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Modification of the linear viscoelastic deformation prediction model of asphalt mixture

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Abstract: A deformation prediction model for the dynamic creep test is deduced based on the linear viscoelastic (LVE) theory. Then, the defect of the LVE deformation prediction model is analyzed by comparing the prediction of the LVE deformation model with the experimental data. To improve accuracy, a modification of the LVE deformation prediction model is made to simulate the nonlinear property of the deformation of asphalt mixtures, and it is verified by comparing its simulation results with the experimental data. The comparison results show that the LVE deformation prediction model cannot simulate the nonlinear property of the permanent deformation of asphalt mixtures, while the modified deformation prediction model can provide more precise simulations of the whole process of the deformation and the permanent deformation in the dynamic creep test. Thus, the proposed modification greatly improves the accuracy of the LVE deformation prediction model. The modified model can provide a better understanding of the rutting behavior of asphalt pavement.

Key words: deformation prediction; rutting; viscoelasticity; dynamic creep test

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Rutting or permanent deformation can greatly reduce the service life of the pavement. In the analysis of permanent deformation, two main aspects are of particular interest: the laboratory test techniques to measure the resistance of asphalt mixtures to permanent deformation and the mechanics models of rutting prediction[1].

Various laboratory test methods are currently used to estimate the permanent deformation property of asphalt mixtures, such as the static creep test, dynamic modulus test, and dynamic creep test[2]. Compared with the dynamic creep test, the static creep test made a larger error due to its static loading condition[3]. The permanent deformation in the dynamic creep test had a good correlation with field performance[4]. Many researchers modified the dynamic creep test to simulate the real condition of road. Jiang et al. [5] proposed an optional multiple repeated load test to consider the axial load spectrum in real pavement. Li et al. [6] modified the dynamic creep test to simulate the layers condition and the temperature gradient of the actual pavement. Also, some studies[7-8] used a smaller platen to better simulate the actual confinements in pavement.

In terms of the mechanics models, the viscoelastic model is widely used in the deformation prediction of asphalt mixtures. The linear viscoelastic (LVE) model, though widely used[9-11], is incapable of long-term rutting prediction due to a lack of consideration of densification and material damage[12]. Thus, some nonlinear viscoelastic models were developed[13-15]. However, they were conducted by some numerical technologies and could not yield a function for deformation prediction. Though many models of the permanent deformation prediction of the dynamic creep test have been developed[13,16], research on the prediction of whole-process deformation (the deformation throughout the dynamic creep test) is rare[9].

The objective of this study is to improve the accuracy of deformation prediction of the dynamic creep test. Specifically, a LVE deformation prediction model for the dynamic creep test is first deduced. Then, a modification to the linear term of the LVE deformation prediction model is made to fit the nonlinear property of permanent deformation. Finally, the accuracy of the modified model in predicting the whole-process deformation and the permanent deformation is verified by comparing the predicted result with the experimental data.

1 LVE Deformation Prediction Model

The load used in the dynamic creep test is a repeated load consisting of a haversine loading period and a rest period, to simulate the traffic load on asphalt pavement. The function of the load form is represented as

$$\sigma(t)=\begin{cases}\dfrac{\sigma_{\max}}{2}\left(1-\cos\dfrac{2\pi}{t_0}t\right) & nT<t\leqslant nT+t_0\\ 0 & nT+t_0<t\leqslant(n+1)T\end{cases}\qquad n\in\mathbf{N} \tag{1}$$

where $\sigma_{\max}$ is the peak value of the haversine load; t_0 is the haversine loading period; T is the duration of a load cycle, $T=t_0+t_d$, and t_d is the rest period.

The creep compliance of the Burgers model, $J(t)$, is

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represented as

$$J(t)=\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}(1-e^{-E_2 t/\eta_2}) \tag{2}$$

where $J(t)$ is the creep compliance at time, GPa^{-1}; t is the test time, s; E_1, E_2, η_1, η_2 are the parameters of the Burgers model.

According to the Boltzmann superposition principle in the linear viscoelastic theory, the strain of asphalt mixtures can be represented by the convolution integral.

$$\varepsilon(t)=\int_0^t J(t-\xi)\frac{d\sigma(\xi)}{d\xi}d\xi \tag{3}$$

where ξ is the integration variable; $\varepsilon(t)$ is the strain of the analyzed material.

