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Optimal decision of retailer for replenishment cycle under a deteriorating product supply chain

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Abstract: In order to minimize the total cost of the retailer, an optimal replenishment cycle is studied by considering the deteriorating product, two-level trade credits, the limited storage capacity of their own warehouse and credit-linked order quantity simultaneously. A two-echelon supply chain model, which consists of a supplier and a retailer, is established. Then, the retailer's optimal replenishment cycle under all the cases are derived by using the optimization theory and method. On the basis of these, the effects of system parameters on the optimal replenishment cycle are examined by using the numerical studies. The results show that, when the retailer's trade credit period is longer (shorter) than the customer's trade credit period, the optimal replenishment cycle should be increased (decreased) as the retailer's trade credit period increases; if the minimum order quantity is high (low), the optimal replenishment cycle should be increased (not changed) as the minimum order quantity increases.

Key words: replenishment cycle; deteriorating product; trade credit; limited storage capacity

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The retailer's replenishment cycle influences the ordering cost, purchasing cost, and holding cost, etc. How to create an optimal replenishment cycle to minimize total cost is important for retailers. In real life, many products will deteriorate during storage, such as fruit, vegetables, and chemicals, etc. Therefore, in recent decades, this issue of deteriorating product has been widely studied.

However, in practice, the retailer faces a more complex decision-making environment. Many factors influence the replenishment cycle. For example, to increase sales, the two-level trade credits are popularly used in the supply chain system. Under this policy, the supplier (retailer) will offer the retailer (customer) a trade credit period. During the trade credit period, the retailer (customer) does not pay for the cost of the products to the supplier (retailer). This policy induces the retailer to further consider the interest earned and interest charged when the retailer makes the decision regarding a replenishment cycle. Meanwhile, many suppliers set a minimum order quantity. Only if the retailer's order quantity is more than the minimum order quantity, will the suppliers provide a trade credit policy for the retailer. The minimum order quantity also influences the retailer's replenishment cycle decision. In addition, the retailer's storage capacity of their own warehouse (OW) is limited in reality which also influences the replenishment cycle decision.

However, to the best of our knowledge, almost all existing research only considers two or three factors in the above-mentioned cases. Therefore, to better fit the actual situation, instead of considering a two or three factors, this paper will consider these four factors simultaneously.

1 Literature Review

In recent decades, inventory problems for deteriorating products have been widely studied. In 1957, Whitin[1] first studied the inventory problem of deteriorating product. Then Ghare and Schrader[2] proposed that the consumption of the deteriorating product was closely related to a negative exponential function of time. According to this, they established the deteriorating product inventory model below:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t}+\theta I(t)=-f(t) \tag{1}$$

where θ represents the deteriorating rate of the product; $I(t)$ represents the inventory level at time t; and $f(t)$ is the demand rate at time t.

In 2006, the inventory problem of non-instantaneous deteriorating product was first studied by Wu et al[3]. Subsequently, Sarkar et al.[4-6] started to study related issues.

Nowadays, trade credit is more and more popular as a crucial proportion of company finance in a supply chain system. Ouyang et al.[7] developed an appropriate inventory model for non-instantaneous deteriorating product with permissible delay payments. Soni and Shah[8] established an inventory model with a stock-dependent demand when the supplier offers two progressive credit periods to the retailer.

Similarly, in turn, the retailer often offers a trade credit period to the customer. Huang[9] first considered this

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condition. Jaggi et al. [10] developed a method to determine the optimal replenishment policy for the retailer under considering a two-level trade credit. Mahata[11] investigated the optimal retailer's replenishment decisions for deteriorating product under two levels of trade credit policy.

However, in practice, the supplier usually offers the retailer a period to delay payments only when the retailer's order quantity is greater than a minimum order quantity. Huang[12] proposed an EOQ model in which the supplier offers the retailer a fully permissible delay of M periods if the retailer orders more than a minimum order quantity. Chen et al. [13] extended Huang's model to complement some shortcomings of the model and proposed an arithmetic-geometric method to solve the inventory problem.

In all above articles, the authors are not concerned with the limited capacity of the retailer's own warehouse. Liang and Zhou[14] considered a two-warehouse inventory model for deteriorating product under constant demand when considering permissible delay in payment. One is an owned warehouse (OW) and the other is a rented warehouse (RW).

2 Model Description and Assumptions

2.1 Model description

In this paper, we propose a two-echelon supply chain model which consists of a supplier and a retailer. The retailer orders the product from the supplier, and needs to pay purchasing cost c per unit and ordering cost A per order. The retailer's unit holding cost in OW and RW are h_1 and h_2, respectively. The retailer faces a constant demand rate D per year, and sells the product to customers at selling price s per unit. Furthermore, in this model, the deteriorating product, two-level trade credit, limited storage capacity of a retailer's own warehouse and credit-linked order quantity are considered simultaneously. First, the product will deteriorate during the storage at deteriorating rate θ. Secondly, the supplier provides the retailer a permissible delay in payments, i. e., retailer's trade credit period M; and the retailer in turn provides customers a permissible delay in payments, i. e., customer's trade credit period N. Thirdly, the retailer's warehouse capacity W is limited. The retailer can rent the warehouse if the retailer needs. Finally, only if the retailer's order quantity is more than a minimum order quantity Q_d, the retailer can enjoy the trade credit period offered by the supplier.

2.2 Assumptions

1) Demand rate is known and constant.

