Estimation of structure crack propagation based on multiple factors correction

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Abstract: By deriving the stress concentration factor of the estimation approach for residual fatigue life, an estimation approach for structure crack propagation based on multiple factors correction is proposed. Then, the quantitative expression among the structure factor, stress ratio, loading type, the manufacture processing factor and the crack propagation is achieved. The proposed approach is implemented in a case study for an instance structure, and the influences of correction factors on the crack propagation are analyzed. Meanwhile, the probabilistic method based on Weibull distribution probability density function is selected to evaluate the precision of the corrected estimation approach, and the probability density of results is calculated by the probability density function. It is shown that the results estimated by the corrected approach is more precise than those estimated by the fracture mechanics, and they are closer to the test data.

Key words: estimation study; crack propagation; multiple factor correction; probability density function

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M any mechanical components experience fatigue failure in service, e. g. the mast in a helicopter, railroad wheels, turbine blades, etc[1]. As an important parameter for reliability assessment, the calculation of crack propagation is of great significance. However, many factors, such as the structure factor, stress ratio, loading type, manufacturing processing and etc. , affect the results of crack propagation.

Therefore, it is necessary to conduct research on crack propagation based on the factors above. Stasevic et al. [2] predicted the crack propagation by the mechanics model. Carter et al. [3] presented the prediction of crack propagation by the finite element method (FEM). However,

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these prediction methods disregard the structure characteristics and other influencing factors.

Also, various correction approaches were proposed. Zhang et al. [4] proposed an approach to predict the crack propagation by using the closed-form solution of the stress concentration factor. Wang et al. [5] built a linear embedded spring model by using the distribution load. Esmaeili et al. [6] studied the crack propagation of specimens under multiaxial loading. Lu et al. [7] presented research on the base of the multiaxial damage model and the concept of equivalent initial flaw size. These corrected prediction approaches can be used effectively to predict the crack propagation under multiaxial loading. However, these approaches did not make comprehensive consideration of the impact factors. As a result, a new approach which considers the impact factors comprehensively should be proposed.

Zhu et al. [8] and Shi et al. [9] proposed an estimation approach of residual fatigue life based on multiple factors correction. The quantitative values of multiple factors were achieved. When fracture is seen as the unified failure standard, both the *S-N* curve and crack propagation curve can be used to reflect the fatigue properties. Cui[10] and Lam et al. [11] proposed that the crack propagation curve can be achieved by transforming the *S-N* curve after finding the threshold between the initiation process and the propagation process. The stress concentration factor is an important parameter in the conversion process. The proposed estimation approach[8–9] includes the factor for reflecting the degree of stress concentration. Therefore, the corrected estimation approach can be used to reflect the process of crack propagation.

In this paper, the corrected prediction approach of crack propagation is proposed, and it is also implemented in a case study. The effects of correction factors on the results of the crack propagation are analyzed. The results calculated by the proposed approach are compared with the results of fracture mechanics and the test data by an acoustic emission device. Furthermore, the probabilistic method based on Weibull distribution is selected as the statistical framework to evaluate the precision of the proposed approach.

1 Estimation of Crack Propagation Based on Multiple Factors Correction

The relationship of structure factor, stress ratio, load-

ing type, manufacture processing factor and the residual fatigue life of crack propagation process can be expressed as

$$\left\{ \frac{\sigma[1-(\sigma_m/\sigma_u)^2]}{K_t}\beta C_L \right\}^m N = C \qquad (1)$$

where σ is the cyclic stress; σ_m is the mean stress; $\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2}$; σ_{max} is the maximum stress; σ_{min} is the minimum stress; σ_u is the ultimate tensile strength; K_t is the stress concentration coefficient; β is the surface quality coefficient; m and C are material constants; C_L is the loading type coefficient; N is the fatigue life.

K_t is the parameter in the calculation, which can be used to represent the crack propagation process.

$$K_t = \frac{\sigma_{max}}{\sigma_n} \qquad (2)$$

Nominal stress σ_n[12] can be expressed as

$$\sigma_n = \frac{\sum_{i=1}^{n-1}\int_{r_i}^{r_{i+1}}\sigma_i dr}{A} = \frac{\int_0^A \sigma_i dr}{A} \qquad (3)$$

where σ_i is the stress amplitude of points in the stress field; r is the distance between arbitrary point under the stress field and the maximum stress point; A is the limit

radius of the stress field, $A = r_{max}$.

