,2,\cdots,m)$, the average double hesitant linguistic preference of the alternative $G_i(i=1,2,\cdots,n)$ over the others derived by Eq. (8) is $d_i^k = \{\langle s_{\theta_{p_i}(d_i^k)}, M_{p_i}(d_i^k)\rangle \mid p_i^k = 1,2,\cdots,t_i^k\}$, where $M_{p_i^k}(d_i^k) = \{([h_{r-}^{p_i^k}(d_i^k), h_{r+}^{p_i^k}(d_i^k)], [g_{r-}^{p_i^k}(d_i^k), g_{r+}^{p_i^k}(d_i^k)]) \mid 0 \leqslant h_{r-}^{p_i^k}(d_i^k) \leqslant h_{r+}^{p_i^k}(d_i^k) \leqslant 1, 0 \leqslant g_{r-}^{p_i^k}(d_i^k) \leqslant g_{r+}^{p_i^k}(d_i^k) \leqslant 1, 0 \leqslant h_{r+}^{p_i^k}(d_i^k) + g_{r+}^{p_i^k}(d_i^k) \leqslant 1, r = 1,2,\cdots,l_{p_i^k}^{d_i^k}\}$. Then by Eq. (1), we compute the score F_i^k of d_i^k:

$$F_i^k = \sum_{p_i^k=1}^{t_i^k} \frac{1}{2t_i^k}\left(\frac{\bar{h}_{p_i^k}^-(d_i^k) + \bar{h}_{p_i^k}^+(d_i^k)}{2} + 1 - \frac{\bar{g}_{p_i^k}^-(d_i^k) + \bar{g}_{p_i^k}^+(d_i^k)}{2}\right) Q(s_{\theta_{p_i}(d_i^k)}) \qquad (8)$$

where $[\bar{h}_{p_i^k}^-(d_i^k), \bar{h}_{p_i^k}^+(d_i^k)] = \left[\sum_{r=1}^{l_{p_i^k}^{d_i^k}} h_{r-}^{p_i^k}(d_i^k)/l_{p_i^k}^{d_i^k}, \sum_{r=1}^{l_{p_i^k}^{d_i^k}} h_{r+}^{p_i^k}(d_i^k)/l_{p_i^k}^{d_i^k}\right]$ and $[\bar{g}_{p_i^k}^-(d_i^k), \bar{g}_{p_i^k}^+(d_i^k)] = \left[\sum_{r=1}^{l_{p_i^k}^{d_i^k}} g_{r-}^{p_i^k}(d_i^k)/l_{p_i^k}^{d_i^k}, \sum_{r=1}^{l_{p_i^k}^{d_i^k}} g_{r+}^{p_i^k}(d_i^k)/l_{p_i^k}^{d_i^k}\right]$. Therefore, we can obtain the preference score vector, $\boldsymbol{F}^k = \{F_1^k, F_2^k, \cdots, F_n^k\}^{\mathrm{T}}$, of the decision organization O_k, and the preference score vector, $\boldsymbol{F}^l = \{F_1^l, F_2^l, \cdots, F_n^l\}^{\mathrm{T}}$, of the decision organization O_l.

Moreover, we calculate the correlation coefficient z_{kl} between $\boldsymbol{F}^k$ and $\boldsymbol{F}^l$:

$$z_{kl} = \frac{\sum_{i=1}^{n}(F_i^k - \overline{F^k})(F_i^l - \overline{F^l})}{\sqrt{\sum_{i=1}^{n}(F_i^k - \overline{F^k})^2 \sum_{i=1}^{n}(F_i^l - \overline{F^l})^2}} \qquad k,l = 1,2,\cdots,m \qquad (9)$$