2) Time horizon is infinite, and replenishments are instantaneous.

3) Shortages are not allowed.

4) If the order quantity is greater than the retailer's capacity, the retailer needs to rent an additional warehouse to hold the inventory.

5) The retailer accumulates the sales revenue from the last customer at time $T+N$, but he/she must pay the entire purchasing cost to the supplier at time M. Consequently, if $M \geqslant T+N$, the retailer can accumulate all revenue and pays the entire purchasing cost to the supplier at time M. There is no interest charge during the replenishment cycle time. If $M < T+N$, the retailer pays the purchasing cost of all the units sold to the supplier at time M. Then the retailer keeps the profit for the use of other activities and starts paying the interest charges on products in stock with rate I_c over an interval of $[M, T+N]$. Since the products have been deteriorating, some products are not sold but lost. These products also require the retailer to pay the interest charges. The interest charges of these products will be paid at time $T+N$.

6) If $M > N$, the retailer can accumulate revenue and earn interest during the time interval of $[N, M]$ with rate I_e. Conversely, if $M \leqslant N$, there is no interest earned for the retailer.

7) The stocks of RW are transported to OW in a continuous release pattern and the transportation cost is ignored.

8) For economic reasons, the products in RW are consumed first and next the products in OW.

3 Model Formulations

Let $I(t)$ be the inventory level at any time $t(0 \leqslant t \leqslant T)$. During the time interval, the inventory level decreases due to demand ad deterioration. Thus, the instantaneous state of the inventory level can be described by the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -D - \theta I(t) \tag{2}$$

We use Q to denote the retailer's order quantity. By solving the above differential equation with boundary condition $I(T)=0$, we can obtain the number of units that deteriorate during one cycle.

$$Q - DT = \frac{D(\mathrm{e}^{\theta T} - \theta T - 1)}{\theta} \tag{3}$$

When $I(0)=W$ and $I(T)=0$, we can obtain the time interval T_W so that the maximum inventory in the retailer's own warehouse is depleted to zero.

$$T_\mathrm{W} = \frac{1}{\theta}\ln\left(\frac{\theta W}{D} + 1\right) \tag{4}$$

Similarly, when $I(0)=Q_\mathrm{d}$ and $I(T)=0$, we can obtain the time interval T_d so that the quantity Q_d is depleted to zero.

$$T_\mathrm{d} = \frac{1}{\theta}\ln\left(\frac{\theta Q_\mathrm{d}}{D} + 1\right) \tag{5}$$

3.1 All kinds of relevant cost

1) Annual ordering cost:

$$\mathrm{OC} = \frac{A}{T} \tag{6}$$

2) Annual deteriorating cost:

$$\mathrm{DC} = \frac{cD(\mathrm{e}^{\theta T} - \theta T - 1)}{\theta T} \tag{7}$$

3) Annual stock holding cost needs to be divided into two scenarios . One is that the order quantity is less than the capacity in the retailer's own warehouse. In this case, the retailer does not need to rent a warehouse. The other is that the order quantity is more than the capacity in own warehouse. In this case, the retailer needs to rent some warehouses, as shown in Fig. 1. Therefore, the annual stock holding cost HC is

$$\mathrm{HC} = \begin{cases} \mathrm{HC}_1 & Q \leqslant W, \text{ i. e., } T \leqslant T_{\mathrm{W}} \\ \mathrm{HC}_2 & Q > W, \text{ i. e., } T > T_{\mathrm{W}} \end{cases} \tag{8}$$

where

$$\mathrm{HC}_1 = \frac{h_1}{T}\int_0^T I(t)\,\mathrm{d}t = \frac{h_1 D(\mathrm{e}^{\theta T} - \theta T - 1)}{\theta^2 T}$$

$$\mathrm{HC}_2 = \frac{h_1 W(T - T_{\mathrm{W}}) + h_1\int_{T-T_{\mathrm{w}}}^{T} I(t)\,\mathrm{d}t + h_2\int_0^{T-T_{\mathrm{w}}} [I(t) - W]\,\mathrm{d}t}{T}$$

4) To calculate the interest earned and interest charged, there are two possible cases which should be considered.

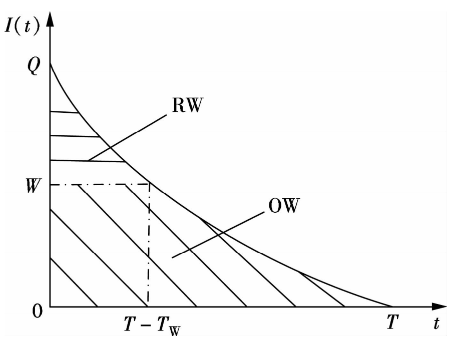


Fig. 1 The storage situation ($Q > W$)

Case 1 $T \geqslant T_{\mathrm{d}}$ (i. e., $Q \geqslant Q_{\mathrm{d}}$). Based on the values of M, N and $N + T$, three subcases are given.