Stress on the crack tip in the direction of the crack extension, σ_i[13], can be expressed as

$$\sigma_i = \frac{\sigma_{max}(a+r)}{\sqrt{2ar+r^2}} \qquad (4)$$

where a is the crack propagation length.

Substituting Eqs. (3) and (4) into Eq. (2), the stress concentration factor can be obtained by

$$K_t = \frac{A}{\int_0^A \frac{(a+r)}{\sqrt{2ar+r^2}}dr} \qquad (5)$$

Finally, substituting Eq. (5) into Eq. (1), the crack length a is expressed as

$$\sigma\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^2\right]\left(\frac{C}{N}\right)^{-1/m}\beta C_L = \frac{A}{\int_0^A \frac{a+r}{\sqrt{2ar+r^2}}dr} \qquad (6)$$

Each processing type has different influences on the crack propagation of the structure. The surface quality coefficient corresponding to the common processing method[14] is shown in Tab. 1. The loading type coefficient is used to modify the influence of working loading on the crack propagation.

Tab. 1   The surface quality coefficient corresponding to the common processing method

Processing method	Strength/MPa	Coefficient of surface quality β		
		$K_t = 0$	$K_t \leqslant 1.5$	$K_t \geqslant 2.0$
High frequency quenching	600-800	1.3-1.5	1.4-1.5	1.8-2.2
	800-1 000	1.5-2.0	1.7-2.7	
Surface nitriding treatment	900-1 200	1.1-1.3	1.5-1.7	1.7-2.1
Carburizing treatment	400-600	1.8-2.0	3.0	3.1-3.7
	700-800	1.4-1.5	2.2-2.6	2.9-3.2
Shot blasting	600-1 500	1.1-1.4	1.4-1.6	1.6-2.0

2   Case Study

In order to verify the prediction accuracy of the proposed approach, the case structure as shown in Fig. 1 is predicted based on multiple factors correction. The dimension of the structure is shown in Tab. 2. The material is Q235, and the performance parameters of the material are shown in Tab. 3.
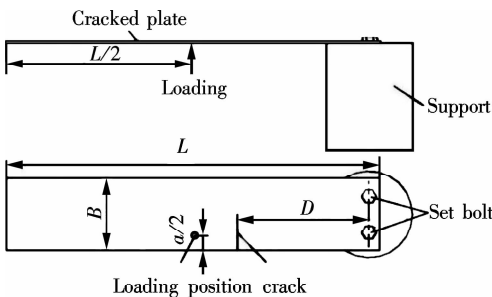

Fig. 1   Case structure

Tab. 2   The size of instance structure   mm

Size type	Value
Plate length L	600
Plate width B	100
Crack length a	50
Length between crack and bolt D	170
Plate thickness t	3

Tab. 3   The mechanical parameters of the material

Parameter	$R_{0.2}$/MPa	R_m/MPa	Elongation A/%
Value	237	500	22

First, the 3D model of the case structure is created by using the Pro-e software. Then, the 3D model is imported into the ANSYS software for the solution of equivalent stress. Finally, the stress nephogram is obtained as shown in Fig. 2. It can be found in Fig. 2 that the maximum stress occurs in the crack tip, and the value of which is 163.49 MPa.

Fig. 2 Stress nephogram of case structure

The relationship between stress and fatigue cycles can be expressed as

$$\lg S = -\frac{1}{m}\lg N + \frac{\lg C}{m} \tag{7}$$

where S is the stress amplitude; m and C are material constants; N is the number of fatigue cycles.

Eq. (7) shows that the linear slope of the logarithmic relationship between stress amplitude and fatigue cycles is $-\frac{1}{m}$, and the intersection value of the logarithmic relationship is $\frac{\lg C}{m}$.