where for $k,l = 1,2,\cdots,m$, $\overline{F}^k = \sum_{i=1}^{n} F_i^k/n$ and $\overline{F}^l = \sum_{i=1}^{n} F_i^l/n$, respectively. Clearly, $-1 \leqslant z_{kl} \leqslant 1$, and the larger z_{kl}, the closer the numerical distribution of $\boldsymbol{F}^k$ is to that of $\boldsymbol{F}^l$; in contrast, the smaller z_{kl}, the more different the numerical distribution of $\boldsymbol{F}^k$ is from that of $\boldsymbol{F}^l$. During the decision making process, it is expected that the difference between the preference information provided by one decision organization and those provided by the others is as small as possible. Thus, we may assign the weight to the decision organization $O_k(k=1,2,\cdots,m)$ by the following rules: the larger $\sum_{l=1}^{m}\sqrt{z_{kl}+1}$, the larger the weight allocated to the decision organization O_k; otherwise, the smaller weight is assigned.

Additionally, it is required to compute the standard deviation σ_k of the preference score vector $\boldsymbol{F}^k$ of the decision organization O_k:

$$\sigma_k = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(F_i^k - \overline{F}^k)^2} \qquad k = 1,2,\cdots,m \qquad (10)$$

where $\overline{F}^k = \sum_{i=1}^{n} F_i^k/n$ for $k = 1,2,\cdots,m$. Clearly, the larger σ_k, the greater difference among the preference information given to alternatives by the decision organization O_k, which indicates the greater the influence of the decision organization O_k on decision making. In this case, a larger weight should be assigned to the decision organization O_k. Alternately, a smaller weight could be assigned.

According to the above analysis, the importance weight v_k of the decision organization O_k can be computed by the following formula:

$$v_k = \frac{a_k}{\sum_{j=1}^{m} a_j} \qquad k = 1,2,\cdots,m \qquad (11)$$

where $a_k = \sigma_k \sum_{l=1}^{m}\sqrt{z_{kl}+1}$ for $k = 1,2,\cdots,m$.

4 Case Studies

We below use the example of the Jiudianxia Reservoir operation alternative selection[16-17] to illustrate the application and implementation process of the proposed method, and then make a comparative analysis to show its advantages.

4.1 Decision background

The Jiudianxia Reservoir is designed for many purposes, such as power generation, irrigation, total water supply for industry, agriculture, residents and the environment. Now four reservoir operation alternatives G_1, G_2, G_3 and G_4 are proposed because there are different requirements for the partition of the amount of water.

G_1: The maximum plant output, sufficient supply of water used in the Tao River basin, larger and lower supply for society and economy.

G_2: The maximum plant output, sufficient supply of water used in the Tao River basin, larger and lower supply for society and economy, lower supply for the ecosystem.

G_3: The maximum plant output, sufficient supply of water used in the Tao River basin, larger and lower supply for society and economy, total supply for the ecosystem and environment, of which 90% is used for flushing out sands during a low water period.

G_4: The maximum plant output, sufficient supply of water used in the Tao River basin, larger and lower supply for society and economy, total supply for the ecosystem and environment, of which 50% is used for flushing out sands during a low water period.

In order to select the optimal alternative, the government invites three organizations O_1, O_2, O_3 from different fields to assess the four alternatives. Suppose that each expert in each organization independently gives his/her preferences for each pair of alternatives in the form of a linguistic term in the linguistic term set $S=\{s_{-3}$: extremely poor, s_{-2}: very poor, s_{-1}: poor, s_0: fair, s_1: good, s_2: very good, s_3: extremely good$\}$ and the membership and non-membership degrees to the linguistic term. For instance, when three experts in the organization O_3 compare the alternative G_1 with the alternative G_2, they give the following preferences: $\langle s_0,([0.8,0.9],[0,0.1])\rangle$, $\langle s_2,(0.2,0.6)\rangle$ and $\langle s_1,(0.7,0.3)\rangle$. Since the experts in the organization O_3 are independent, we may think of the preferences given by the organization O_3 as the DHLE $\{\langle s_0,([0.8,0.9],[0,0.1])\rangle,\langle s_1,(0.7,0.3)\rangle,\langle s_2,(0.2,0.6)\rangle\}$. The preferences provided by the three organizations for the four alternatives are presented in matrices $\boldsymbol{R}_1$ to $\boldsymbol{R}_3$, all of which are DHLPRs. For convenience, we denote the matrix $\boldsymbol{R}_k$ $(k=1,2,3)$ by $\boldsymbol{R}_k=(d_{ij}^k)_{4\times4}$. Due to limited space, we merely list the DHLEs in the upper triangular part of each matrix and the rest can be correspondingly obtained according to Definition 11.