Sub-case 1 $N < N + T \leqslant M$. In this subcase, the retailer does not only earn the interest in the time interval $[N, N + T]$, but also can earn the interest in the time interval $[N + T, M]$, as shown in Fig. 2. Since $M \geqslant N + T$, the retailer does not need to pay any interest. Therefore, the retailer's interest earned IE_{11} and interest charged IC_{11} are

$$\mathrm{IE}_{11} = \frac{I_{\mathrm{e}} sDT}{2} + I_{\mathrm{e}} sD(M - T - N) \tag{9}$$

$$\mathrm{IC}_{11} = 0 \tag{10}$$

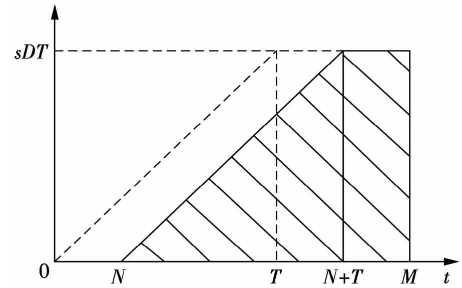


Fig. 2 Interest earned ($N < N + T \leqslant M$)

Subcase 2 $N < M \leqslant N + T$. In this subcase, the retailer earns the interest only in the time interval $[N, M]$, as shown in Fig. 3. Since $M < N + T$, the retailer needs to pay interest when the time is greater than M, as shown in Fig. 4. Therefore, the retailer's interest earned IE_{12} and interest charged IC_{12} are

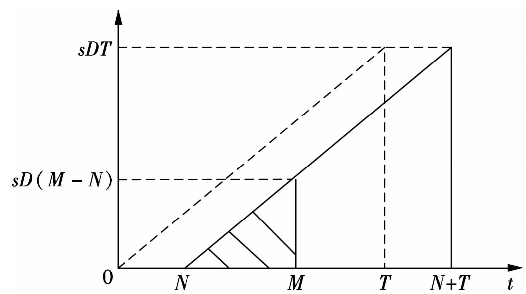
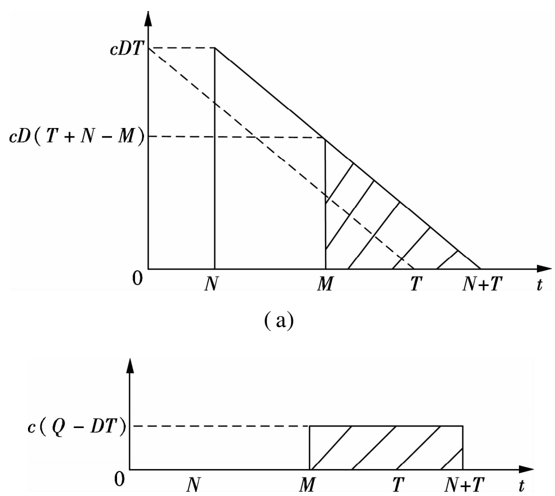


Fig. 3 Interest earned ($N < M \leqslant N + T$)



(a)

(b)

Fig. 4 Interest charged ($N < M \leqslant N + T$). (a) For unsold products; (b) For deteriorated products

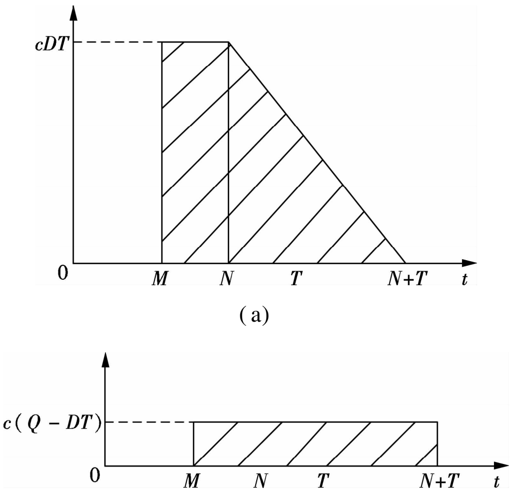
$$\mathrm{IE}_{12} = \frac{I_{\mathrm{e}} sD(M - N)^2}{2T} \tag{11}$$

$$\mathrm{IC}_{12} = \frac{I_{\mathrm{c}} cD(T + N - M)^2}{2T} + \frac{I_{\mathrm{c}} c(Q - DT)(T + N - M)}{T} \tag{12}$$

Subcase 3 $M \leqslant N < N + T$. In this subcase, since $M < N$, the retailer does not earn any interest. Meanwhile, the retailer needs to pay the interest when the time is greater than M, as shown in Fig. 5. Therefore, the retailer's interest earned IE_{13} and interest charged IC_{13} are

$$\mathrm{IE}_{13}=0 \tag{13}$$

$$\mathrm{IC}_{13}=\frac{I_c cD(T+2N-2M)}{2}+\frac{I_c c(Q-DT)(T+N-M)}{T} \tag{14}$$



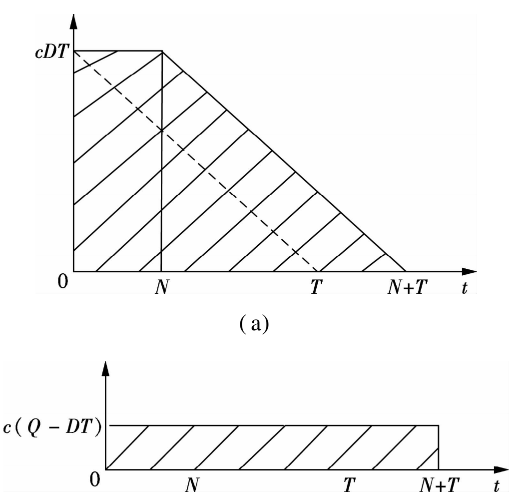
(b)

Fig. 5 Interest charged ($M \leqslant N < N+T$)). (a) For unsold products; (b) For deteriorated products