Substituting Eq. (2) into Eq. (3), the strain for the 1st load cycle ($0<t\leqslant T$) can be represented as

$$\varepsilon(t)=\begin{cases}\varepsilon_1(t) & 0<t\leqslant t_0\\ \varepsilon_2(t) & t_0<t\leqslant T\end{cases} \tag{4}$$

where

$$\varepsilon_1(t)=\frac{\sigma_{max}}{2}+\frac{2\sin\left(\frac{\pi t}{t_0}\right)^2(E_1+E_2)}{E_1E_2}+\frac{t-\frac{t_0\sin\left(\frac{2\pi t}{t_0}\right)}{2\pi}}{\eta_1}-\frac{\omega\sin(\omega t)\eta_2}{E_2^2+\omega^2\eta_2^2}+\frac{\omega^2\left(e^{E_2t/\eta_2}+\cos\left(\frac{2\pi t}{t_0}\right)\right)\eta_2^2}{E_2^3+\omega^2E_2\eta_2^2} \tag{5}$$

$$\varepsilon_2(t)=\frac{\pi\sigma_{max}}{\omega\eta_1}+\frac{e^{-E_2t/\eta_2}(-1+e^{t_0E_2/\eta_2})\omega^2\eta_2^2\sigma_{max}}{2(E_2^3+\omega^2E_2\eta_2^2)} \tag{6}$$

where $\omega=2\pi/t_0$.

Based on the Boltzmann superposition principle, the LVE deformation prediction model in the $(n+1)$-th load cycle ($nT<t\leqslant(n+1)T$) can be deduced as

$$\varepsilon(t)=\begin{cases}\sum_{i=1}^{n}\varepsilon_2(t-(i-1)T)+\varepsilon_1(t-nT) \\ \qquad nT<t\leqslant nT+t_0\\ \sum_{i=1}^{n+1}\varepsilon_2(t-(i-1)T) \\ \qquad nT+t_0<t\leqslant(n+1)T\end{cases} \tag{7}$$

2 Material and Test Method

2.1 Material properties

The commonly used dense-grade asphalt mixture AC-13 was selected in this study. No. 70 asphalt was used as the asphalt binder, the properties of which are listed in Tab. 1. Optimal asphalt content was determined to be 5.0%. The aggregate gradation was designed as shown in Tab. 2.

All specimens were fabricated by a superpave gyratory compactor (SGC). The cylindrical specimens were 100 mm in diameter and 100 mm in height.

Tab. 1 Main technical indices of asphalt binder

Binder	$P_{25\,℃}$/0.1 mm	PI	$T_{R\&B}$/℃	$D_{15\,℃}$/cm	γ
No. 70 asphalt	63.4	−0.88	47.6	>100	1.035

Notes: $P_{25\,℃}$ is the penetration at 25 ℃; PI is the penetration index; $T_{R\&B}$ is the softening point; $D_{15\,℃}$ is the ductility at 15 ℃; γ is the relative density.

Tab. 2 Aggregate gradation of AC-13 asphalt mixture

Sieve size/mm	19	16	13.2	9.5	4.75	2.36
Passing percentage/%	100	100	92.7	77.4	46.6	32.1
Sieve size/mm	1.18	0.6	0.3	0.15	0.075	
Passing percentage/%	21.0	14.6	8.2	6.4	5.4	

2.2 Dynamic creep test

The dynamic creep test was conducted by a universal testing machine. The repeated load described in Eq. (1) was applied. t_0 and T were chosen as 0.45 and 6 s based on the intersection traffic survey. The temperature was maintained at 60 ℃. The vertical deformation of specimen was measured by linear variable differential transducers (LVDTs). The first test was conducted with a stress level of 0.7 MPa and 100 cycles of deformation were measured in the first test to verify the accuracy of the whole-process deformation prediction. Three other tests with different stress levels (0.6, 0.7 and 0.8 MPa) were conducted to verify the accuracy of the permanent deformation prediction.

3 Fitting Parameters of Burgers Model

The experimental data in the 1st cycle of the dynamic creep test is used to fit the Burgers parameters. The nonlinear deformation equation for the 1st cycle is deduced and shown in Eqs. (5) and (6). The parameters of the Burgers model are regressed and listed in Tab. 3, with the coefficient of correlations R^2 being equal to 0.986. Fig. 1 shows that the fitted curve matches well with the experimental data. It can be concluded that the LVE deformation prediction model can precisely simulate the recoverable deformation in a single load cycle.