The logarithmic relationship between stress amplitude and fatigue cycles as shown in Fig. 3 is obtained through the fatigue test. The basic performance parameters C and m are calculated according to Eq. (7).
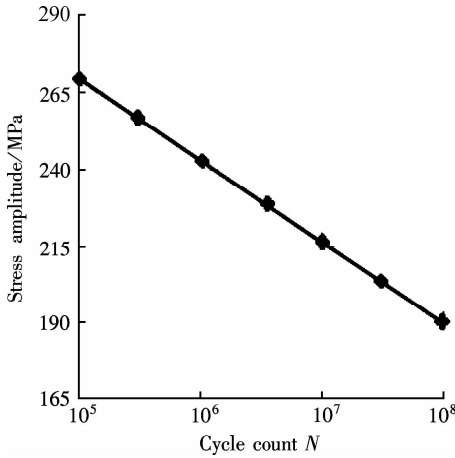

Fig. 3 Logarithmic relationship between stress amplitude and fatigue cycles

Also, the calculation of correction coefficient is important for the study. The surface quality coefficient (SQC) of the case is 1.6 according to Tab. 1. According to Ref. [15], the loading type coefficient (LTC) corresponding to uniaxial load is 1, and the coefficient corresponding to the bending load is 0.8. Finally, the result as shown in Fig. 4 is obtained by substituting the initial parameters into a program.

3 Influence of Correction Factors on Crack Propagation

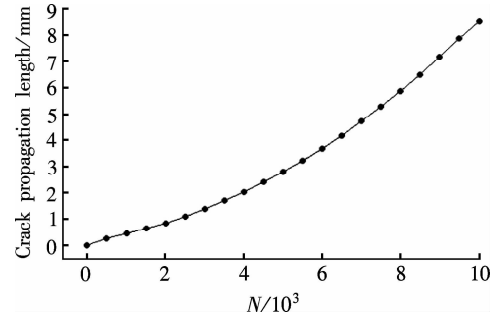The LTC and SQC are the principal correction factors in

Fig. 4 Simulation result of crack propagation

the numerical procedure. Therefore, it is necessary to analyze the influence of factors on the result. Different groups of SQC parameter and LTC parameter are utilized for the analysis. The SQC is selected as 1.4, 1.6, 1.8, 2.0, 2.2, and the LTC is selected as 0.6, 0.8, 1.0, 1.2 and 1.4. Then, the influence of the SQC on the crack propagation is achieved by the corrected approach, as shown in Fig. 5. It can be found from Fig. 5 that the crack propagation decreases with the increase of the SQC when the LTC is determined.

Also, the influence of the LTC on the crack propagation is obtained by the corrected approach, as shown in Fig. 6. It can be found that the crack propagation length of the case structure decreases with the increase of the LTC when the SQC is determined.

4 Experimental Validation

4.1 Experimental plan

The SAMOS PCI-8 AE device is used for the long term monitoring of the crack propagation process in the experiment study. The system parameters of the AE experiment are shown in Tab. 4.

Tab. 4 The system parameters of AE experiment

Parameter	Value
Sampling frequency/MHz	2
Threshold value/dB	32
Digital filter frequency/kHz	1-400
Wavelength/nm	4 096
Trigger rate/(bit·s^{-1})	512

A broadband, resonant-type, electric transducer from the physical acoustics corporation (PAC), called PICO, is used as the AE sensor. The sensor has a resonance frequency of 480.58 kHz and an optimum operating range of 100-750 kHz. To improve the signal transmission between the instance structure and sensor, vacuum silicon grease is used. The location of the sensor arrangement is shown in Fig. 7. The arrangement of sensors, the instance structure, the loading device and the acoustic emission device is shown in Fig. 8.