$\boldsymbol{R}_1$: $d_{12}^1=\{\langle s_1,([0.8,0.9],[0.05,0.1])\rangle,\langle s_2,(0.2,0.5)\rangle\}$, $d_{13}^1=\{\langle s_2,(0.7,0.2)\rangle,\langle s_3,([0.7,0.8],[0.1,0.2]),(0.9,0.1)\rangle\}$, $d_{14}^1=\{\langle s_{-1},(0.5,0.5)\rangle,\langle s_0,(0.7,0.2),(0.6,0)\rangle\}$, $d_{23}^1=\{\langle s_0,([0.6,0.8],[0.1,0.2]),(1,0)\rangle\}$, $d_{24}^1=\{\langle s_1,(0.9,0.1)\rangle,\langle s_2,(0.7,0.2)\rangle\}$, $d_{34}^1=\{\langle s_{-2},(0.6,0.2),([0.8,0.9],[0,0.1])\rangle,\langle s_{-1},(0.7,0.1)\rangle\}$.

$\boldsymbol{R}_2$: $d_{12}^2=\{\langle s_{-3},([0.1,0.3],[0.6,0.7])\rangle,\langle s_{-2},(0.8,0.1)\rangle\}$, $d_{13}^2=\{\langle s_1,([0.7,0.8],[0.1,0.2])\rangle,\langle s_2,(0.6,0.2)\rangle,\langle s_3,(0.9,0.1)\rangle\}$, $d_{14}^2=\{\langle s_{-2},(1,0)\rangle,\langle s_{-1},(0.9,0.1),(0.6,0.3)\rangle\}$, $d_{23}^2=\{\langle s_0,(0.9,0.1)\rangle,\langle s_2,([0.6,0.7],[0.1,0.2])\rangle\}$, $d_{24}^2=\{\langle s_0,(0.8,0.1)\rangle,\langle s_1,(0.6,0.2)\rangle,\langle s_2,([0.1,0.5],[0.3,0.4])\rangle\}$, $d_{34}^2=\{\langle s_1,([0.7,0.9],[0,0.1])\rangle,\langle s_2,(0.9,0.1)\rangle\}$.

$\boldsymbol{R}_3$: $d_{12}^3=\{\langle s_0,([0.8,0.9],[0,0.1])\rangle,\langle s_1,(0.7,0.3)\rangle,\langle s_2,(0.2,0.6)\rangle\}$, $d_{13}^3=\{\langle s_1,([0.7,0.8],[0.1,0.2]),(0.9,0.1)\rangle\}$, $d_{14}^3=\{\langle s_0,(0.9,0.1)\rangle,\langle s_1,([0.1,0.3],[0.5,0.7])\rangle\}$, $d_{23}^3=\{\langle s_0,([0.7,1],[0,0]),(0.8,0.1),(0.7,0.3)\rangle\}$, $d_{24}^3=\{\langle s_1,([0.7,0.9],[0,0.1])\rangle,\langle s_2,([0.3,0.6],[0.1,0.2]),(0.7,0.1)\rangle\}$, $d_{34}^3=\{\langle s_{-1},([0.6,0.8],[0.1,0.2])\rangle,\langle s_1,(0.9,0.1)\rangle,\langle s_3,(0.2,0.4)\rangle\}$.