Case 2 $T < T_d$ (i. e., $Q < Q_d$). In this case, since $Q < Q_d$, the retailer does not obtain trade credit which is offered by the supplier. Therefore, the retailer does not earn any interest and needs to pay the interest in time interval $[0, N+T]$, as shown in Fig. 6. Therefore, the retailer's interest earned IE_2 and interest charged IC_2 are

$$\mathrm{IE}_2=0 \tag{15}$$

$$\mathrm{IC}_2=I_c cDN+\frac{I_c c\mathrm{DT}}{2}+\frac{I_c c(Q-\mathrm{DT})(T+N)}{T} \tag{16}$$



(b)

Fig. 6 Interest charged ($T < T_d$) (a) For unsold products; (b) For deteriorated products

3.2 Annual total relevant cost

From sub-section 3.1, we can obtain the total cost per year, and the details are as follows:

Case 1 $T \geqslant T_d$ (i. e., $Q \geqslant Q_d$).

In this case, the retailer can enjoy the trade credit period. Therefore, the total cost per year $\mathrm{TC}_1(T)$ is

$$\mathrm{TC}_1(T)=\begin{cases}\mathrm{TC}_{11}(T) & T \leqslant T_{\mathrm{W}}\\ \mathrm{TC}_{12}(T) & T > T_{\mathrm{W}}\end{cases} \tag{17}$$

where

$$\mathrm{TC}_{11}(T)=\begin{cases}\mathrm{TC}_{11\text{-}1}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_1+\\ \qquad \mathrm{IC}_{11}-\mathrm{IE}_{11} \qquad N<N+T\leqslant M\\ \mathrm{TC}_{11\text{-}2}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_1+\\ \qquad \mathrm{IC}_{12}-\mathrm{IE}_{12} \qquad N<M\leqslant N+T\\ \mathrm{TC}_{11\text{-}3}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_1+\\ \qquad \mathrm{IC}_{13}-\mathrm{IE}_{13} \qquad M\leqslant N<N+T\end{cases}$$

$$\mathrm{TC}_{12}(T)=\begin{cases}\mathrm{TC}_{12\text{-}1}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_2+\\ \qquad \mathrm{IC}_{11}-\mathrm{IE}_{11} \qquad N<N+T\leqslant M\\ \mathrm{TC}_{12\text{-}2}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_2+\\ \qquad \mathrm{IC}_{12}-\mathrm{IE}_{12} \qquad N<M\leqslant N+T\\ \mathrm{TC}_{12\text{-}3}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_2+\\ \qquad \mathrm{IC}_{13}-\mathrm{IE}_{13} \qquad M\leqslant N<N+T\end{cases}$$

Case 2 $T < T_d$ (i. e., $Q < Q_d$).

In this case, the retailer cannot enjoy the trade credit period. Therefore, the total cost per year $\mathrm{TC}_2(T)$ is

$$\mathrm{TC}_2(T)=\begin{cases}\mathrm{TC}_{21}(T) & T \leqslant T_{\mathrm{W}}\\ \mathrm{TC}_{22}(T) & T > T_{\mathrm{W}}\end{cases} \tag{18}$$

where

$$\mathrm{TC}_{21}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_1+\mathrm{IC}_2-\mathrm{IE}_2$$
$$\mathrm{TC}_{22}(T)=\mathrm{OC}+\mathrm{DC}+\mathrm{HC}_2+\mathrm{IC}_2-\mathrm{IE}_2$$

4 Replenishment Cycle Decision

In this section, we mainly study the optimal replenishment cycle to minimize the retailer's total cost of the inventory. To this end, we first need to analyze the model.

Lemma 1

1) If $x>0$, $x^2\mathrm{e}^x-2x\mathrm{e}^x+2\mathrm{e}^x-2>0$;

2) If $x>0$, $\mathrm{e}^{\theta x}-\theta x-1>0$.

Lemma 2

1) $\mathrm{TC}'_{11\text{-}1}(T)$, $\mathrm{TC}'_{11\text{-}3}(T)$, $\mathrm{TC}'_{12\text{-}1}(T)$, $\mathrm{TC}'_{12\text{-}3}(T)$, $\mathrm{TC}'_{21}(T)$ and $\mathrm{TC}'_{22}(T)$ increase on $[0, +\infty)$;

2) $\mathrm{TC}'_{11\text{-}2}(T)$ and $\mathrm{TC}'_{12\text{-}2}(T)$ increase on $[M-N, +\infty)$.

4.1 Optimal replenishment cycle under the condition of $T \geqslant T_d$

In this sub-section, the retailer can enjoy the trade credit period.

Case 1 $M > N$

In this case, since the trade credit period offered by the supplier is longer than that offered by the retailer, the retailer can earn interest.

