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Three-way group decision making with linguistic evaluations

Liu Shuli[1,2] Liu Xinwang[1]

([1]School of Economics and Management, Southeast University, Nanjing 211189, China)
([2]School of Mathematics and Physics, Anhui Polytechnic University, Wuhu 241000, China)

Abstract: Based on linguistic evaluations, a linguistic three-way decision method is proposed. First, the alternatives are rated in linguistic forms and divided into acceptance, rejection and uncertainty regions. Secondly, the linguistic three-way group decision steps are provided. Specifically, the experts determine the lower bound and upper bound of the uncertainty region, respectively. When the evaluation is superior to the upper bound, the corresponding alternative is put into the acceptance region directly. Similarly, when the evaluation is inferior to the lower bound, the corresponding alternative is put into the rejection region directly, and the remaining alternatives are put into the uncertain region. Moreover, the objects in the uncertainty region are especially discussed. The linguistic terms are transformed into fuzzy numbers and then aggregated. Finally, a recommendation example is provided to illustrate the practicality and validity of the proposed method.

Key words: linguistic evaluation; uncertainty; triangular fuzzy sets; three-way group decision making; recommendation

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The theory of three-way decisions (3WD) is the extension of two-way decisions (2WD)[1-3]. Yao[3] introduced three-way decisions with probabilistic rough sets. The main purpose of three-way decisions is to interpret the positive, negative and boundary regions of rough sets as three decision outcomes, acceptance, rejection and uncertainty in a ternary classification, respectively[4]. Recently, the method of three-way decisions has received much attention. Meanwhile, in some decision-making problems, it is difficult to rate the objects in numerical values. In these occasions, the decision makers prefer to apply the linguistic terms to evaluate the alternatives. Additive linguistic evaluation scales and multiplicative linguistic evaluation scales are defined, respectively, through which linguistic information can be aggregated[5]. Also, the fuzzy linguistic methodology used to deal with linguistic term sets is presented[6-8].

However, the existing research seldom combines linguistic assessments into three-way decisions. To the best of our knowledge, Liang et al.[9] discussed three-way decisions based on decision-theoretic rough sets under linguistic assessment with the aid of group decision making. Therefore, a novel method on linguistic three-way decisions is proposed in this paper. In linguistic three-way decisions, the alternatives are assessed in linguistic terms and separated into three parts: the acceptance region, rejection region and uncertainty region. Specifically, linguistic three-way decisions are constructed though a pair of thresholds (s_L, s_U) on the linguistic evaluation function, in which s_L and s_U denote the lower bound and upper bound of uncertainty region, respectively. When the evaluation is not inferior to the upper bound, the corresponding element is put into the acceptance region directly. Similarly, when the evaluation is inferior to the lower bound, the corresponding element is put into the rejection region directly, and the remaining elements are put into the uncertain region.

Moreover, for the alternatives in the uncertainty region, further analysis must be developed. The decision makers have to compute with words. Unfortunately, this is extremely difficult. One of the reasons is that words mean different things to different people[10]. The words must be transformed before computation. Therefore, in this paper, the linguistic terms are transformed into fuzzy numbers, and then the fuzzy operational rules are applied to aggregate the achieved information. Ultimately, the alternatives are ranked from good to bad.

1 Decision Methods

In this section, we propose the concept of linguistic three-way decisions and the decision steps.

1.1 Linguistic three-way decisions

According to the method of three-way decisions, objects are classified into three regions, called the positive, negative and boundary regions, respectively. However, in real decision making, the alternatives are often rated in linguistic variables (i.e. qualitative description). In recent years, some linguistic computational models for dealing with real-world decision making problems have been proposed[8, 11-15].

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Biographies: Liu Shuli (1980—), male, graduate; Liu Xinwang (corresponding author), male, doctor, professor, xwliu@seu.edu.cn.

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Therefore, we provide the concept of linguistic three-way decisions, which can be considered as the fusion of linguistic evaluation and three-way decisions.