4.2 Result and precision comparison

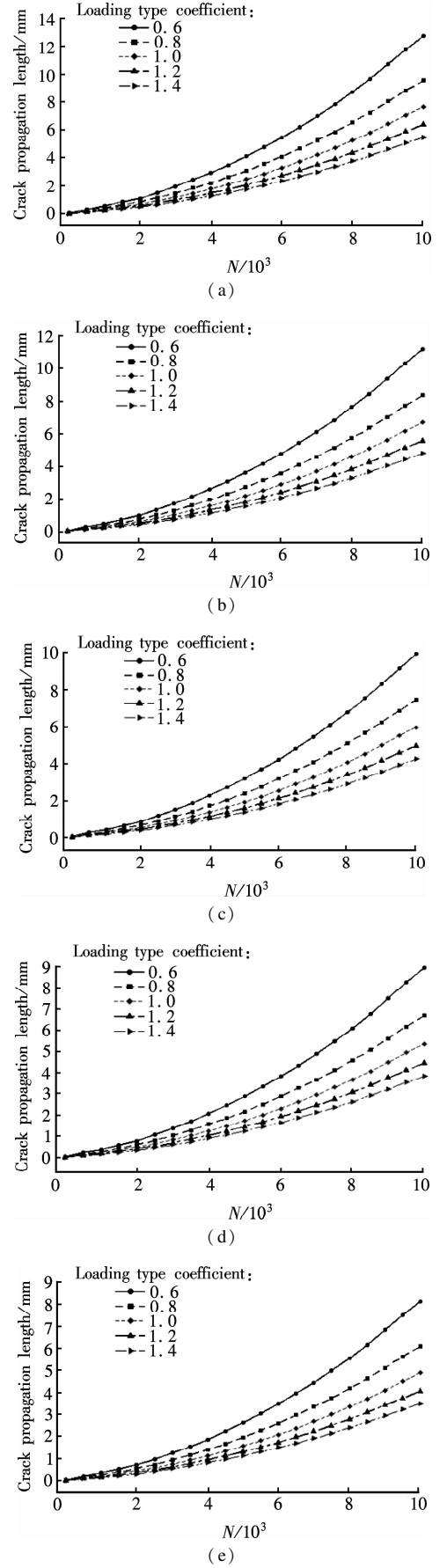After the process of processing the emission signal, the

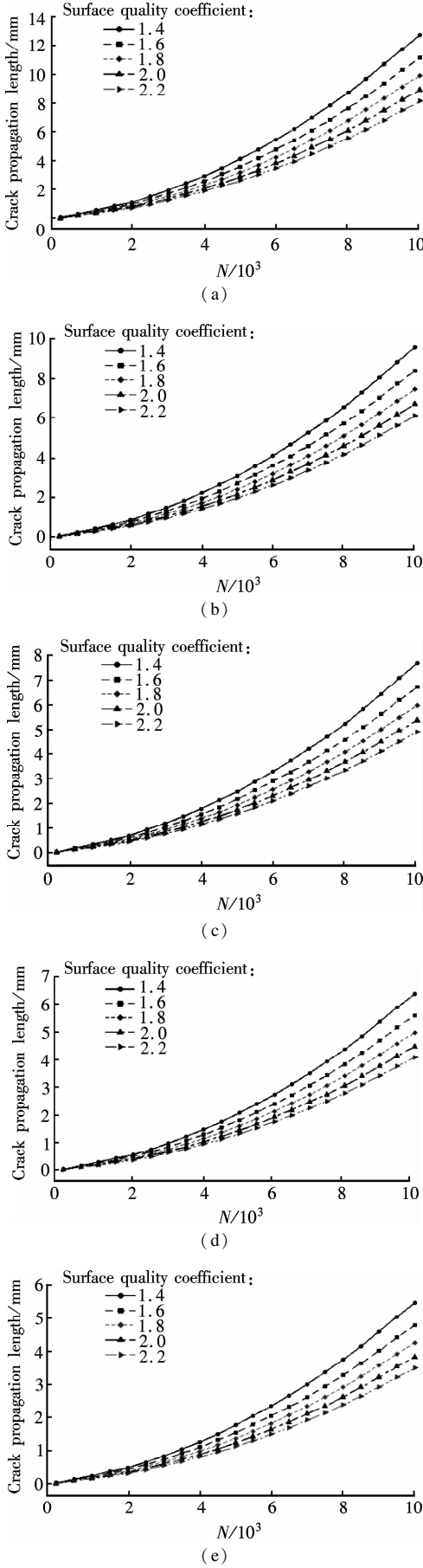**Fig. 5** Influence of SQC on crack propagation. (a) LTC = 0.6; (b) LTC = 0.8; (c) LTC = 1.0; (d) LTC = 1.2; (e) LTC = 1.4

Fig. 6 Influence of LTC on crack propagation. (a) SQC = 1.4; (b) SQC = 1.6; (c) SQC = 1.8; (d) SQC = 2.0; (e) SQC = 2.2
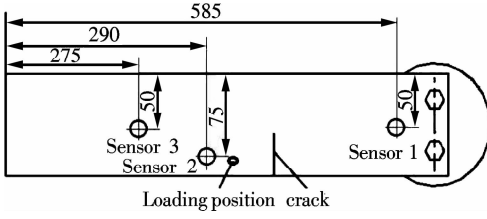
Fig. 7 Location of the sensor arrangement（unit：mm）

Fig. 8 Process of crack propagation test

test data of the crack propagation is obtained. The test data shows some uncertainty corresponding to the fixed load cycle. The probabilistic method based on Weibull distribution is selected as the statistical framework to evaluate the precision of the corrected estimation approach, which is more suitable for the estimation of crack propagation[16].