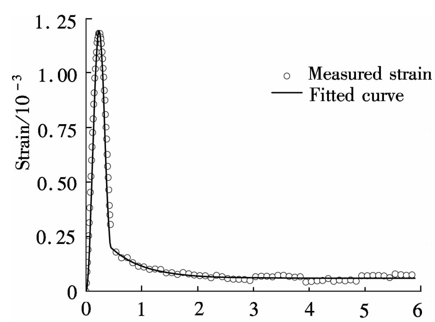


Fig. 1 Experimental data and the fitted curve in the first load cycle

Tab. 3 The parameters of Burgers model

Coefficients	E_1/MPa	E_2/MPa	η_1/(MPa·s)	η_2/(MPa·s)	R^2
Values	449.259	930.999	1 850.735	555.648	0.986

4 Modified Deformation Prediction Model

4.1 Defect of LVE deformation prediction model

Fig. 2 compares the experimental results and the prediction of the LVE deformation prediction model within the first 60 s (10 cycles) in the dynamic creep test. However, the prediction error becomes significant as the number of loading cycles increases.

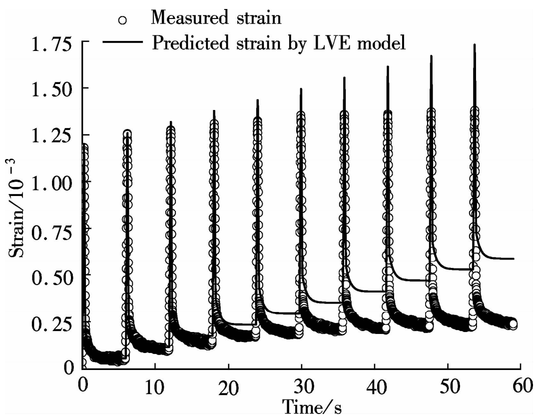


Fig. 2 Experimental data in the first 10 loading cycles and the predicted deformation by the LVE model

The increase of permanent deformation in every load cycle (from the $(n-1)$-th to the n-th load cycle) is calculated as

$$\Delta\varepsilon_{pd}(n)=\varepsilon_{pd}(n)-\varepsilon_{pd}(n-1)=\varepsilon(nT)-\varepsilon((n-1)T) \tag{8}$$

where $\varepsilon_{pd}(n)$ is the increase of permanent deformation from the $(n-1)$-th to the n-th load cycle; $\varepsilon_{pd}(n)$ is the permanent deformation at the end of the n-th load cycle where $t=nT$, so $\varepsilon_{pd}(n)=\varepsilon(nT)$.

Substituting Eq. (7) into Eq. (8), the following equation is obtained:

$$\Delta\varepsilon_{pd}(n)=\varepsilon_2(nT)=\frac{\pi\sigma_{\max}}{\omega\eta_1}+\frac{e^{-E_2nT/\eta_2}(-1+e^{t_0E_2/\eta_2})\omega^2\eta_2^2\sigma_{\max}}{2(E_2^3+\omega^2E_2\eta_2^2)} \tag{9}$$

The first term of Eq. (9) is a constant term and the second one is an exponential term. Clearly, the exponential term decreases to 0 as the number of loading cycles n increases, so the value of $\varepsilon_{pd}(n)$ will converge to the constant term. Tab. 4 compares the exponential term with the constant term. The exponential term becomes smaller as the number of cycles increases. Therefore, the constant term is the main factor contributing to the permanent deformation. The permanent deformation in the LVE deformation prediction model can be calculated as

$$\varepsilon_{pd}(n)=\sum_{i=1}^{n}\varepsilon_{pd}(n)=\sum_{i=1}^{n}\varepsilon_2(nT)=\frac{\pi\sigma_{\max}}{\omega\eta_1}n \tag{10}$$

From Eq. (10), it is clear that the predicted permanent deformation is a linear equation of the number of loading cycles n. Also, it is reasonable to conclude that, in the Burgers model, η_1 is the parameter which has the most significant influence on the prediction of permanent deformation.

Tab. 4 Comparison between the constant term and the exponential term

Cycle number	Constant term	Exponential term	Ratio
1	1.22×10^{-4}	2.84×10^{-8}	4.29×10^{3}
2	1.22×10^{-4}	1.35×10^{-12}	9.00×10^{7}
3	1.22×10^{-4}	6.43×10^{-17}	1.89×10^{12}
4	1.22×10^{-4}	3.06×10^{-21}	3.97×10^{16}
5	1.22×10^{-4}	1.46×10^{-25}	8.35×10^{20}

Note: Ratio represents the ratio of the value of constant term to the value of the exponential term.