Subcase 1 $T_d < T_W \leqslant M - N$. In this subcase, when $T_d \leqslant T < T_W$, the retailer does not need to rent the additional warehouse and pay the interest charges; when $T_W \leqslant T < M - N$, the retailer needs to rent the additional warehouse, but does not need to pay the interest charges. When $M - N \leqslant T$, the retailer needs to rent the additional warehouse and pay the interest charges. Therefore, in this subcase, the total cost function is

$$\mathrm{TC}(T) = \begin{cases} \mathrm{TC}_{11\text{-}1} & T_d \leqslant T < T_W \\ \mathrm{TC}_{12\text{-}1} & T_W \leqslant T < M - N \\ \mathrm{TC}_{12\text{-}2} & M - N \leqslant T \end{cases} \tag{19}$$

Clearly, $\mathrm{TC}_{11\text{-}1}(T_W) = \mathrm{TC}_{12\text{-}1}(T_W)$ and $\mathrm{TC}_{12\text{-}1}(M - N) = \mathrm{TC}_{12\text{-}2}(M - N)$, so $\mathrm{TC}(T)$ is continuous on $[T_d, +\infty)$. Meanwhile, it is easy to obtain $\mathrm{TC}'_{11\text{-}1}(T_W) = \mathrm{TC}'_{12\text{-}1}(T_W)$. Subsequently, according to Lemma 2, we can obtain $\mathrm{TC}'_{11\text{-}1}(T_d) < \mathrm{TC}'_{11\text{-}1}(T_W) < \mathrm{TC}'_{12\text{-}1}(M - N)$. According to 2) of Lemma 1, we also obtain $\mathrm{TC}'_{12\text{-}1}(M - N) < \mathrm{TC}'_{12\text{-}2}(M - N)$. Based on the above, we then can obtain $\mathrm{TC}'_{11\text{-}1}(T_d) < \mathrm{TC}'_{11\text{-}1}(T_W) < \mathrm{TC}'_{12\text{-}1}(M - N) < \mathrm{TC}'_{12\text{-}2}(M - N)$. According to Lemma 2, we can demonstrate that $\mathrm{TC}'(T)$ increases on $[T_d, +\infty)$ in this subcase.

Proposition 1 When $T \geqslant T_d$, $M > N$ and $T_d < T_W \leqslant M - N$, the optimal replenishment cycle T^* is as follows:

1) If $\mathrm{TC}'_{11\text{-}1}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $\mathrm{TC}'_{11\text{-}1}(T_d) < 0$ and $\mathrm{TC}'_{11\text{-}1}(T_W) \geqslant 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{11\text{-}1}(T) = 0$.

3) If $\mathrm{TC}'_{11\text{-}1}(T_W) < 0$ and $\mathrm{TC}'_{12\text{-}1}(M - N) \geqslant 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{12\text{-}1}(T) = 0$.

4) If $\mathrm{TC}'_{12\text{-}1}(M - N) < 0$ and $\mathrm{TC}'_{12\text{-}2}(M - N) \geqslant 0$, then $T^* = M - N$.

5) If $\mathrm{TC}'_{12\text{-}2}(M - N) < 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{12\text{-}2}(T) = 0$.

Subcase 2 $T_d < M - N \leqslant T_W$. In this subcase, when $T_d \leqslant T < M - N$, the retailer does not need to rent the additional warehouse and pay the interest charges; when $M - N \leqslant T < T_W$, the retailer needs to pay the interest charges, but does not need to rent the additional warehouse. When $T_W \leqslant T$, the retailer needs to rent the additional warehouse and pay the interest charges. Therefore, in this subcase, the total cost function is

$$\mathrm{TC}(T) = \begin{cases} \mathrm{TC}_{11\text{-}1} & T_d \leqslant T < M - N \\ \mathrm{TC}_{11\text{-}2} & M - N \leqslant T < T_W \\ \mathrm{TC}_{12\text{-}2} & T_W \leqslant T \end{cases} \tag{20}$$

Clearly, $\mathrm{TC}_{11\text{-}1}(M - N) = \mathrm{TC}_{11\text{-}2}(M - N)$ and $\mathrm{TC}_{11\text{-}2}(T_W) = \mathrm{TC}_{12\text{-}2}(T_W)$, so $\mathrm{TC}(T)$ is continuous on $[T_d, +\infty)$. Using the similar proof method of Proposition 1, we can obtain that $\mathrm{TC}'(T)$ increases on $[T_d, +\infty)$ in this subcase.

Proposition 2 When $T \geqslant T_d$, $M > N$ and $T_d < M - N \leqslant T_W$, the optimal replenishment cycle T^* is as follows:

1) If $\mathrm{TC}'_{11\text{-}1}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $\mathrm{TC}'_{11\text{-}1}(T_d) < 0$ and $\mathrm{TC}'_{11\text{-}1}(M - N) \geqslant 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{11\text{-}1}(T) = 0$.

3) If $\mathrm{TC}'_{11\text{-}1}(M - N) < 0$ and $\mathrm{TC}'_{11\text{-}2}(M - N) \geqslant 0$, then $T^* = M - N$.

4) If $\mathrm{TC}'_{11\text{-}2}(M - N) < 0$ and $\mathrm{TC}'_{12\text{-}2}(T_W) \geqslant 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{11\text{-}2}(T) = 0$.

5) If $\mathrm{TC}'_{12\text{-}2}(T_W) < 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{12\text{-}2}(T) = 0$.

Subcase 3 $T_W < T_d \leqslant M - N$. In this subcase, when $T_d \leqslant T < M - N$, the retailer needs to rent the additional warehouse, but does not need to pay the interest charges; when $M - N \leqslant T$, the retailer needs to rent the additional warehouse and pay the interest charges. Therefore, in this subcase, the total cost function is

$$\mathrm{TC}(T) = \begin{cases} \mathrm{TC}_{12\text{-}1} & T_d \leqslant T < M - N \\ \mathrm{TC}_{12\text{-}2} & M - N \leqslant T \end{cases} \tag{21}$$

Clearly, $\mathrm{TC}_{12\text{-}1}(M - N) = \mathrm{TC}_{12\text{-}2}(M - N)$, so $\mathrm{TC}(T)$ is continuous on $[T_d, +\infty)$. Using the similar proof method of Proposition 1, we can obtain that $\mathrm{TC}'(T)$ increases on $[T_d, +\infty)$ in this subcase.