Definition 1 Let A be a linguistic term set of U with evaluation function $A(x)$ and $E(A)(x) = A(x)$. If $A = \{s_0, s_1, \ldots, s_k\}$ and $s_0 \prec s_L \prec s_U \prec s_k$, then linguistic three-way decisions (L3WDs) are defined as follows:

Acceptance region

$$\mathrm{ACP}_{(s_L, s_U)}(E, A) = \{x \in U \mid E(A)(x) \succ s_U\}$$

Rejection region

$$\mathrm{REJ}_{(s_L, s_U)}(E, A) = \{x \in U \mid E(A)(x) \prec s_L\}$$

Uncertainty region

$$\mathrm{UNC}_{(s_L, s_U)}(E, A) = (\mathrm{ACP}_{(s_L, s_U)}(E, A) \cup \mathrm{REJ}_{(s_L, s_U)}(E, A))^C$$

where s_L, s_U are called lower and upper uncertain bound, respectively; $s_L \prec s_U$ means that the linguistic term s_L is inferior to the linguistic term s_U.

Linguistic three-way decisions extend three-way decisions into the linguistic evaluation environment, which is more friendly and common in real decision making.

1.2 Linguistic three-way group decision steps

For some problems, the objects are assessed by a group of experts in the form of linguistic terms. Therefore, we provide the linguistic three-way group decision steps.

Assume that the alternative x_i is assessed as $u_{ij}(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ by the expert e_j. The assessment values u_{ij} are from the linguistic term set $S = \{$extremely bad, very bad, bad, moderate, good, very good, excellent$\}$; i.e., $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{(0, 0, 0.17), (0, 0.17, 0.33), (0.17, 0.33, 0.5), (0.33, 0.5, 0.67), (0.5, 0.67, 0.83), (0.67, 0.83, 1), (0.83, 1, 1)\}$. Now, we plan to select k proper alternatives from m alternatives. The group decision-making steps are provided as follows.

Step 1 Construct the linguistic group decision matrix.

Step 2 Determine the lower bound and the upper bound, and establish the corresponding rejection region, acceptance region and uncertainty region, respectively.

Assume that the decision group has reached a consensus about the lower bound s_L and the upper bound s_U of the uncertainty region. For the alternative $x_i(i = 1, 2, \ldots, m)$ if $\forall\ u_{ij} \prec s_L$ then $x_i \in \mathrm{REJ}_{(s_L, s_U)}$, i.e. reject the alternative x_i. If $\forall\ u_{ij} \succ s_U$, then $x_i \in \mathrm{ACP}_{(s_L, s_U)}$, i.e. accept the alternative x_i. The other alternatives belong to the uncertainty region $\mathrm{UNC}_{(s_L, s_U)}$. It is notable that if the elements in the acceptance region are sufficient, choose k objects randomly, then the mission is over. Otherwise, turn to the next step.

Step 3 Construct the fuzzy decision matrix.

Step 4 Aggregate the fuzzy information of the uncertainty region.

Assume that the weight of the expert e_j is w_j and $\sum_{j=1}^{n} w_j = 1$. Aggregate all the $u_{ij}(j = 1, 2, \ldots, n)$ to obtain the value V_i of the alternative x_i in the form:

$$V_i = \mathrm{WA}(u_{i1}, u_{i2}, \ldots, u_{in}) = \sum_{j=1}^{n} w_j u_{ij}$$

Step 5 Compute the defuzzified value m_i with the formula: $m_i = (a_i^L + 2a_i^M + a_i^U)/4$, rank the alternatives in descending order and select the first $k - k_1$ alternatives.

Step 6 Combine the two part alternatives from the acceptance region X_3 in Step 2 and the uncertain region X_2 in Step 5, then we can obtain k alternatives that we need.

2 Illustration Example

In this section, the methods on linguistic three-way group decision making are illustrated by an example.

Assume that there are ten alternatives $x_i(i = 1, 2, \ldots, 10)$ and three decision makers $e_j(j = 1, 2, 3)$ have the same weight, respectively. The decision makers $e_j(j = 1, 2, 3)$ plan to select three proper alternatives. So, they evaluate the alternatives with linguistic terms s_i which are from the linguistic term set S. The linguistic decision matrix is shown in Tab. 1. Linguistic three-way group decision making steps are as follows.

Step 1 The linguistic decision matrix $\boldsymbol{U} = (u_{ij})_{10 \times 3}$ is constructed as shown in Tab. 1.