4.2 Decision model

In the following, we apply the proposed method to solve the above group decision making problem with DHLPRs. The solution process and computation results are summarized as follows:

Step 1 For the organizations O_k $(k=1,2,3)$ and the alternative G_i $(i=1,2,3,4)$, we aggregate all d_{ij}^k $(j=1,2,3,4,\ j\neq i)$ to obtain the average double hesitant linguistic preference of the alternative G_i over the others by using the DHLA operator:

$$d_i^k=\frac{1}{3}\bigoplus_{j=1,j\neq i}^{4}d_{ij}^k \qquad i=1,2,3,4;\ k=1,2,3$$

For example, for the organization O_1 and the alternative, we obtain

$$d_3^1=\frac{1}{3}(d_{31}^1\oplus d_{32}^1\oplus d_{34}^1)=\{\langle s_{-1.6667},([0.6366,0.748],[0.126,0.2]),([0.7116,0.8413],[0,0.1587]),([0.748,0.8],[0.126,0.1587]),([0.8,0.874],[0,0.126]),(1,0)\rangle,\langle s_{-1.3333},([0.6366,0.7116],[0.1587,0.2]),([0.6698,0.7711],[0.1,0.1587]),([0.7116,0.8183],[0,0.1587]),([0.7711,0.8183],[0.1,0.126]),(1,0)\rangle,\langle s_{-1},([0.6698,0.7379],[0.126,0.1587]),(1,0)\rangle\}$$

Similarly, other overall results can be obtained. Due to limited space, we here do not list them one by one.

Step 2 According to Eq. (8), we compute the score F_i^k of d_i^k $(i=1,2,3,4)$, shown as follows:

O_1: $F_1^1=0.8967$, $F_2^1=0.0032$, $F_3^1=-1.2063$, $F_4^1=0.1363$.

O_2: $F_1^2=-0.6008$, $F_2^2=1.0448$, $F_3^2=-0.4287$, $F_4^2=-0.2632$.

O_3: $F_1^3=0.5939$, $F_2^3=0.1463$, $F_3^3=-0.0124$, $F_4^3=-0.7032$.

Then, we obtain the preference score vector $\boldsymbol{F}^k$ of the organization $O_k(k=1,2,3)$ as

$$\boldsymbol{F}^1=\{0.8967, 0.0032, -1.2063, 0.1363\}^{\mathrm{T}}$$
$$\boldsymbol{F}^2=\{-0.6008, 1.0448, -0.4287, -0.2632\}^{\mathrm{T}}$$
$$\boldsymbol{F}^3=\{0.5939, 0.1463, -0.0124, -0.7032\}^{\mathrm{T}}$$

Moreover, by Eqs. (9) and (10), we obtain the following correlation coefficient matrix $\boldsymbol{Z}=(z_{kl})_{3\times3}$ and the standard deviation of the preference score vector $\boldsymbol{F}^k$ of the organization $O_k(k=1,2,3)$, respectively:

$$\boldsymbol{Z}=\begin{bmatrix} 1 & -0.0331 & 0.3227 \\ -0.0331 & 1 & -0.0099 \\ 0.3227 & -0.0099 & 1 \end{bmatrix}$$
$$\sigma_1=0.7534,\ \sigma_2=0.6501,\ \sigma_3=0.466$$

By Eq. (11), we obtain the importance weight v_k of the organization $O_k(k=1,2,3)$ as

$$v_1=0.4089,\ v_2=0.3374,\ v_3=0.2537$$

Step 3 For the alternatives $G_i(i=1,2,3,4)$, we aggregate all $d_i^k(k=1,2,3)$ by the DHLWA operator to obtain the collective double hesitant linguistic preference of the alternative G_i over the others:

$$d_i=\bigoplus_{k=1}^{4} v_k d_i^k \qquad i=1,2,3,4$$

Due to limited space, here we merely show how to obtain one element in d_1. Since $\langle s_{0.6667}, ([0.6893, 0.7534], [0.171, 0.2154])\rangle \in d_1^1$, $\langle s_0, (0.8, 0.1442), (0.874, 0.1)\rangle \in d_1^2$ and $\langle s_{0.3333}, ([0.8183, 0.874], [0, 0.126]), ([0.874, 0.9], [0, 0.1])\rangle \in d_1^3$, then by Eq. (14), we obtain $\langle s_{0.3572}, ([0.7663, 0.8062], [0, 0.1642]), ([0.787, 0.8173], [0, 0.1548]), ([0.8, 0.8342], [0, 0.1451]), ([0.8178, 0.8436], [0, 0.1369])\rangle \in d_1$.

Step 4 Compute the score F_i of $d_i(i=1,2,3,4)$ by Eq. (1) as follows:

$$F_1=0.4006,\ F_2=0.5018$$
$$F_3=-0.6691,\ F_4=-0.2579$$

by which we obtain the rankings $d_2>d_1>d_4>d_3$. Thus, the priority of alternatives is $G_2>G_1>G_4>G_3$, and the alternative G_2 is the best among the four alternatives.

4.3 Comparative analysis

In this subsection, we conduct a specific comparison of our method with the method based on interval-valued intuitionistic trapezoidal fuzzy numbers (IITFNs). To hold the same known information in the comparison process, we here assign the same importance weights to organizations as those obtained in Subsection 4.2, that is, the importance weight vector of organizations is $\boldsymbol{v}=\{0.4089, 0.3374, 0.2537\}^{\mathrm{T}}$. The detailed comparison process is shown as follows.

Step 1 Transform the DHLEs in matrices $\boldsymbol{R}_1$ to $\boldsymbol{R}_3$ into the corresponding IITFNs. Here we take the DHLE $d_{13}^1=\{\langle s_2, (0.7, 0.2)\rangle, \langle s_3, ([0.7, 0.8], [0.1, 0.2]), (0.9, 0.1)\rangle\}$ for an example. First, according to Ref. [18], we represent the linguistic term s_α in the linguistic term set $S=\{s_\alpha \mid \alpha=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ by a trapezoidal fuzzy number $\tilde{d}_\alpha$ using the following formula:

$$\tilde{d}_\alpha=[d_\alpha^1, d_\alpha^2, d_\alpha^3, d_\alpha^4]=\left[\max\left\{\frac{2(\alpha+\tau)-1}{4\tau+1}, 0\right\},\right.$$
$$\left.\frac{2(\alpha+\tau)}{4\tau+1}, \frac{2(\alpha+\tau)+1}{4\tau+1}, \min\left\{\frac{2(\alpha+\tau)+2}{4\tau+1}, 1\right\}\right]$$

So the trapezoidal fuzzy number corresponding to s_2 is $[0.6923, 0.7692, 0.8462, 0.9231]$, and the one corresponding to s_3 is $[0.8462, 0.9231, 1, 1]$. Then the DHLE d_{13}^1 is translated into $\{\langle[0.6923, 0.7692, 0.8462, 0.9231], (0.7, 0.2)\rangle, \langle[0.8462, 0.9231, 1, 1], ([0.7, 0.8], [0.1, 0.2]), (0.9, 0.1)\rangle\}$. Furthermore, we average the interval-valued intuitionistic fuzzy numbers in $\langle[0.8462, 0.9231, 1, 1], ([0.7, 0.8], [0.1, 0.2]), (0.9, 0.1)\rangle$ by the interval-valued intuitionistic fuzzy arithmetic averaging operator[19]:

$$\frac{1}{2}([0.7, 0.8], [0.1, 0.2]) \oplus \frac{1}{2}(0.9, 0.1)=$$
$$([0.8268, 0.8586], [0.1, 0.1414])$$