Proposition 3 When $T \geqslant T_d$, $M > N$ and $T_W < T_d \leqslant M - N$, the optimal replenishment cycle T^* is as follows:

1) If $\mathrm{TC}'_{12\text{-}1}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $\mathrm{TC}'_{12\text{-}1}(T_d) < 0$ and $\mathrm{TC}'_{12\text{-}1}(M - N) \geqslant 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{12\text{-}1}(T) = 0$.

3) If $\mathrm{TC}'_{12\text{-}1}(M - N) < 0$ and $\mathrm{TC}'_{12\text{-}2}(M - N) \geqslant 0$, then $T^* = M - N$.

4) If $\mathrm{TC}'_{12\text{-}2}(M - N) < 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{12\text{-}2}(T) = 0$.

Subcase 4 $T_W < M - N \leqslant T_d$ or $M - N < T_W \leqslant T_d$. In this subcase, the retailer needs to rent the additional warehouse and pay the interest charges. Therefore, the total cost function is

$$\mathrm{TC}(T) = \mathrm{TC}_{12\text{-}2} \qquad T_d \leqslant T \tag{22}$$

According to Lemma 2, we can obtain following proposition.

Proposition 4 When $T \geqslant T_d$, $M > N$ and $T_W < M - N \leqslant T_d$ or $M - N < T_W \leqslant \mathrm{T}_d$, the optimal replenishment cycle T^* is as follows:

1) If $\mathrm{TC}'_{12\text{-}2}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $\mathrm{TC}'_{12\text{-}2}(T_d) < 0$, there exists a unique T^* which is given by $\mathrm{TC}'_{12\text{-}2}(T) = 0$.

Subcase 5 $M - N < T_d \leqslant T_W$. In this subcase, when $T_d \leqslant T < T_W$, the retailer needs to pay the interest charges, but does not need to rent the additional warehouse; when $T_W \leqslant T$, the retailer needs to rent the additional warehouse and pay the interest charges. Therefore, in this subcase, the total cost function is

$$\mathrm{TC}(T) = \begin{cases} \mathrm{TC}_{11\text{-}2} & T_d \leqslant T < T_W \\ \mathrm{TC}_{12\text{-}2} & T_W \leqslant T \end{cases} \tag{23}$$

Clearly, $\mathrm{TC}_{11\text{-}2}(T_W) = \mathrm{TC}_{12\text{-}2}(T_W)$, so $\mathrm{TC}(T)$ is continuous on $[T_d, +\infty)$. Using the similar proof method of Proposition 1, we can demonstrate that $TC'(T)$ increases on $[T_d, +\infty)$ in this subcase.

Proposition 5 When $T \geqslant T_d$, $M > N$ and $M - N < T_d \leqslant T_W$, the optimal replenishment cycle T^* is as follows:

1) If $TC'_{11\text{-}2}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $TC'_{11\text{-}2}(T_d) < 0$ and $TC'_{11\text{-}2}(T_W) \geqslant 0$, there exists a unique T^* which is given by $TC'_{11\text{-}2}(T_d) = 0$.

3) If $TC'_{11\text{-}2}(T_W) < 0$, there exists a unique T^* which is given by $TC'_{12\text{-}2}(T) = 0$.

Case 2 $M \leqslant N$

In this case, because the trade credit period offered by supplier is shorter than that offered by retailer, the retailer cannot earn interest, meanwhile, they also need to pay the interest charges.

Subcase 1 $T_d < T_W$. In this subcase, when $T_d \leqslant T < T_W$, the retailer does not need to rent the additional warehouse; when $T_W \leqslant T$, the retailer needs to rent the additional warehouse. Therefore, in this subcase, the total cost function is

$$TC(T) = \begin{cases} TC_{11\text{-}3} & T_d \leqslant T < T_W \\ TC_{12\text{-}3} & T_W \leqslant T \end{cases} \qquad (24)$$

Clearly, $TC_{11\text{-}3}(T_W) = TC_{12\text{-}3}(T_W)$, and hence $TC(T)$ is continuous on $[T_d, +\infty)$. Using the similar proof method of Proposition 1, we can demonstrate that $TC'(T)$ increases on $[T_d, +\infty)$ in this subcase. We then can obtain following proposition.

Proposition 6 When $T \geqslant T_d$, $M \leqslant N$ and $T_d < T_W$, the optimal replenishment cycle T^* is as follows:

1) If $TC'_{11\text{-}3}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $TC'_{11\text{-}3}(T_d) < 0$ and $TC'_{11\text{-}3}(T_W) \geqslant 0$, there exists a unique T^* which is given by $TC'_{11\text{-}3}(T_d) = 0$.

3) If $TC'_{11\text{-}3}(T_W) < 0$, there exists a unique T^* which is given by $TC'_{12\text{-}3}(T) = 0$.

Subcase 2 $T_d \geqslant T_W$. In this subcase, the retailer needs to rent the additional warehouse. Therefore, in this subcase, the total cost function is

$$TC(T) = TC_{12\text{-}3} \quad T_d \leqslant T \qquad (25)$$

According to Lemma 2, we can obtain following proposition.