4.2 Modification of the LVE deformation prediction model

As mentioned above, the constant term in the expression of $\varepsilon_2(t)$ causes the linear increase of the predicted permanent deformation. A modification of the constant term should be made to fit the nonlinear property of the permanent deformation of the asphalt mixtures.

The constant term in $\varepsilon_2(t)$ is modified by an exponential function,

$$\varepsilon_2^{\mathrm{mod}}(t)=\frac{\pi\sigma_{\max}}{\omega\eta_1}(C_1+(1-C_1)e^{C_2(2\pi-\omega t)})+\frac{e^{-E_2t/\eta_2}(-1+e^{2\pi E_2/(\omega\eta_2)})\omega^2\eta_2^2\sigma_{\max}}{2(E_2^3+\omega^2E_2\eta_2^2)} \tag{11}$$

where C_1 and C_2 are two additional coefficients.

Two other factors are considered in this modification. First, the continuity of the model (when $t=t_0$, $\varepsilon_2^{\mathrm{mod}}(t_0)$ should be equal to $\varepsilon_1(t_0)$) is ensured by this modification. It can be verified by substituting $t_0=\frac{2\pi}{\omega}$ into Eq. (5) and Eq. (11).

Secondly, this modification also guarantees the compatibility of $\varepsilon_2^{\mathrm{mod}}(t)$ with $\varepsilon_2(t)$. It can be verified, in the special case when $C_1=C_2=0$, $\varepsilon_2^{\mathrm{mod}}(t)$ degrades to $\varepsilon_2(t)$. Therefore, $\varepsilon_2^{\mathrm{mod}}(t)$ is a generalization of $\varepsilon_2(t)$, so $\varepsilon_2^{\mathrm{mod}}(t)$ is bound to have a better applicability than $\varepsilon_2(t)$.

Due to these reasons, the modified deformation prediction model can be acquired by replacing $\varepsilon_2(t)$ with $\varepsilon_2^{\mathrm{mod}}(t)$. The mathematical expression is

$$\varepsilon_{\mathrm{mod}}(t)=\begin{cases}\displaystyle\sum_{i=1}^{n}\varepsilon_2^{\mathrm{mod}}(t-(i-1)T)+\varepsilon_1(t-nT) \\ \qquad nT<t\leqslant nT+t_0 \\ \displaystyle\sum_{i=1}^{n+1}\varepsilon_2^{\mathrm{mod}}(t-(i-1)T) \\ \qquad nT+t_0<t\leqslant(n+1)T\end{cases} \tag{12}$$

As discussed in Section 4.1, when simulating permanent deformation, the exponential term in ε_2 is ignored, so the modified permanent deformation prediction model can be deduced and simplified.

$$\varepsilon_{pd}^{mod}(n) = \sum_{i=1}^{n} \varepsilon_2^{mod}(nT) = \sum_{i=1}^{n} \frac{\pi\sigma_{max}}{\omega\eta_1}(C_1 + (1 - C_1)e^{C_2(2\pi-\omega nT)}) = P_1 n + P_2(1 - e^{-P_3 n}) \tag{13}$$

where P_1, P_2, P_3 are three interim coefficients used to simplify the nonlinear regression.

$$P_1 = \frac{\pi\sigma_{max} C_1}{\omega\eta_1}$$

$$P_2 = \frac{(1 - C_1)e^{C_2(2\pi - \omega T)}}{1 - e^{\omega T C_2}}, \quad P_3 = C_2\omega T$$

5 Verification of the Modified Deformation Prediction Model

Using the experimental data in the first 10 loading cycles of the test, the additional coefficients of the modified model C_1, C_2 can be regressed based on the nonlinear equation in Eq. (12), where $C_1 = 2.5121 \times 10^{-3}$, $C_2 = 8.0102 \times 10^{-3}$, and $R^2 = 0.973$.

The modified prediction model and the experimental data in 100 loading cycles are compared in Fig. 3. The enlarged figure shows that even at the end of the test, the prediction matches well with the experimental data in each load cycle. Therefore, the modified model is believed to be capable of providing an accurate prediction for the whole process of the dynamic creep test.