The distribution function $F(x)$ and the probability density function（PDF）$f(x)$ of the Weibull distribution can be expressed as

$$F(x) = 1 - \exp\left[-\left(\frac{x-a_0}{\beta}\right)^{\alpha}\right] \qquad (8)$$

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x-a_0}{\beta}\right)^{\alpha-1}\exp\left[-\left(\frac{x-a_0}{\beta}\right)^{\alpha}\right] \qquad (9)$$

where α is the shape parameter; β is the scale parameter; a_0 is the position parameter.

The Weibull parameters of the test data corresponding to the fixed load cycle can be calculated by the probability weighted moment method[17]. The probability weighted moment function can be expressed as

$$M_{1,0,K} = \frac{a_0}{1+K} + \frac{\beta\Gamma\left(1+\dfrac{1}{\alpha}\right)}{(1+K)^{1+1/\alpha}} \qquad (10)$$

where M is the probability weighted moment function; Γ is the Gamma function; $K = 0$, 1, 3.

$$M_{1,0,0} = a_0 + \beta\Gamma\left(1+\frac{1}{\alpha}\right) \qquad (11)$$

$$M_{1,0,1} = \frac{a_0}{2} + \frac{\beta\Gamma\left(1+\dfrac{1}{\alpha}\right)}{2^{1+1/\alpha}} \qquad (12)$$

$$M_{1,0,3} = \frac{a_0}{4} + \frac{\beta\Gamma\left(1+\dfrac{1}{\alpha}\right)}{4^{1+1/\alpha}} \qquad (13)$$

Therefore, the Weibull distribution parameter α, β, a_0 can be expressed as

$$\alpha = \frac{\ln 2}{\ln\left[\dfrac{M_{1,0,0}-2M_{1,0,1}}{2(M_{1,0,1}-2M_{1,0,3})}\right]} \qquad (14)$$

$$\beta = \frac{M_{1,0,0}-a_0}{\Gamma\left[\ln\left(\dfrac{M_{1,0,0}-2M_{1,0,1}}{M_{1,0,1}-2M_{1,0,3}}\right)\middle/ \ln 2\right]} \qquad (15)$$

$$a_0 = \frac{4(M_{1,0,3}M_{1,0,0}-M_{1,0,1}^2)}{4M_{1,0,3}+M_{1,0,0}-4M_{1,0,1}} \qquad (16)$$

Then, the Weibull parameters of the test data corresponding to the fixed load cycle are calculated and shown in Tab. 5. Also, the probabilistic density of the test data is shown in Fig. 9. In order to further evaluate the precision of the corrected estimation approach, fracture mechanics is also used. Different results are compared as shown in Fig. 9.

Tab. 5 The Weibull distribution parameter of test data

Cycles	Shape parameter	Scale parameter	Position parameter
2 000	1.79	0.11	0.79
4 000	1.44	0.14	1.92
6 000	4.50	0.64	3.25
8 000	2.82	0.28	5.67
10 000	1.76	0.57	7.99

The probability density of the different results is shown in Tab. 6. It can be found from Tab. 6 that the probability density of the results estimated by the corrected approach is greater than that by fracture mechanics. Therefore, the results estimated by the corrected approach are more precise than those estimated by fracture mechanics, and they are closer to the test data.