Then d_{13}^1 can be approximately written as $\{\langle[0.6923, 0.7692, 0.8462, 0.9231], (0.7, 0.2)\rangle, \langle[0.8462, 0.9231, 1, 1], ([0.8268, 0.8586], [0.1, 0.1414])\rangle\}$. Finally, we average the IITFN $\langle[0.6923, 0.7692, 0.8462, 0.9231], (0.7, 0.2)\rangle$ and the IITFN $\langle[0.8462, 0.9231, 1, 1], ([0.8268, 0.8586], [0.1, 0.1414])\rangle$ by using the interval-valued intuitionistic trapezoidal fuzzy arithmetic averaging operator[20]:

$$\frac{1}{2}\langle[0.6923, 0.7692, 0.8462, 0.9231], (0.7, 0.2)\rangle+$$
$$\frac{1}{2}\langle[0.8462, 0.9231, 1, 1], ([0.8268, 0.8586],$$
$$[0.1, 0.1414])\rangle=\langle[0.7693, 0.8462, 0.9231, 0.9616], ([0.7721, 0.7940], [0.1414, 0.1682])\rangle$$

Therefore, we transform the DHLE $d_{13}^1=\{\langle s_2, (0.7, 0.2)\rangle, \langle s_3, ([0.7, 0.8], [0.1, 0.2]), (0.9, 0.1)\rangle\}$ into the IITFN $\langle[0.7693, 0.8462, 0.9231, 0.9616], ([0.7721, 0.794], [0.1414, 0.1682])\rangle$. In a similar way, we transform the rest of DHLEs in matrices $\boldsymbol{R}_1$ to $\boldsymbol{R}_3$ into the corresponding IITFNs. Due to limited space, we here omit their transformed results.

Step 2 For convenience, we denote $\boldsymbol{I}^k=(I_{ij}^k)_{4\times4}(k=1,2,3)$, and for the organizations $O_k(k=1,2,3)$, aggregate $I_{ij}^k(j=1,2,3,4, j\neq i)$ to obtain the averaged I_i^k of the alternatives $G_i(i=1,2,3,4)$ over the others by the interval-valued intuitionistic trapezoidal fuzzy arithmetic averaging operator[20]:

$$I_i^k = \sum_{j=1, j\neq i}^{4} \frac{1}{3} I_{ij}^k \qquad i = 1,2,3,4;\ k = 1,2,3$$

The overall aggregation results are listed as follows:

O_1: $I_1^1 = \{\langle [0.5641, 0.641, 0.718, 0.7821], ([0.664, 0.7106], [0,0])\rangle\}$, $I_2^1 = \{\langle [0.3846, 0.4615, 0.5385, 0.6154], (1,0)\rangle\}$, $I_3^1 = \{\langle [0.1923, 0.2564, 0.3333, 0.4103], (1,0)\rangle\}$, $I_4^1 = \{\langle [0.4103, 0.4872, 0.5641, 0.641], ([0.7241, 0.7396], [0,0])\rangle\}$.

O_2: $I_1^2 = \{\langle [0.2949, 0.359, 0.4359, 0.5043], (1,0)\rangle\}$, $I_2^2 = \{\langle [0.6154, 0.6932, 0.7693, 0.8333], ([0.672, 0.7191], [0.1645, 0.1956])\rangle\}$, $I_3^2 = \{\langle [0.3162, 0.3846, 0.4616, 0.5385], ([0.8006, 0.8487], [0, 0.1309])\rangle\}$, $I_4^2 = \{\langle [0.3333, 0.4102, 0.4872, 0.5641], (1,0)\rangle\}$.

O_3: $I_1^3 = \{\langle [0.5128, 0.5897, 0.6667, 0.7436], ([0.7337, 0.779], [0, 0.214])\rangle\}$, $I_2^3 = \{\langle [0.4103, 0.4872, 0.5641, 0.641], ([0.6719, 1], [0,0])\rangle\}$, $I_3^3 = \{\langle [0.3846, 0.4615, 0.5385, 0.6068], ([0.7566, 1], [0,0])\rangle\}$, $I_4^3 = \{\langle [0.2393, 0.3077, 0.3846, 0.4615], ([0.6719, 0.7685], [0, 0.1846])\rangle\}$.