Step 2 First, assume that the lower and upper bounds are s_3 and s_6, respectively. For each alternative x_i, if all the evaluations are inferior to the lower bound s_3, then alternative x_i belongs to the rejection region; else if the evaluations are not inferior to the upper bound s_6, then alternative x_i belongs to the acceptance region. The remaining alternatives belong to the uncertainty region, which will be analyzed in the following steps.

Because the elements in the acceptance region are not sufficient, the alternatives in the uncertainty region should be analyzed in the next steps.

Step 3 Similar to Refs. [8, 12, 16 − 18], we can translate the linguistic term set S into a triangular fuzzy set. For convenience, we denote the corresponding fuzzy numbers in the same notations as the linguistic terms. Afterward, the fuzzy decision matrix $\boldsymbol{U} = (u_{ij})_{5 \times 3}$ is constructed.

Step 4 For the remaining alternatives x_3, x_4, x_6, x_7 and x_8, aggregate all the $u_{ij}(j = 1, 2, 3)$ to obtain the weighted value V_i of the alternative x_i in the form:

$$V_i = \sum_{j=1}^{3} w_j u_{ij}$$

The weighted values of the alternative x_4, x_6, x_7 and x_8 are obtained. The result is illustrated in Tab. 2.

Step 5 Based on the weighted value V_i, the defuzzified value m_i is computed as follows:

$$m_i = \frac{a_i^{\mathrm{L}} + 2a_i^{\mathrm{M}} + a_i^{\mathrm{U}}}{4} \qquad i = 3, 4, 6, 7, 8$$

Tab. 1 Linguistic decision matrix

x_i	e_1	e_2	e_3
x_1	s_0	s_2	s_1
x_3	s_6	s_3	s_1
x_4	s_5	s_6	s_5
x_5	s_2	s_0	s_2
x_6	s_3	s_5	s_5
x_7	s_5	s_4	s_2
x_8	s_4	s_5	s_5
x_9	s_6	s_6	s_6
x_{10}	$s2$	s_0	s_1

Tab. 2 Decision results

Alternative x_i	Weighted V_i	m_i	Rank
x_3	(0.39, 0.56, 0.67)	0.54	5
x_4	(0.72, 0.89, 1.00)	0.87	1
x_6	(0.56, 0.72, 0.89)	0.72	3
x_7	(0.45, 0.61, 0.78)	0.61	4
x_8	(0.61, 0.78, 0.94)	0.78	2

The computational result is illustrated in Tab. 2. Thus, the first two alternatives (i. e. x_4 and x_8) are selected.

Step 6 Combine both the alternative (x_9) from the acceptance region and the alternatives (x_4 and x_8) from the uncertainty region. Ultimately, the alternatives x_4, x_8 and x_9 are chosen as the most proper choices.

By the proposed method, the alternatives x_4, x_8 and x_9 are chosen. The rejection region has 4 alternatives (x_1, x_2, x_5 and x_{10}); the acceptance region has a single alternative (x_9) and the uncertainty region has 5 alternatives (x_3, x_4, x_6, x_7 and x_8).

In the example, the evaluations to the alternatives take the form of linguistic terms, so the proposed methods on linguistic three-way decisions are valid and their effectiveness is illustrated.

3 Conclusion

In this paper, we introduce a novel extension of three-way decisions, called linguistic three-way group decisions, which introduces linguistic rating information into three-way decisions. Moreover, the uncertain region is specifically analyzed. An example is provided to illustrate the validity of the proposed methods.

References

[1] Li T R, Chen H M, Yao J A, et al. A multifaceted analysis of probabilistic three-way decisions [J]. *Fundamenta Informaticae*, 2014, **132**(3): 291 − 313. DOI: 10.3233/FI-2014 − 1045.

[2] Deng X F, Yao Y Y. Decision-theoretic three-way approximations of fuzzy sets [J]. *Information Sciences*, 2014, **279**: 702 − 715. DOI: 10.1016/j.ins.2014.04.022.

[3] Yao Y Y. Three-way decisions with probabilistic rough sets [J]. *Information Sciences*, 2010, **180**(3): 341 − 353. DOI: 10.1016/j.ins.2009.09.021.

[4] Hu B Q. Three-way decisions space and three-way decisions [J]. *Information Sciences*, 2014, **281**: 21 − 52. DOI: 10.1016/j.ins.2014.05.015.

[5] Xu Z S. *Linguistic decision making*: *Theory and methods* [M]. Berlin: Springer, 2012.