Proposition 7 When $T \geqslant T_d$, $M \leqslant N$ and $T_d \geqslant T_W$, the optimal replenishment cycle T^* is as follows:

1) If $TC'_{12\text{-}3}(T_d) \geqslant 0$, then $T^* = T_d$.

2) If $TC'_{12\text{-}3}(T_d) < 0$, there exists a unique T^* which is given by $TC'_{12\text{-}3}(T) = 0$.

4.2 Optimal replenishment cycle under the condition of $T < T_d$

In this sub-section, the retailer cannot enjoy the trade credit period.

Case 1 $T_d < T_W$

In this case, the retailer does not need to rent the additional warehouse. Therefore, the total cost function is

$$TC(T) = TC_{21} \quad 0 < T < T_d \qquad (26)$$

According to Lemma 2, we can obtain the following proposition.

Proposition 8 When $T < T_d$ and $T_d < T_W$, the optimal replenishment cycle T^* is as follows:

1) If $TC'_{21}(T_d) \geqslant 0$, there exists a unique T^* which is given by $TC'_{21}(T) = 0$.

2) If $TC'_{21}(T_d) < 0$, then $T^* = T_d$.

Case 2 $T_W \leqslant T_d$

In this case, when $0 < T \leqslant T_W$, the retailer does not need to rent the additional warehouse. When $T_W < T < T_d$, the retailer needs to rent the additional warehouse. Therefore, the total cost function is

$$TC(T) = \begin{cases} TC_{21} & 0 < T \leqslant T_W \\ TC_{22} & T_W < T < T_d \end{cases} \qquad (27)$$

Clearly, $TC_{21}(T_W) = TC_{22}(T_W)$, and hence $TC(T)$ is continuous on $(0, T_d)$. Using the similar proof method of Proposition 1, we can demonstrate that $TC'(T)$ increases on $[0, +\infty)$ in this subcase. We then can obtain the following proposition.

Proposition 9 When $T < T_d$ and $T_W \leqslant T_d$, the optimal replenishment cycle T^* is as follows:

1) If $TC'_{21}(T_d) < 0$, then $T^* = T_d$.

2) If $TC'_{21}(T_W) \geqslant 0$, there exists a unique T^* which is given by $TC'_{21}(T) = 0$.

3) If $TC'_{21}(T_W) < 0$ and $TC'_{22}(T_d) \geqslant 0$, there exists a unique T^* which is given by $TC'_{22}(T) = 0$.

4) If $TC'_{22}(T_d) < 0$, then $T^* = T_d$.

5 Numerical Examples

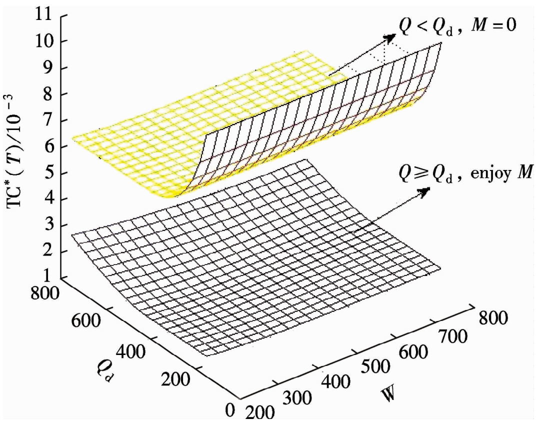
We use the following numbers as the base values of the parameters: $A = 500$, $D = 2\,500$, $s = 80$, $c = 50$, $k = 5$, $h = 2$, $I_c = 0.15$, $I_e = 0.12$, $M = 0.25$, $N = 0.05$, $\theta = 0.05$, $W = 400$, $Q_d = 300$.

5.1 Impact of (W, Q_d) on the total cost and replenishment cycle

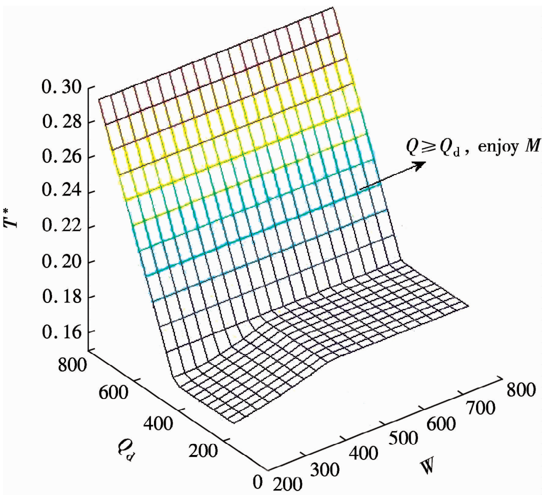
From Fig. 7(a), we can see that the retailer will order more goods than Q_d to enjoy the trade credit period. Therefore, we only need to analyze the results when $Q \geqslant Q_d$. Fig. 7(b) shows the variation of T^* when W and Q_d change under the condition of $Q \geqslant Q_d$.

Observation 1 If the values of Q_d and W are low, the optimal replenishment cycle T^* increases when W increases; otherwise, the optimal replenishment cycle T^* does not change as W changes.

From Fig. 7(b), we can find that when the values of Q_d and W are less than 400, the optimal replenishment cycle T^* increases in W. When W is low, increasing W means that the retailer can rent less space from the RW to satisfy the demand of storage, which will reduce the average unit holding cost for the retailer. Meanwhile, since Q_d is also low, increasing T^* does not lead to any extra interest charged. Therefore, under this condition, increasing T^* can decrease the annual total cost, thus reducing the annual ordering cost.