Tab. 6 The probability density of results

Cycles	Corrected approach	Fracture mechanics
2 000	6.20	1.88
4 000	4.81	1.20
6 000	2.71	1.27
8 000	3.20	1.50
10 000	1.22	0.36

It can be found from Tab. 6 that the results estimated by fracture mechanics show some discrepancy with the test data. The original estimation method based on fracture mechanics can be used effectively for the estimation under constant loading. However, the original fracture mechanic method takes less account of the influencing factors, and cannot be quantitatively corrected by the influencing factors at the same time.
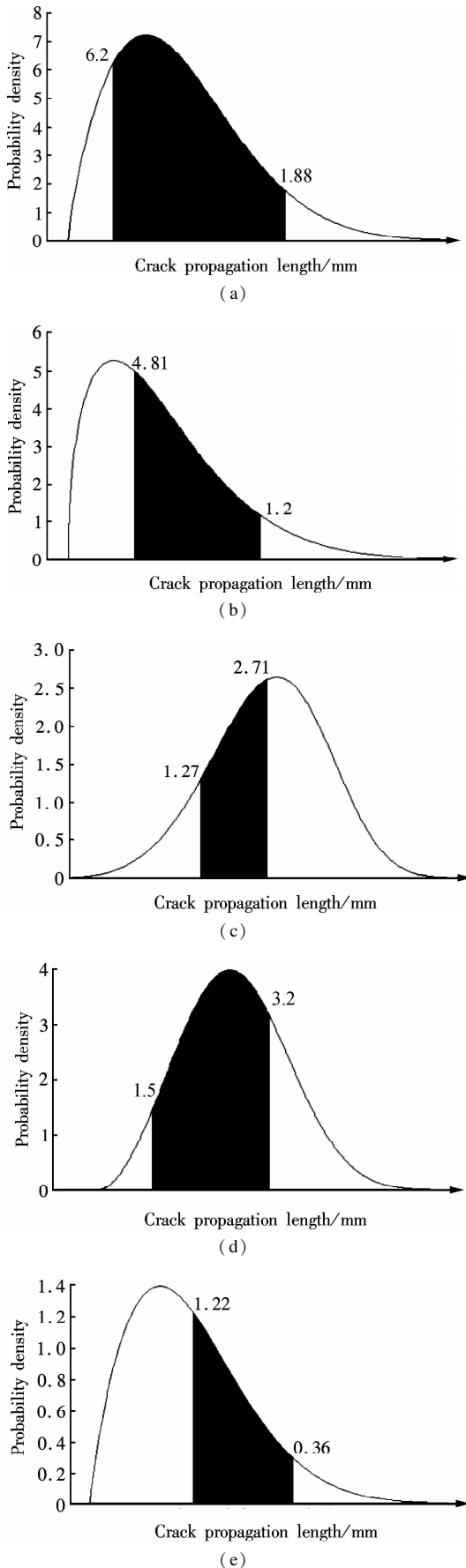
Fig. 9 The comparison of results obtained by the corrected model and fracture mechanics. (a) 2 000 cycles; (b) 4 000 cycles; (c) 6 000 cycles; (d) 8 000 cycles; (e) 10 000 cycles

5 Conclusions

1) A crack propagation prediction approach based on multiple factors correction is achieved by the estimation approach of residual fatigue life based on multiple factors correction. Then, the quantitative expressions among the structure factor, stress ratio, loading type, the manufacturing processing method and the crack propagation are achieved.

2) The crack propagation of a case is calculated by the prediction approach based on multiple factors correction. The influence of the correction factor on the crack propagation is studied. It can be found from the results that the crack propagation length decreases with the increase of LTC and SQC.

3) The AE device is used for the long term monitoring of the crack propagation in this experiment study. The probabilistic method based on the Weibull distribution is selected as the statistical framework to process the data. Then, the probability densities of the results estimated by the corrected approach and the results estimated by fracture mechanics are calculated by the probability density function. The comparisons show that the results estimated by the corrected approach are more precise than those estimated by fracture mechanics.

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一种新的基于多因素修正的结构件裂纹扩展预测方法

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摘要:通过对基于多因素修正的剩余寿命预测方法中的应力集中因子进行推导，提出了一种基于多因素修正的结构件裂纹扩展预测方法,从而实现了结构因素、应力比、加载方式、表面质量系数与构件裂纹扩展之间的量化关系表达. 将所提方法应用于具体的实例构件的裂纹扩展预测中，并定量分析了所提出的各具体修正因素对实例构件裂纹扩展预测结果的影响. 同时,运用基于 Weibull 分布概率密度方程的概率方法对修正后方法预测结果的精度进行评价,通过概率密度方程对实际预测结果所对应的概率密度值进行了求解. 结果表明,该方法的预测结果比断裂力学的预测结果更精确,更接近试验结果.

关键词:预测研究;裂纹扩展;多因素修正;概率密度方程

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