[6] Ma J, Ruan D, Xu Y, et al. A fuzzy-set approach to treat determinacy and consistency of linguistic terms in multi-criteria decision making [J]. *International Journal of Approximate Reasoning*, 2007, **44**(2): 165 − 181.

[7] Herrera F, Herrera-Viedma E, Martinez L. A fuzzy linguistic methodology to deal with unbalanced linguistic term sets [J]. *IEEE Transactions on Fuzzy Systems*, 2008, **16**(2): 354 − 370. DOI: 10.1109/tfuzz.2007.896353.

[8] Massanet S, Riera J V, Torrens J, et al. A new linguistic computational model based on discrete fuzzy numbers for computing with words [J]. *Information Sciences*, 2014, **258**: 277 − 290. DOI: 10.1016/j.ins.2013.06.055.

[9] Liang D C, Liu D. Three-way decisions based on decision-theoretic rough sets under linguistic assessment with the aid of group decision making [J]. *Applied Soft Computing*, 2015, **29**: 256 − 269. DOI: 10.1016/j.asoc.2015.01.008.

[10] Mendel J M, Wu D R. *Peceptual computing*: *Aiding people in making subjective judgments* [M]. New Jersey: John Wiley and Sons, Inc., 2010.

[11] Yang, X J, Yan L L, Peng H, et al. Encoding words into Cloud models from interval-valued data via fuzzy statistics and membership function fitting [J]. *Knowledge-Based Systems*, 2014, **55**: 114 − 124. DOI: 10.1016/j.knosys.2013.10.014.

[12] Meng F Y, Chen X H, Zhang Q. Multi-attribute decision analysis under a linguistic hesitant fuzzy environment [J]. *Information Sciences*, 2014, **267**: 287 − 305. DOI: 10.1016/j.ins.2014.02.012.

[13] Kacprzyk J, Zadrozny S. Computing with words is an implementable paradigm: Fuzzy queries, linguistic data summaries, and natural-language generation [J]. *IEEE Transactions on Fuzzy Systems*, 2010, **18**(3): 461 − 472. DOI: 10.1109/tfuzz.2010.2040480.

[14] Cao Y Z, Chen G Q. A fuzzy Petri-nets model for computing with words [J]. *IEEE Transactions on Fuzzy Systems*, 2010, **18**(3): 486 − 499. DOI: 10.1109/tfuzz.2009.2035816.

[15] Wang J H, Hao J Y. A new version of 2-tuple fuzzy linguistic, representation model for computing with words [J]. *IEEE Transactions on Fuzzy Systems*, 2006, **14**(3): 435 − 445.

[16] Liu F, Mendel J M. Encoding words into interval type-2 fuzzy sets using an interval approach [J]. *IEEE Transactions on Fuzzy Systems*, 2008, **16**(6): 1503 − 1521.

[17] Qin J D, Liu X W. Multi-attribute group decision making using combined ranking value under interval type-2 fuzzy environment [J]. *Information Sciences*, 2015, **297**: 293 − 315. DOI: 10.1016/j.ins.2014.11.022.

[18] Qin J D, Liu X W, Pedrycz W. An extended VIKOR method based on prospect theory for multiple attribute decision making under interval type-2 fuzzy environment [J]. *Knowledge-Based Systems*, 2015, **86**: 116 – 130. DOI: 10.1016/j.knosys.2015.05.025.

基于语言评价的三支群决策

刘树利[1,2] 刘新旺[1]

([1] 东南大学经济管理学院, 南京 211189)
([2] 安徽工程大学数理学院, 芜湖 241000)

摘要:基于语言评价信息,提出了语言三支决策方法. 该方法采用语言术语对被择选项进行评价,将其分为3个区域:接受域,拒绝域和不确定域. 然后,基于三支决策方法,给出了语言三支群决策方法及其具体的决策步骤. 先由专家给出不确定域的上下限,当目标选项的评价优于上限时,将被分配到接受域;当目标选项的评价劣于下限时,将被分配到拒绝域;其余选项被分配到不确定域. 其中,针对不确定域中的选项,将语言评价术语转化为三角模糊数,然后加以集成分析. 最后,通过一个推荐应用中的例子验证了该方法的有效性和实用性.

关键词:语言评价;不确定性;三角模糊集;三支群决策;推荐
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