(a)



(b)

Fig. 7 Impact of W and Q_d on TC and T^*. (a) TC; (b) T^*

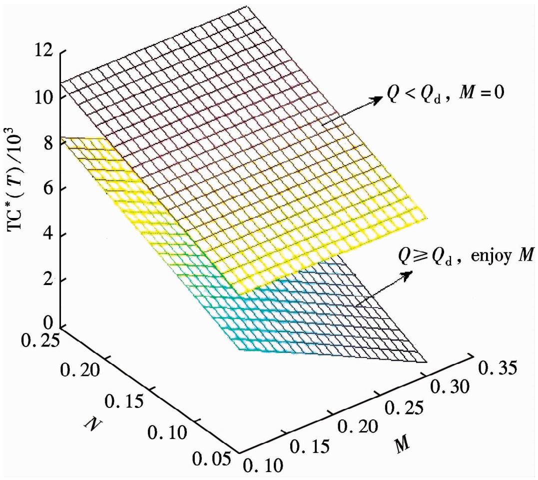
From Fig. 7(b), we can find that when $W<400$ and $Q_d>400$, the optimal replenishment cycle T^* does not change as W changes. Since Q_d is high, increasing T^* will lead to a higher interest charged. Therefore, the retailer does not increase the replenishment cycle T^* even if W increases.

Observation 2 If the value of Q_d is low, T^* does not change as Q_d changes. If the value of Q_d is high, T^* increases as Q_d increases.

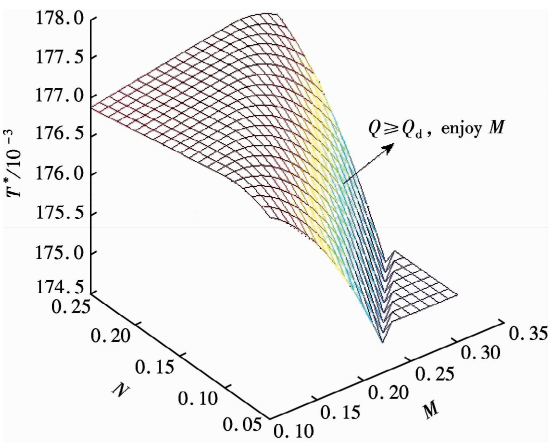
From Fig. 7(b), we can find that when $Q_d<400$, T^* does not change as W changes. Since T^* is longer than T_d when Q_d is not too high, increasing Q_d cannot affect the optimal replenishment cycle. However, from Fig. 7(b), we also find that when $Q_d>400$, T^* increases as fast as Q_d does. Under this condition, the influence of Q_d becomes significant. To enjoy the trade credit period, the retailer has to order more products, which leads to increasing the replenishment cycle.

5.2 Impact of (M, N) on the total cost and replenishment cycle

From Fig. 8(a), it is clear that under the same situation, ordering more goods than the minimum order quantity Q_d can lead to a lower annual total cost. Therefore, we only need to analyze the results when $Q\geqslant Q_d$. Fig. 8(b) shows the variation of T^* when M and N change under the condition of $Q\geqslant Q_d$. Then we can have the following observations.



(a)



(b)

Fig. 8 Impact of M and N on TC and T^*. (a) TC; (b) T^*

Observation 3 If $N\geqslant M$, T^* increases as M increases. If $N<M$, T^* increases as M decreases.

Under the condition of $N\geqslant M$, when M increases, the retailer needs to lengthen the replenishment cycle to fully enjoy the trade credit period, thus lowering the average total cost.

Under the condition of $N<M$, since the retailer offers the customers a short trade credit period, the retailer can obtain the sales money relatively quickly. Therefore, the retailer will shorten the replenishment cycle to enjoy more trade credit periods when M increases.

Observation 4 If $N\geqslant M$, T^* does not change when N changes.

Under the condition of $N \geqslant M$, the influence of N is small, while the influence of M is significant. Therefore, the retailer will keep the replenishment cycle according to the trade credit period offered by the supplier.

6 Conclusion

This paper mainly studies the optimal replenishment cycle to minimize the retailer's total cost under considering the deteriorating product, two-level trade credits, limited storage capacity of their own warehouse and credit-linked order quantity simultaneously. Furthermore, our study also leads to a few more managerial insights. First, when the minimum order quantity and the storage capacity of OW are low, the retailer should increase the replenishment cycle if the storage capacity of OW increases. Secondly, when the minimum order quantity is high, the retailer should increase the replenishment cycle if the minimum order quantity increases.

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折损产品供应链下零售商最优补货周期

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摘要:综合考虑产品折损性、两阶段信用周期、库存能力有限及信用周期与订购量相关等现实情形,以最小化零售商总成本为目标,对零售商最优补货周期进行了研究. 建立了一个由单个供应商与单个零售商组成的二级供应链模型,利用优化理论与方法得到了所有情形下的零售商最优补货周期决策. 基于此,采用数值仿真的方法进一步研究了系统参数变动对于最优补货周期的影响. 研究发现,当零售商信用周期长于(短于)消费者信用周期时,最优补货周期应随着零售商信用周期的增加而增加(减少);当供应商制定的最低订购量较高(较低)时,最优补货周期应随着最低订购量的增加而增加(不变).

关键词:补货周期;折损产品;信用周期;库存能力限制

中图分类号:C934