Step 3 For the alternatives $G_i\ (i = 1,2,3,4)$, by using the interval-valued intuitionistic trapezoidal fuzzy weighted arithmetic averaging operator[20] and the weight vector $\boldsymbol{v} = \{0.4089, 0.3374, 0.2537\}^{\mathrm{T}}$, we aggregate all $I_i^k\ (k=1,2,3)$ to obtain a collective I_i of the alternative G_i over the others:

$$I_i = \mathrm{IITFWAA}(I_i^1, I_i^2, I_i^3) = \sum_{k=1}^{3} v_k I_i^k \qquad i = 1,2,3,4$$

The followings are the computation results:

$$I_1 = \{\langle [0.4603, 0.5328, 0.6098, 0.6786], (1,0)\rangle\}$$
$$I_2 = \{\langle [0.469, 0.5459, 0.6229, 0.6954], (1,0)\rangle\}$$
$$I_3 = \{\langle [0.2829, 0.3517, 0.4286, 0.5034], (1,0)\rangle\}$$
$$I_4 = \{\langle [0.3409, 0.4157, 0.4926, 0.5695], (1,0)\rangle\}$$

Step 4 By ranking these IITFNs according to the method in Ref. [20], we obtain the rankings of these alternatives: $G_2 \succ G_1 \succ G_4 \succ G_3$.

It can be clearly seen that the rankings of the alternatives obtained by the two methods are the same, which demonstrates that our method is reasonable and effective. Furthermore, from the solution processes of the two methods, we can see that our method has some advantages, as shown below:

1) Our method can reduce the loss of information since it does not need a transformation of DHLEs into IITFNs and does not need to use the average interval-valued intuitionistic trapezoidal fuzzy information obtained by incorporating the preferences of the decision makers in one decision organization to represent the group's preference; but uses the DHLEs to represent the group's preferences directly, which is more intuitive and reasonable.

2) Our method provides a technique to objectively determine the weights of decision organizations, which is more rational.

5 Conclusion

A method is proposed to solve the group decision making problems in which the weights of decision organizations are unknown and the preferences for the alternatives are provided by double hesitant linguistic preference relations. In this method, the weights of decision organizations are determined objectively by using the standard deviation of scores of preferences provided by the individual decision organization and the correlation coefficient between the scores of preferences and those provided by the other decision organizations; then the preferences of decision organizations are aggregated by the double hesitant linguistic weighted averaging operator. Compared with the existing methods, the proposed method is much simpler, has less information loss and can deal with the group decision making problems with double hesitant linguistic preference relations in a more objective way.

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基于双重犹豫语言偏好关系的群决策方法

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摘要: 针对决策小组权重未知、方案的偏好信息以双重犹豫语言偏好关系形式给出的群决策问题,提出了一种简单决策方法. 首先,为准确、全面地描述群决策过程中的不确定评估信息,定义双重犹豫语言数,并根据定义的运算法则,提出双重犹豫语言加权平均算子. 其次,定义双重犹豫语言偏好关系,并利用单个决策小组对方案偏好信息得分值的标准差和其偏好信息得分值与其他决策小组偏好信息得分值的相关系数,提出一种客观确定决策小组权重的方法,进而提出一种基于双重犹豫语言偏好关系的群决策方法. 同时,通过九甸峡水库运行方案选择实例说明该方法的可行性和有效性. 最后,将该方法与现有方法进行比较,结果表明,所提出的方法能够直接处理双重犹豫语言偏好信息,不需要进行信息转化,从而可以减少原始决策信息的丢失.

关键词: 群决策;双重犹豫语言数;双重犹豫语言偏好关系;双重犹豫语言加权平均算子

中图分类号: C934