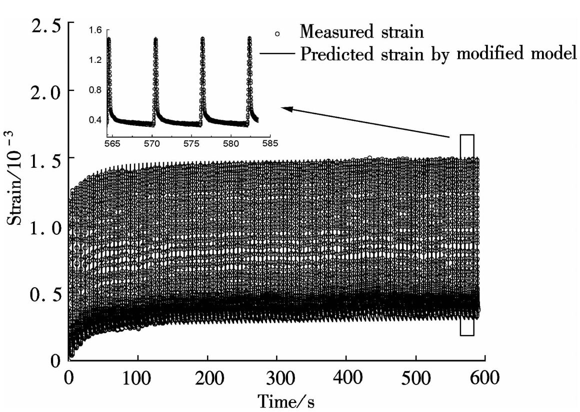


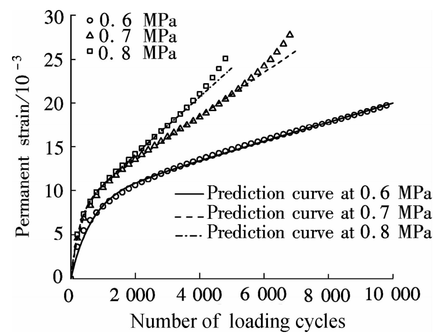
Fig. 3 Experimental data in the 100 loading cycles and the predicted deformation by the modified model

To verify the prediction of the permanent deformation, dynamic creep tests with three different stress levels of 0.6, 0.7, 0.8 MPa were conducted. The values of the regression coefficients in Eq. (13) are listed in Tab. 5.

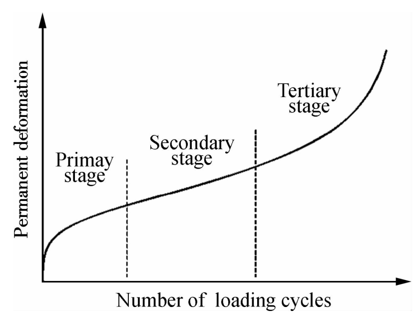
The prediction curve of the permanent deformation and the experimental data of the dynamic creep test are compared in Fig. 4. The permanent deformation curve can be divided into three stages: 1) Decelerating in the primary stage; 2) Increasing linearly in the secondary stage; 3) Accelerating in the tertiary stage [2]. It can be concluded that the modified model can provide an accurate prediction for the permanent deformation in the primary and secondary stage, but it cannot simulate the accelerating in the tertiary stage.

Tab. 5 Regression coefficients of the modified permanent deformation prediction model ε_{pd}^{mod}

Stress/MPa	Regression coefficients			R^2
	P_1	P_2	P_3	
0.6	1.12×10^{-6}	8.90×10^{-3}	1.74×10^{-3}	0.9977
0.7	2.75×10^{-6}	7.73×10^{-3}	3.5×10^{-3}	0.9963
0.8	3.59×10^{-6}	6.94×10^{-3}	4.87×10^{-3}	0.9976



(a)



(b)

Fig. 4 Permanent deformation at three different stress levels. (a) Permanent deformation at three different stress levels; (b) Three stages of the permanent deformation

6 Conclusions

1) The LVE deformation prediction model can precisely simulate the deformation in a single load cycle, but it cannot simulate the nonlinear property in permanent deformation.

2) The constant term of $\varepsilon_2(t)$ in the LVE deformation prediction model leads to the linear increase of the permanent deformation prediction, and it is this defect that needs to be modified in the LVE model.

3) In all the Burgers model parameters, η_1 has the most significant influence on the prediction of permanent deformation.

4) The modified deformation prediction model can accurately predict the whole-process deformation in dynamic creep tests.

5) The modified deformation prediction model can accurately predict the permanent deformation in the primary

and secondary stages, but it cannot simulate the permanent deformation in the tertiary stage, so further research needs to be conducted.

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沥青混合料线性黏弹性变形预估模型修正

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摘要:基于线性黏弹性理论(LVE)推导出沥青混料在动态蠕变实验中的变形预估模型. 然后,将线性黏弹性变形预估模型和实验结果对比,分析说明了线性黏弹性预估模型的不足. 最后,为了提高预估准确性,对线性黏弹性预估模型进行了修正,使其具有与沥青混合料变形特性相符的非线性特性,并用实验数据对修正模型进行了验证. 结果表明,线性黏弹性变形预估模型无法模拟沥青混合料的永久变形的非线性特性,而修正变形预估模型可以准确地预测动态蠕变实验中变形的全过程以及永久变形. 说明了所提出的修正方法可以有效地提高线性黏弹性变形预估模型的准确性,该修正模型可以为沥青路面的车辙预估提供指导.

关键词:变形预估;车辙;黏弹性力学;动态蠕变实验

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